FAST MULTILAYER RADOME COMPUTATION WITH GAUSSIAN BEAMS

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Abstract: We propose a technique to compute the effect of a dielectric multilayer radome. This technique uses a gaussian beam expansion associated with an evaluation of the physical optics spectral integrals using the asymptotic steepest descent path method.

INTRODUCTION

A conventional method used to compute the transmitted and reflected fields by a radome combines plane wave spectrum (PWS), Physical Optics (PO) and Kottler integrals. This technique is however limited by its computation time and by the local planar multilayer approximation. We propose a new approach where the incident field and its interaction with the radome use a gaussian beam formalism. To reduce computation time, two new improvements are developed in this communication: the asymptotic steepest descent path method applied to thin dielectric multilayers and a fast process to take into account multiple reflections inside the radome. Finally, we present the radiation computation of a multilayer spherical radome. The results are compared to those obtained by the PWS-PO method.

GAUSSIAN BEAMS EXPANSION

Gaussian beams. Gaussian beams are approximate analytical solution to Maxwell’s equations. They depend on the paraxial approximation which requires low diverging beams. Generally, the maximum divergence angle $\theta$ from the propagation axis is assumed to be around 20°. The vectorial analytical formulation of a beam propagating along the z direction with an E field polarised along Ox is given by:

\[
\begin{align*}
\hat{E}(x,y,z) &= u(x,y,z)\hat{e}_z - j\frac{k}{k} \frac{\partial u(x,y,z)}{\partial x} \hat{e}_z \\
\hat{H}(x,y,z) &= \frac{1}{Z_0} u(x,y,z) \hat{e}_y - j\frac{k}{k} \frac{\partial u(x,y,z)}{\partial y} \hat{e}_y
\end{align*}
\]

with $u(x,y,z) = \frac{1}{\sqrt{4\pi k}} \exp \left(-\frac{j k z}{2} \left[x^2 + y^2\right] Q(z)\right)$

$k$ is the wave number. $Z_0$ is the free space impedance. $Q(z)$ is the complex curvature matrix of the beam:

\[
Q^{-1}(z) = Q^{-1}(0) + Z_0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

For a circular symmetric beam with a $W_0$ waist:

\[
Q^{-1}(0) = \begin{bmatrix} \frac{j k W_0^2}{2} & 0 \\ 0 & \frac{j k W_0^2}{2} \end{bmatrix}
\]

The amplitude evolution of a symmetric circular beam along propagation is shown on figure 1.

Expansion. A novel approach has been developed to compute field radiation with an expansion through fundamental Gaussian beams on a conformal surface (GBE) [1,2]. The characteristic parameters of each elementary beam are defined in order to optimise the expansion efficiency. Like in the Gabor decomposition [3], the electrical field $\hat{E}$ and the magnetic field $\hat{H}$ on the aperture are expressed as a sum of gaussian beams shifted in both position and direction (figure 2).

\[
\begin{align*}
\hat{E} &= \sum_n \left( a_{n0} \hat{E}_{n0} + a_{n1} \hat{E}_{n1} \right) \\
\hat{H} &= \sum_n \left( a_{n0} \hat{H}_{n0} + a_{n1} \hat{H}_{n1} \right)
\end{align*}
\]

$(\hat{E}_{n0}, \hat{H}_{n0})$ and $(\hat{E}_{n1}, \hat{H}_{n1})$ are the co- and cross-polarisation components, defined with respect to the main polarisation of the field on the aperture. Then, the characteristic parameters of each elementary beam have to be determined. The beam centres $P_n$ are uniformly distributed on the aperture, on a grid of step $d$ (figure 2). Only one beam (one direction) is selected on each point $P_n$. In order to have a proper description of the local properties of the electromagnetic fields, each elementary beam is oriented along the local Poynting vector $\hat{E} \times \hat{H}$. The distance $d$ depends on the waist $W_0$. An efficient distribution of the beams over the aperture is obtained for $d=0,9W_0$. Hence, $W_0$ is characteristic of both the speed and the accuracy of our method. $W_0$ is also limited by the paraxial approximation and must satisfy the condition $W_0 > 0,7\lambda$. Finally, the decomposition coefficients of each beam in co- and cross-polarisation are determined with a point-matching technique, combined if necessary with a least square optimisation.
TANGENTIAL FIELD COMPUTATION

