ELECTROMAGNETIC MODELING OF COMPOSITE FINITE STRUCTURES
BY EMBEDDING

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Abstract: Recently, an embedding technique has been proposed to incorporate the presence of an environment in scattering problems. As an example of embedding, multiple scattering between adjacent objects is described in terms of scattering and reflection operators that account for the respective scattering from an object and its environment (the other object). The operators are constructed through repeated application of the equivalence principle to auxiliary problems involving single scatterers. The operators can subsequently be combined, providing the recipe to evaluate the scattering operator of large composite structures, consisting of building blocks that have been characterised electromagnetically at an earlier stage. The embedding concept is illustrated numerically.

INTRODUCTION
Embedding is a technique that has been introduced by Franchois et al.[1], in order to incorporate the presence of an environment in inverse scattering problems, involving sources and receivers on a circular observation contour between the scatterer and the environment. In Tijhuis et al.[2] the embedding concept has been generalised to allow for observation contours of general shape. Eventually, embedding may be used to evaluate the scattering operator of large composite structures, consisting of building blocks that have been characterised electromagnetically at an earlier stage. Below, we expand on the theory in Tijhuis et al.[2] and present a numerical result as a proof of principle. To illustrate the two-dimensional (2D) idea of connecting elementary building blocks, we consider two observation domains, \( D_1 \) and \( D_2 \), bounded by the respective contours \( C_1 \) and \( C_2 \), which contain scattering objects as depicted in Fig. 1. Free space surrounds the two scatterers. The complement of a domain \( D \) is denoted as \( \bar{D} \). We describe the Transverse Magnetic (TM) polarized case with \( E_z \) being the only non-vanishing component of the electric field.

![Figure 1: The two observation domains \( D_1 \) and \( D_2 \) containing the scattering objects under consideration.](image)

SCATTERING AND REFLECTION OPERATORS
As a first step towards determining the total scattered field, we introduce a scattering operator \( S(\ell, \ell_S) \) for each building block. For \( D_1 \), this operator allows us to reproduce the scattered field in \( D_1 \) in the absence of the scattering object in \( D_2 \), for an arbitrary field incident on \( D_1 \). Instead of the pertaining incident field, we employ an equivalent current distribution \( J_{\text{eq}}(\rho_S) \) on this contour that generates the prescribed incident field inside \( D_1 \). The scattering operator converts \( J_{\text{eq}}(\rho_S) \) into an equivalent current distribution \( J_{\text{eq}}(\rho_S) \) on the same contour, which in turn, reproduces the scattered field in \( D_1 \). Upon defining the field in the configuration depicted in Fig. 1 with \( D_2 \) empty, as the first auxiliary problem, the scattering operator follows from the electric-field integral equation (EFIE),

\[
\frac{s}{\mu} \int_{C_1} G(\rho, \rho') \frac{S(\ell', \ell_S) d\ell'}{d'} = -E_{\text{eq}}(\rho, \rho_S),
\]
for \( \boldsymbol{\rho} \in \mathcal{C}_1 \), where \( G(\boldsymbol{\rho}, \boldsymbol{\rho}') \) represents the 2D Green’s function. The associated scattered field, \( E^s_z(\boldsymbol{\rho}, \boldsymbol{\rho}_S) \), due to a line source on the contour, can be determined via a domain integral equation as described in Peng et al [4], or alternatively, using a boundary integral equation. Since this is a linear problem, we may introduce an inverse operator \( Q(\ell, \ell') \) such that

\[
S(\ell, \ell_S) = -\int_{\mathcal{C}_1} Q(\ell, \ell') E^s_z(\boldsymbol{\rho}', \boldsymbol{\rho}_S) d\ell',
\]  

which converts a field distribution on \( \mathcal{C}_1 \) into an equivalent current distribution along \( \mathcal{C}_1 \) that generates the field \( E^s_z(\boldsymbol{\rho}, \boldsymbol{\rho}_S) \) for \( \boldsymbol{\rho} \in \mathcal{D}_1 \). To define a reflection operator \( R(\ell, \ell_S) \), we consider a second auxiliary problem by removing the object from \( \mathcal{D}_1 \), instead of \( \mathcal{D}_2 \). The reflection operator corresponds to the equivalent surface current density on \( \mathcal{C}_1 \) in homogeneous space that accounts for the field inside \( \mathcal{D}_1 \) scattered by objects in the complement \( \mathcal{D}_2 \), instead of \( \mathcal{D}_1 \). To obtain this operator, we propagate the field due to a line source at \( \boldsymbol{\rho}_S \in \mathcal{C}_1 \) to all \( \boldsymbol{\rho} \in \mathcal{C}_2 \), using a propagator \( P_{12}(\boldsymbol{\rho}, \boldsymbol{\rho}') \), determine the equivalent source density on \( \mathcal{C}_2 \) that would generate the same field incident on the scatterers in \( \mathcal{D}_2 \), employ \( S(\ell, \ell_S) \) for \( \mathcal{D}_2 \), propagate the resulting current distribution back to \( \mathcal{D}_1 \), and finally obtain the equivalent current distribution that accounts for the field scattered by the environment of \( \mathcal{D}_1 \). After discretisation, this process is expressed in terms of the matrix-counterparts of the operators defined above according to,

\[
R = QP^{T}_{12}SQP_{12},
\]

where the superscript \( T \) denotes the transpose of a matrix.

