RECONSTRUCTION OF 3-D OBJECTS BURIED UNDER A ROUGH SURFACE USING INVERSE SCATTERING METHOD

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Abstract: A three-dimensional (3-D) time-domain inverse scattering technique called forward-backward time-stepping (FBTS) method for reconstructing lossy media is presented. This method utilizes all wave fields including direct wave, reflections and diffractions. The reconstructions of a mine buried in soil with a rough surface from both synthetic noise-free and noisy data illustrates the feasibility of detecting landmines with the FBTS method.

INTRODUCTION

Ground penetrating radars (GPR) are used to detect targets buried in the ground [1-4]. There are two general ways to detect shallowly buried objects, one is image and signal processing [5-9] and the other is inverse scattering [10]. It is not easy to separate direct wave and reflections from targets if the buried objects are shallow, particularly when the background media are inhomogeneous and the surface is not flat. Therefore, an inverse scattering method, which can use total wave fields including direct couplings, reflections or diffractions, may be easy to use in detection of shallowly buried objects.

Forward-backward time-stepping (FBTS) method [10-11] can utilize the whole wave fields. We have discussed it for reconstructing 3-D objects in free space background [11] and in homogeneous half space with flat surface [10]. In this paper we apply this inversion technique to rebuild a 3-D mine shallowly buried in homogeneous half space with a 2-D rough surface.

THE FBTS METHOD

Maxwell’s equation is expressed as

\[ L v = j, \]

where \( v = (E, H, \eta H_x, \eta H_y, \eta H_z) \), \( j = (\eta j_x, \eta j_y, \eta j_z, 0, 0, 0) \), \( E_x, E_y, E_z \) and \( H_x, H_y, H_z \) are components of electric and magnetic fields, \( \eta \) is the impedance in free space, and the superscript \( t \) represents transpose. The differential operator \( L \) is given by

\[ L = A \frac{\partial}{\partial x} + B \frac{\partial}{\partial y} + C \frac{\partial}{\partial z} - F \frac{\partial}{\partial (ct)} - \mathcal{G}, \]

where \( A, B, C \) are 6x6 constant matrixes, \( c \) is the velocity of light in free space, \( \epsilon_r \) is relative permittivity, \( \mu_r \) is relative permeability, and \( \sigma \) is conductivity, \( \mathcal{T} \) is a 3x3 unit matrix.

Suppose that in a homogeneous background there is an inhomogeneous object. Ideal dipole transmitters in the -y direction at \( \mathbf{r}_m = (x_m, y_m, z_m) \) illuminate the object by a current pulse \( J(t) \) in the -y direction. For each illumination the -y component of time-domain electric field data are collected by ideal dipoles in the -y direction at \( \mathbf{r}_n = (x_n, y_n, z_n) \). The distributions of the electrical parameters \( p = (\epsilon, \mu, \eta \sigma) \) are reconstructed from the received data by minimizing the functional

\[ F(p) = \int \left( \sum_{m=1}^{M} \sum_{n=1}^{N} K_{mn}(t) \left| E_{yn}^m(\mathbf{p}; \mathbf{r}_n^m, t) - \tilde{E}_{yn}^m(\mathbf{r}_n^m, t) \right|^2 \right) d(ct), \]

where \( \tilde{E}_{ym}^m(\mathbf{r}_n^m, t) \) is the observed -y component of electric field at \( \mathbf{r}_n^m \) exited by the source \( \mathbf{j}_m \), \( E_{yn}^m(\mathbf{p}; \mathbf{r}_n^m, t) \) is the calculated -y component of electric field for guessed parameters \( \mathbf{p} \), \( K_{mn}(t) \) is a continuous non-negative weighting function with a value of 0 at \( t = T, T \) is the duration of observed data.

The Fréchet differential of the functional is

\[ F'(p)\delta p = 2 \int \mathcal{T} \sum_{m=1}^{M} \sum_{n=1}^{N} u_{yn}^m(\mathbf{p}; \mathbf{r}_n^m, t) \delta E_{yn}^m(\mathbf{p}; \mathbf{r}_n^m, t) d(ct), \]

where \( u_{yn}^m(\mathbf{p}; \mathbf{r}_n^m, t) \) is the sensitivity of the functional to \( p \).
where \( u_m(p; r', t) \) is the weighted residual, \( \delta E_m(p; r', t) \) is the \( y \)-component of \( \delta v_m(p; r', t) \) satisfying Eqs. (6) and (7) and 
\[
\lim_{\tau \to \infty} \delta v_m(p; r, t) = 0,
\]
\[
L(\delta v_m) = \delta F \frac{\partial v_m}{\partial (ct)} + \delta G v_m,
\]
where \( \delta F \) and \( \delta G \) are the variations of \( F \) and \( G \).

