ELECTROMAGNETIC SCATTERING OF A DIPOLE EXCITED WEDGE 
WITH ANISOTROPIC IMPEDANCE AND PEC FACES

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Abstract: Three-dimensional dyadic Green’s function is derived for the electromagnetic diffraction of a point electric current source radiating in the vicinity of a wedge with anisotropic impedance and PEC faces. The anisotropic impedance face is characterized by surface impedances along directions parallel and orthogonal to the edge axis. The final asymptotic result exhibits three different scattering mechanisms: edge-guided waves, surface waves, and guided waves in the classical sense. The resulting expressions can be used to study electromagnetic scattering from artificially hard and soft surfaces due to an electric dipole excitation.

INTRODUCTION
The electromagnetic fields due to a source near a PEC wedge have been studied in the past [1]-[3]. In all of these works, edge-guided wave behavior of the scattered field was observed. When one of the PEC faces of the wedge is characterized by an anisotropic impedance boundary condition (IBC), a different scattering phenomenon occurs due to electric dipole excitation. This scattering mechanism is studied in this work. Anisotropic IBC’s are utilized to represent corrugated metallic planes with dielectric filled corrugations [4]. Most of the analytical works for scattering from an impedance wedge consider plane wave excitation of the wedge and they are centered around Maliuzhinets theory of Sommerfeld integral representation of fields [5]. On the other hand, Felsen et al.[6] considered the spectral integral representation of Green’s function in all coordinates and the physics of the problem is better revealed under such formulation. However, the asymptotic and/or numerical evaluation of discrete (continuous) infinite spectral summations (integrals) presents a major challenge for the derivations. In this work, the asymptotic evaluations of scalar Green’s functions are performed in the paraxial region and the dyadic Green’s function is constructed there from.

PROBLEM STATEMENT
The face of the wedge at \( \phi = \phi_e \) is a perfect electric conductor (PEC) and the face at \( \phi = 0 \) has an anisotropic surface impedance as illustrated in Fig. 1. The point source is arbitrarily oriented and located at \( r'=(\rho', \phi', z') \) and the field is observed at \( r=(\rho, \phi, z) \). The particular case studied here is that the source and observation points are located in close vicinity of the edge, but are widely separated. The \( e^{int} \) time convention is employed. At the wedge surface \( \phi = \phi_e \), the electric field, \( \mathbf{E} \) satisfies

\[
\mathbf{n} \times \mathbf{E} = 0
\] (1)

where \( \mathbf{n} \) is the unit vector normal to the wedge surface. On the other face, at \( \phi = 0 \), the surface impedance boundary condition is specified as either

\[
E_z = -Z_\eta \eta_1 H_\rho \quad \text{and} \quad E_\rho = 0 \quad (\eta_1 = 0)
\] (2)

or

\[
E_\rho = Z_\eta \eta_1 H_z \quad \text{and} \quad E_z = 0 \quad (\eta_3 = 0)
\] (3)

where \( \eta_1 \) and \( \eta_3 \) represent surface impedances along \( \rho \) and \( z \) principal axis, respectively. The scattering problem that corresponds to boundary conditions specified in (1) and (2) is termed as Case A. Paraxial fields and scalar Green’s functions corresponding to Case A are discussed in the subsequent sections.

DERIVATION OF PARAXIAL FIELDS
Following Levine-Schwinger[7] scalarization procedure, the following relations hold between the transverse and longitudinal fields away from the sources,

\[
\left( \frac{\partial^2}{\partial z^2} + k^2 \right) \mathbf{H}_t = i \omega \mathbf{\hat{z}} \times \nabla \times \mathbf{E}_z + \nabla \times \frac{\partial \mathbf{H}_z}{\partial z}
\] (4)

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Once, the longitudinal components are solved the transverse components can be obtained using (4) and (5). The following scalar Green’s function problems must be solved for the scalar Helmholtz equations:

\[ (\nabla^2 + k^2) g_{E,u}^A (r,r') = -\delta (r-r') \]  \hspace{1cm} (6a)

subject to

\[ g_E^A = 0 \text{ at } \phi = \phi_0 \quad \text{and} \quad \frac{1}{\rho} \frac{\partial g_E^A}{\partial \phi} + \frac{ik}{\eta_3} g_E^A = 0 \text{ at } \phi = 0 \]  \hspace{1cm} (6b)

\[ \left. \frac{\partial g_E^A}{\partial \phi} \right|_{\phi=\phi_0} = \left. \frac{\partial g_E^A}{\partial \phi} \right|_{\phi=\phi_0} = 0 . \]  \hspace{1cm} (6c)

The transverse fields can also be described in terms of longitudinal part of the dyadic Green’s functions, \( g_{E,u}^A (r,r') \) and \( g_{H,u}^A (r,r') \), as follows

\[ \left( \frac{\partial^2}{\partial z^2} + k^2 \right) H^A_i (r) = \int dV' \left[ i\omega \varepsilon \hat{z} \times \nabla, \Gamma^A_i (r,r') + \nabla, \frac{\partial \Gamma^A_u (r,r')}{\partial z} \right] J(r') \]  \hspace{1cm} (7)

and

\[ \left( \frac{\partial^2}{\partial z^2} + k^2 \right) E^A_i (r) = \int dV' \left[ -i\omega \mu \hat{z} \times \nabla, \Gamma^A_i (r,r') + \nabla, \frac{\partial \Gamma^A_u (r,r')}{\partial z} \right] J(r') . \]  \hspace{1cm} (8)

