POWER TRANSFER BETWEEN CORES IN WAVEGUIDE SYSTEMS WITH RANDOM IMPERFECTIONS

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Abstract: New equations describing the propagation of light in random waveguide systems with a short correlation length are theoretically derived based on the coupled mode theory and the transfer process of the average power between cores is discussed. The average power of light is transferred from core to core as a wave at a short propagation distance. The nature of a wave disappears with distance and the behaviour of the average power is diffusive. Then, the equations derived result in the well-known coupled power equations.

INTRODUCTION

An image fiber is composed of a large number of cores and a single cladding and is used to transmit directly an optical image. In such a fiber the crosstalk between cores takes place. The observed crosstalk can be described by the coupled power equations[1]. The equations include two coupling coefficients. One describes the power transfer between modes in a single core and the discussion on the mode coupling in a random waveguide[2,3,4] is applicable to the coefficient. The other describes the power transfer between neighboring cores. The dependence of the coefficient on the correlation length has been discussed based on the numerical solutions of the coupled mode equations[5]. When the correlation length is long, the average power of the mode behaves diffusively and can be described by the coupled power equations,

\[
\frac{dP_n}{dz} = d_c \sum_{m \neq n} (P_m - P_n)
\]  

where \( P_n \) is the average power of the mode and \( d_c \) is the power coupling coefficient. The equations mean that the power is only incoherently transferred from core to core. A value of the coefficient can be estimated from the distance dependence of the average power obtained numerically. However, the behaviour of the average power for a short correlation length is different from that for a long correlation length and cannot be described by the coupled power equations(1).

In this paper, new equations describing the propagation of light in random waveguide systems with a short correlation length are theoretically derived based on the coupled mode theory and the transfer process of the average power between cores is discussed.

POWER TRANSFER BETWEEN CORES

We assume a one-dimensional waveguide system composed of cores of equal spacing as a model of an image fiber. Sizes of the cores change randomly along the fiber axis. Each core supports only one mode and the coupling between modes in a neighbouring cores is taken into account. We have the following coupled mode equations,

\[
\frac{dc_n}{dz} = -j\beta_n c_n - j\kappa \sum_{m \neq n} c_m
\]

where \( z \) is the distance along the fiber axis. The summation is limited to the two cores next to the \( n \)-th core. \( \kappa \) is the mode coupling coefficient. \( c_n \) and \( \beta_n \) are the amplitude and the propagation constant of the mode in the \( n \)-th core, respectively. \( \beta_n \) is a random function with the average \( \langle \beta_n(z) \rangle = 0 \) and the variance \( \delta\beta^2 \) where the symbol\( \langle \rangle \) indicates the ensemble average. The correlation function for \( \beta_n \) has a Gaussian form.

The amplitude \( c_n \) can be divided into two parts,

\[
c_n = c_n^e + c_n^i
\]
where $c_n^c$ is the coherent part and $c_n^i$ is the incoherent part with the average $<c_n^i >= 0$. We can derive the following equation for the coherent part $c_n^c$ of the amplitude based on the perturbation solution of Eq.(2),

$$\frac{dc_n^c}{dz} = (-\frac{1}{2}\alpha - j\beta)c_n^c - j\kappa \sum_{m \neq n} c_m^c$$

(4)

The damping factor $\alpha$ is given by $\alpha = \sqrt{\pi}D\delta\beta^2$ where $D$ is the correlation length. The equation means that the propagation properties of the coherent part are identical with the propagation properties of light in an ordered waveguide system except for the exponential decay of the amplitude. We obtain from Eq.(4) the following equations for the coherent power $|c_n^c|^2$:

$$\frac{d|c_n^c|^2}{dz} = -\alpha |c_n^c|^2 - j\kappa \sum_{m \neq n} (c_m^c c_n^c^* - c_m^c c_n^c)$$

(5)

$$\frac{dc_m^c c_n^c^*}{dz} = -\alpha c_m^c c_n^c^* - j\kappa \sum_{l \neq m} c_l^c c_n^c^* + j\kappa \sum_{l \neq n} c_m^c c_l^c^*$$

(6)

c_m^c c_n^c^* is the interaction between the $m$-th core and the $n$-th core.