To express the gradient of \( F(p) \) with respect to \( p \) explicitly, an adjoint operator \( L^* \) is introduced. It is defined as
\[
L^* = \sum_j A_j B_j C_j D_j E_j F_j G_j x_j y_j z_j = \sum_j A_j B_j C_j D_j E_j F_j G_j x_j y_j z_j
\]
By using an adjoint field \( w_m(p; r, t) \) satisfying
\[
L w_m = i \mu_n(p; r', t) \delta (r - r')
\]
and considering Eqs. (6) and (9), we can express the Fréchet differential (5) as the form of
\[
F'(p) \delta p = \{ g_x, \delta e_x \} + \{ g_y, \delta e_y \} + \{ g_y, \delta \eta \}
\]
Finally, the gradient \( g_x, g_y, g_y, g_y \) of \( F(p) \) with respect to \( e_x, e_y, \mu, \eta, \) and \( \sigma \) are given explicitly, for example
\[
g_x = 2 \int_0^t \sum_{m=1}^N \sum_{i=1}^i \frac{w_m(p; r, t)}{\partial (ct)} \partial v_m(p; r, t) d(ct)
\]
where \( w'_m(p; r, t) = \sum_{m=1}^N w'_m(p; r, t) \), \( w'_m(p; r, t) \) is the \( i \)-th component of \( w_m(p; r, t) \).

**NUMERICAL EXAMPLE**

The configuration of transmitters and receivers is shown in Fig. 1. Transmitting and receiving antennas, which are assumed to be ideal dipoles orientated in the \( y \)-direction, are set on a plane 3.6 cm above the average plane of the rough ground surface. The number of transmitters is 16, and that of receivers is 64. The transmitter interval is 9.6 cm, the receiver interval is 4.8 cm in horizontal directions. Incident current pulse is expressed as
\[
J(t) = \frac{d^3}{dt^3} \exp \left[ -\alpha^2 (t - \tau)^2 \right]
\]
where \( \tau = \beta \Delta t, \alpha = 4/\tau, \beta = 145, \Delta t \) is time interval. The highest frequency of the incident pulse is 1.0 GHz. For both forward and backward calculations using FDTD, spatial and time steps are \( \Delta x = \Delta y = \Delta z = 1.2 \) cm and \( \Delta t = 22.65 \) ps, and the measurement duration is \( T = 550 \Delta t \).

The relative permittivity of the soil is 6 and a cylindrical mine of relative permittivity 4 is buried 3.6 cm beneath the average plane of the rough surface. The mine is 10 cm in height and 15 cm in diameter. The FDTD space is a cube with a side length of 42 cells, and
the reconstruction area is the region between the 10th-32nd, 10th-32nd, 8th-26th grid points in the x, y, and z-directions, respectively. The average surface is located at grid point 9 in the z-direction. The projection of the reconstruction region on the transmitter-receiver plane is shown in Fig. 1 as the area surrounded by the gray rectangular.

The initial guess of relative permittivity of the reconstruction region is that of the background. The shape of the rough surface is supposed known by laser measurement. Fig. 2(a) shows the actual distribution of relative permittivity of the reconstruction region. Fig. 2(b) is the reconstructed result using noise-free data after 20 iterations. Fig. 3(a) is the reconstructed result using noisy data with signal to noise ratio of 15 dB. Fig. 3(b) is the reconstructed result using noisy data with the same signal to noise ratio by applying three filters during the iterations. From Figs. 2 and 3 we learn that although the reconstructed size of the mine after 20 iterations is smaller than the true one, the existence of a mine-like object in the soil can be clearly confirmed.

CONCLUSIONS
The reconstruction of a 3-D mine buried in soil with rough surface has been succeeded using synthetic noise-free and noisy data in the time-domain. After 20 iterations, it can be confirmed whether there is something different from the soil.

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