The transverse dyadic in spectral domain can now be written as

\[ \tilde{G}^A_{E,u} = \frac{-i\omega \mu}{\gamma^2} \left( \hat{z} \times \nabla, \right) \tilde{G}^A_{H,u} = \frac{i k}{i\omega \varepsilon \gamma^2} \nabla, \left( \gamma^2 \hat{z} - i k \gamma V^l \right) \tilde{G}^A_E \]  \hspace{1cm} (9)

where \( \tilde{G}^A_E \) and \( \tilde{G}^A_H \) represent the spectral scalar Green’s functions of \( g^A_E \) and \( g^A_H \), respectively. The spectral representation of \( \tilde{G}^A_E \) can be asymptotically evaluated to determine the dyadic representations.

**ASYMPTOTIC FORM OF THE DYADIC GREEN’S FUNCTION**

Although not shown here, the series representations of scalar Green’s functions converge rapidly for either source or observation point is in close vicinity of the apex. Hence, only the first terms of the series are used in the asymptotic analysis. The transverse dyadics is derived as

\[ \tilde{G}^A_{E,u} = \frac{\omega \mu}{2 \phi_0} e^{i(k z - \omega t)} \]

\[ \left\{ (\rho \rho')^{-\frac{1}{2}} \sin \nu_l \phi + \cos \nu_l \phi \left( \frac{(-\tilde{A}_l + \tilde{B}_l)}{\sin \xi_l (\phi_o - \phi)} \right) \sin \nu_l \phi' \tilde{\rho} + \left(-\tilde{A}_l + \tilde{B}_l\right) \cos \nu_l \phi' \tilde{\rho} \right\} \]

\[ + \frac{1}{1 - \eta_3 \cos^2 \xi_l / \phi_0} \sin \xi_l (\phi_o - \phi) \tilde{\rho} - \cos \xi_l (\phi_o - \phi) \tilde{\rho} \]

\[ \left\{ \left(-\tilde{A}_l + \tilde{B}_l\right) \sin \xi_l (\phi_o - \phi') \tilde{\rho} + \left(-\tilde{A}_l + \tilde{B}_l\right) \cos \xi_l (\phi_o - \phi') \tilde{\rho} \right\} \]

\[ - \frac{i \kappa \gamma (\rho \rho')^{-\frac{k}{2}}}{1 - \eta_3 \cosh^2 \kappa \phi_o / \phi_0} \left\{ \sinh \kappa (\phi_o - \phi) \tilde{\rho} - \cosh \kappa (\phi_o - \phi) \tilde{\rho} \right\} \]

\[ \left\{ \left(-\tilde{A}_l + \tilde{B}_l\right) \sinh \kappa (\phi_o - \phi') \tilde{\rho} - i \left(-\tilde{A}_l + \tilde{B}_l\right) \cosh \kappa (\phi_o - \phi') \tilde{\rho} \right\} \]
where $v_i = \pi/\phi_i$, $\xi_i$ and $\kappa$ are the smallest real solutions to the following transcendental equations,

$$\tan (\xi_i \eta_i) = \frac{\xi_i}{[\eta_i]} \quad \text{and} \quad \tanh (\kappa \eta_i) = \frac{\kappa}{[\eta_i]}.$$  \hspace{1cm} (11)

The $\tilde{A}$'s and $\tilde{B}$'s represent the branch cut and branch point contributions in the asymptotic evaluation of the spectral integrals (the SDP shown in Fig. 2), respectively, and they are defined as follows

$$\tilde{A}_i = -\frac{k_v^{-1} e^{-i\kappa z^2}}{(z-z')^{2\kappa} \Gamma(v_i)} \quad ; \quad \tilde{B}_i = -\frac{k_v^{-1} e^{-i\kappa z^2}}{(z-z')^{\kappa} \Gamma(v_i)} \quad ; \quad \tilde{B}_x = -\frac{\xi_i}{k} \left( \frac{1}{\rho'} \right)^{2^{\xi_i}}$$ \hspace{1cm} (12)

and

$$\tilde{A}_x = \frac{k^{-1} e^{-i\kappa z^2}}{(z-z')^{2\kappa} \Gamma(1-i\kappa)} \quad ; \quad \tilde{B}_x = \frac{1}{k} \left( \frac{1}{\rho'} \right)^{-2i\kappa}$$ \hspace{1cm} (13)

where $\Gamma(.)$ is the gamma function. The asymptotic form (10) reduces to PEC wedge result of [1] if the surface impedances are made zero ($\eta_i \to 0$ or $\eta_i \to 0$).

**CONCLUSIONS**

The asymptotic fields of a point electric current source radiating in close proximity of a wedge with PEC and anisotropic impedance walls have been derived. The asymptotic result consists of three different wave phenomena: edge-guided waves, surface waves, and guided waves in the classical sense. The appearance of guided waves in present asymptotic result suggests that the anisotropic impedance wall introduces additional scattering phenomenon which is entirely different than a wedge with PEC faces.

**REFERENCES**