By subtracting Eq.(4) from Eq.(2) we obtain

$$\frac{dc_n^{ic}}{dz} = -j\beta c_n^{ic} - j\kappa \sum_{m \neq n} c_n^{ic} - j\delta\beta c_n^c + \frac{1}{2}\alpha c_n^c$$

(7)

where $\delta\beta = \beta_n - \beta$. Then, we can derive from Eq.(7) the following equations for the incoherent part of the average power $<|c_n^{ic}|^2>$:

$$\frac{d <|c_n^{ic}|^2>}{dz} = -j\kappa \sum_{m \neq n} \{ <c_m^{ic} c_n^{ic^*}> - <c_m^{ic^*} c_n^{ic}> \} + \alpha |c_n^c|^2$$

(8)

$$\frac{d <c_m^{ic} c_n^{ic^*}>}{dz} = -\alpha <c_m^{ic} c_n^{ic^*}> - j\kappa \sum_{l \neq m} <c_l^{ic} c_n^{ic^*}> + j\kappa \sum_{l \neq n} <c_m^{ic} c_l^{ic^*}>$$

(9)

For small $z$, the contribution of the damping term in Eq.(9) can be neglected. This means that the incoherent power is transfered as a wave from core to core and the behaviour is identical with that of the coherent part. Equation(5) for the coherent power includes the damping term. On the other hand, there is no damping term in Eq.(8). The lack of the damping term makes a large difference in its behaviour. For sufficiently large $z$, Eqs.(8) and (9) result in the well-known coupled power equations,

$$\frac{d <|c_n^{ic}|^2>}{dz} = d_c \sum_{m \neq n} \{ <|c_m^{ic}|^2> - <|c_n^{ic}|^2> \}$$

(10)

where $d_c$ is the power coupling coefficient which is given by $d_c = 2\kappa^2/\alpha$. Thus, the power transfer can be interpreted as a diffusion process.

**NUMERICAL EXAMPLES**

We assume in our model that a core diameter is $5\mu m$ on average, a spacing between cores is $8\mu m$ and the numerical aperture(NA) of the fiber is 0.24. Then each core can support six $LP$ modes at wavelengths in red. We treat only the coupling between the $LP_{12}$ modes[6]. The mode coupling coefficient $\kappa$ has a value of $8.68 \times 10^{-4}$ ($1/\mu m$) at a wavelength of 0.633($\mu m$).

The coherent part of the average power is shown in Fig.1. The number of the cores is 65 and the central core( the 33rd core ) is illuminated. The fluctuation of the propagation constants is $\delta\beta/\kappa = 4$ and the correlation length is $D = 10/\sqrt{\pi}$$($$\mu m$$). The coherent power is calculated from
the theoretical solution of Eq.(4). The coherent part is propagating mainly in a certain direction. The propagation angle $\vartheta_c$ to the $z$-axis is given by $\vartheta_c = \tan^{-1}2\kappa d$ where $d$ is the spacing between cores. The propagation angle is independent of the randomness of the waveguide system and is equal to that for an ordered waveguide system.

The incoherent part of the average power is shown in Fig.2. Equations (8) and (9) are numerically solved by the Runge-Kutta method. Then, the interaction terms of the higher order $\langle c_n^{ic} c_{n+3}^{ic*} \rangle, \langle c_n^{ic} c_{n-3}^{ic*} \rangle \cdots$ are neglected. The incoherent power spreads out over the entire system. The spreading angle is equal to the propagation angle $\vartheta_c$ of the coherent power. When the distance $z$ is large the incoherent power varies gradually from core to core in the cross-section and the nature of a wave disappears.

CONCLUSIONS

New equations describing the propagation of light in random waveguide systems with a short correlation length have been theoretically derived based on the coupled mode theory and the transfer process of the average power between cores has been discussed. The coherent part of the average power is transformed into the incoherent part with distance and decreases exponentially. Except for the exponential damping the propagation properties are identical with those of light in an ordered waveguide system. The incoherent power is transferred from core to core as a wave at a short distance. The nature of a wave disappears with distance and the behaviour is diffusive. Then, the equations derived result in the well-known coupled power equations.

REFERENCES