**EMBEDDING**

Although we have only discussed the case where the field in the complete configuration is generated by a line source located at \( \boldsymbol{\rho}_S \in \mathcal{C}_1 \), the procedure described below can be generalized immediately to fields generated by sources in \( \mathcal{D}_1 \) or \( \mathcal{D}_2 \). We now express the actual total field in terms of the total fields in the respective first and second auxiliary problems, \( E_z^{ob}(\boldsymbol{\rho}, \boldsymbol{\rho}_S) \) and \( E_z^{ev}(\boldsymbol{\rho}, \boldsymbol{\rho}_S) \),

\[
E_z(\boldsymbol{\rho}, \boldsymbol{\rho}_S) = \begin{cases} 
E_z^{ob}(\boldsymbol{\rho}, \boldsymbol{\rho}_S) + \int_{\mathcal{C}_1} v(\ell', \ell_S) E_z^{ob}(\boldsymbol{\rho}', \boldsymbol{\rho}) d\ell', & \boldsymbol{\rho} \in \mathcal{D}_1, \\
E_z^{ev}(\boldsymbol{\rho}, \boldsymbol{\rho}_S) + \int_{\mathcal{C}_1} w(\ell', \ell_S) E_z^{ev}(\boldsymbol{\rho}', \boldsymbol{\rho}) d\ell', & \boldsymbol{\rho} \in \mathcal{D}_2.
\end{cases}
\]

In both cases, the field is written as the sum of the fields due to the original line source and a continuous superposition of line sources in the pertaining auxiliary problem. Now, we can use the scattering and reflection operators introduced above to relate the distributions \( v(\ell, \ell') \) and \( w(\ell, \ell') \). In region \( \mathcal{D}_1 \), \( v(\ell, \ell_S) \) represents the additional total field due to reflections in \( \mathcal{D}_1 \). This field is generated by the original line source located at \( \boldsymbol{\rho} = \boldsymbol{\rho}_S \) and a continuous superposition of line sources with distribution \( w(\ell, \ell_S) \) that radiate into \( \mathcal{D}_1 \). Applying the reflection operator results in

\[
v(\ell, \ell_S) = R(\ell, \ell_S) + \int_{\mathcal{C}_1} R(\ell, \ell') w(\ell', \ell_S) d\ell'.
\]

Similarly, in region \( \mathcal{D}_1 \), \( w(\ell, \ell_S) \) generates the field scattered in \( \mathcal{D}_1 \), which irradiates the embedding. In conjunction with the scattering operator, this leads to

\[
w(\ell, \ell_S) = S(\ell, \ell_S) + \int_{\mathcal{C}_1} S(\ell, \ell') v(\ell', \ell_S) d\ell'.
\]

By substituting \( w(\ell, \ell_S) \) from (6) into (5) we obtain,

\[
v(\ell, \ell_S) - \int_{\mathcal{C}_1} R(\ell, \ell') \int_{\mathcal{C}_1} S(\ell', \ell''') v(\ell'', \ell_S) d\ell'' d\ell' = R(\ell, \ell_S) + \int_{\mathcal{C}_1} R(\ell, \ell') S(\ell', \ell_S) d\ell'.
\]
This equation is of the second kind, and the integrals can be evaluated to second order by the midpoint rule, because of the periodicity in $\ell$. The first term on the right-hand side of (7) represents the field directly reflected from $\mathcal{D}_1$, while the second term represents the field scattered from $\mathcal{D}_1$ and subsequently reflected from $\mathcal{D}_1$. The operator on the left-hand side of (7) generates the multiples.

**NUMERICAL EXAMPLE**

As a proof of principle, let us consider two dielectric homogeneous circular cylinders in free space separated by a distance $a$. The relative permittivity in the cylinders is $\varepsilon_r = 11.4$ and their radius $r = 0.45a$. The frequency is normalized such that $fa/c = 0.45$. For reference, we have calculated the fields using a boundary integral equation on the cylinder boundaries (cf. Fig. 2a) with 1000 equivalent electric and magnetic surface currents on each cylinder boundary. For an honest comparison with the theory presented above, we have employed analytical solutions for the fields scattered by a single cylinder, from which the scattering operator is determined. The associated observation domains $\mathcal{D}_1$ and $\mathcal{D}_2$ are hexagonals. The field is excited by a unit line source located on one of the observation contours. The resulting total electric field strength inside both observation domains is shown in Fig. 2b, computed with a mere 42 unknowns on each hexagonal contour.

![Figure 2a: Total electric field of the scattering from two cylinders (highlighted) using a boundary integral equation with 1000 unknowns for both electric and magnetic currents on each cylinder.](image)

![Figure 2b: Total electric field inside both hexagonal observation domains using the embedding approach. The line source is located on the contour of the left hexagonal.](image)

Nearly no visual deviation can be observed between both field plots. At the horizontal cross-section the average relative deviation is 0.18% with a maximum deviation of 1.05% at the transition between the hexagonal observation domains.

**CONCLUSIONS**

The embedding technique can be adapted for problems involving multiple scattering. A modest number of equivalent currents suffices to take the effect of multiple scattering between two high-contrast dielectric cylinders into account. It is possible to apply the theory presented above to handle large finite structures consisting of simple and reusable building blocks.

**ACKNOWLEDGEMENTS**

The research presented above has been financially supported by ESA/ESTEC, under Contract No. 14897/00/NL/LvH.

**REFERENCES**


