TOLERANCE STUDY FOR A RANGE LIMITED SENSOR UTILIZING THE EXTENDED NEAR-FIELD OF A CIRCULAR ARRAY

Walter K. Kahn

Department of Electrical and Computer Engineering, The George Washington University, Washington, DC 20052, U.S.A.

Abstract: Precise phasing of the signals received by each element in a circular array sensor is essential if the potential degree of range discrimination of which the sensor configuration is capable is to be realized. This discrimination is a function of the near field of a circular array of dipoles that can exhibit strong reactive attenuation within an extended radial distance from the antenna circle. The attenuation is utilized to produce a sensor array with sharply range-limited sensitivity characteristic. The performance of this arrangement, subject to perturbations of the nominally required phasing is investigated.

INTRODUCTION

A sensor that achieves sharp range discrimination utilizing the extended near-field of a circular array of antennas was described recently. [1]. Electromagnetic theory points to one general method to achieve the effect of a range discrimination characteristic substantially in excess of the discrimination due to the inverse square law. It is based on selective excitation of spherical modes comprising spherical Bessel (Hankel) functions of large orders that are comparable to or larger than the arguments. The current study examines the possibilities for utilizing this feature as a practical discriminant for preferential detection of radiation that originates at points located inside the “near field” of an array antenna specifically designed for this purpose. A threshold may be set so that only sources within a limited radius are detected. A simulation of this principle is presented uses a circular array of vertical receiving dipoles excited by a dipole transmitter whose position is varied. The phase relations nominally required to achieve the desired performance within an extended near-field region are then perturbed and the resulting degradation of discrimination computed.

ELECTROMAGNETIC BASIS

The field outside a sphere immediately enclosing any antenna in free space can always be regarded as a sum of spherical modes or complete field patterns each of which can, in principle, exist entirely independently [2,3]. As is well known, when applied to the wave equation (Helmholtz equation) in spherical coordinates the separation of variables leads to the differential equation for the so-called spherical Bessel functions for the radial coordinate

\[
\frac{d^2}{d(kr)^2} + \frac{2}{kr} \frac{d}{d(kr)} + \left(1 - \frac{n(n+1)}{(kr)^2}\right)z_n(kr) = 0, \quad \text{or} \quad \frac{d^2}{d(kr)^2} + \left(1 - \frac{n(n+1)}{(kr)^2}\right)(kr)z_n(kr) = 0.
\]

The amplitude of each mode, \(V_{nm}(r)\) below, has radial variation proportional to \(krz_n(kr)\) that depends on a mode index “n”. This second form of the differential equation is easily solved in the limits \((kr)^2 >> n(n+1)\) and \((kr)^2 << n(n+1)\). The first limiting value leads to

\[V_{nm}(r) \sim \exp\{\mp jkr\}\]

while the second leads to
\[ V_{m_n}(r) \sim j\left(\frac{n - 1}{kr}\right)^n r, n \gg kr, kr > kr_n; \quad V_{m_n}(r) \sim (\frac{kr}{n + \frac{1}{2}})^{n+1}, n \gg kr, kr < kr_n, \]

where \( kr_n \) is the radius of the circular array responsible for excitation of the mode. The vector electric and magnetic fields (tangential to the sphere) transverse to the radial coordinate \( E_t, H_t \) are given by [2,3]

\[
\begin{align*}
E_t(r) &= \sum_{m,n} V_{m_n}(r) e_{m_n}(\theta, \phi); \\
H_t(r) &= \sum_{m,n} I_{m_n}(r) h_{m_n}(\theta, \phi).
\end{align*}
\]

An additional index distinguishing the two possible polarizations has been suppressed as only vertical [TE] polarization is considered in the sequel. The index \( m \) with range \( -n < m < n \) is characteristic of the azimuthal (\( \phi \) coordinate) variation

\[
\begin{align*}
h_{m_n}(\theta, \phi) &= r^\theta \times e_{m_n} = -r \nabla_i \Psi_{m_n}; \\
\Psi_{m_n}(\theta, \phi) &= N_{m_n} P_n^m(\cos \theta) \exp\{jm\phi\};
\end{align*}
\]

where the \( P_n^m(\cos \theta) \) are associated Legendre Polynomials.

**THE CIRCULAR ARRAY OF DIPOLES**

A circular array of \( N \) elementary electric dipoles is employed as receiver for the selected high \( n \) index spherical mode complex. The basis for this selection is the angular (\( \phi \) dependence) of the spherical mode, i.e., the index \( m \) that is determined by the relative phasing of the dipoles. Since the azimuthal index \( m \) must satisfy \( -n \leq m \leq n \), and for large \( m \) can only subsist in conjunction with large \( n \). For a circular array of \( N \) elements considered as a transmitter, the value of the index \( m \) is forced (modulo \( N \)) by the index \( q \) associated with any one of a set of \( N \) orthogonal possible excitation vectors (for values of the perturbation parameter, \( \theta = 0 \)),

\[
a_v(q) = \frac{1}{\sqrt{N}} \exp(\frac{2\pi}{N} q\nu) \exp(j\nu), \quad 0 < q < N - 1,
\]

and where the index \( \nu, 1 \leq \nu \leq N \) denotes the various dipole ports. [4] Invoking the reciprocity principle, this index may be considered to label the input ports of a physical \( N \times N \) (Butler Matrix) feed network the outputs of which are each connected to one dipole of the circular receiving array. [5].

**RELATIVE TRANSMISSION FROM A DIPOLE ANTENNA EXTERNAL TO THE CIRCULAR ARRAY**

The power received by an array of vertical electric dipoles is studied with the objective of determining the relative strength or discrimination of the received signals emanating from the source when it is located in the near field region compared to the received signals when the source is located (immediately) outside the near field region. The general problem is simulated employing a circular array comprising \( N = 10 \) elementary vertical dipoles located on a circle of radius \( kr_a = 0.5 \). The response (for \( q = 6 \)) is shown in the following two figures. The transmitting vertical dipole traverses radial locations from \( 0 < kr < 10 \). The angular cuts are taken at \( \phi = 0.1 \) and \( \phi = \pi / 10 \) radians. The values of the perturbation parameter, \( \theta \), associated with each curve are as follows: \( \theta = 0.0001 \) solid----, \( 0.001 \) dots----, \( 0.01 \) dash-dot -.-.-., \( 0.1 \) dashed ----. In the simulations it is assumed that the transmitting dipole is conjugate matched to free space. When the transmitting antenna is sufficiently close to the array radius, near field coupling to the array results in transmitter detuning causing the receive power to decrease. This effect has been remarked in [6].
ACKNOWLEDGEMENT

The author should like to acknowledge helpful comments from Dr. W. Wasylkiwskyj in the preparation of this paper.

REFERENCES