**Dielectric interface** The reflected and transmitted fields on a dielectric interface $S$ illuminated by a gaussian beam can be computed using the plane wave spectrum expansion associated with an asymptotic steepest descent path (SDP) integration method. Using the Fresnel dyadic coefficients $(R, T)$ and the physical optics approximation, the reflected and transmitted fields on the surface $S$ stand as:

$$
\tilde{E}_r(l) = \frac{1}{4\pi^2} \iint R(k_x, k_y, l) \tilde{E}(k_x, k_y) e^{-\jmath k_y d} dk_x dk_y
$$

$$
\tilde{E}_t(l) = \frac{1}{4\pi^2} \iint T(k_x, k_y, l) \tilde{E}(k_x, k_y) e^{-\jmath k_y d} dk_x dk_y
$$

Equation (4)

$k_i$ is the incident plane wave vector. Thanks to the gaussian form of the incident plane wave spectrum $\tilde{E}$, these integrals can analytically be evaluated by the SDP method [2].

$$
\tilde{E}_r(l) = \frac{1}{4\pi^2} R(k_x, k_y, l) \tilde{E}_i(l)
$$

$$
\tilde{E}_t(l) = \frac{1}{4\pi^2} T(k_x, k_y, l) \tilde{E}_i(l) \quad \text{with} \quad \left[ \begin{array}{c} k_x \\ k_y \\ \end{array} \right] = k_i \Omega(x, y)
$$

Equation (5)

The analytical results (5) are limited by paraxial approximation on the incident beam and by its incidence which must be lower than the total reflection angle.

**Extension to thin multilayer dielectric** For dielectric multilayer with parallel interfaces, the reflected and transmitted fields respectively on internal and external surfaces can still be computed with (4). $R$ and $T$ are then global reflection and transmission coefficients of the multilayer. So, the analytic result (5) can still be used. However, to evaluate correctly the propagation in the layers, the SDP method needs slowly varying $R$ and $T$ coefficients which imposes very thin layers. On figure 3, we show the average quadratic error on the external interface between the analytical solution (5) and the numerical computation of (4). In all cases, the error rapidly increases with layer thickness but is always negligible for layer thickness under $\lambda_o/3$.

RADOME COMPUTATION

**Iterative process** The two previous methods can be combined for radome computation. The incident field is first expanded on gaussian beams. The transmitted and reflected fields on the first interface (or thin multilayer) are then computed via (5) and expanded on gaussian beams with our method. The same process is applied when transmitted or reflected field impacts a new interface (or thin multilayer). This process is repeated until field powers are under a fixed threshold (figure 4).

**Improved process** The iterative process is time consuming for radomes with many layers since the number of expansions rapidly increases with the number of interfaces. That’s why we develop an improved process which is illustrated on figure 5. The incident field is first expanded on gaussian beams. Then, on the first interface, the transmitted and reflected fields are still computed with (5) but only the transmitted field is expanded. The reflected field is stored at level $\{I\}$. On the following interfaces, the same procedure is applied. On the last one, the procedure is reversed, the transmitted field is stored at $\{N\}$ whereas the reflected field is expanded. The process then crosses the radome in the other direction. On the last but one interface, the transmitted field is added to $\{N-1\}$ and expanded. After that, the reflected field is stored at $\{N-1\}$. This process is repeated until field powers are under a fixed threshold. Finally, the sum of $\{I\}$ and the sum of $\{N\}$ respectively contain the fields on the first and last interfaces. These fields are expanded on gaussian beams in order to obtain the fields reflected and transmitted by the radome. This process allows an important decrease of computation time which now linearly depends on the number of layers.
APPLICATION

As an application, we present the radiation computation of a cosine circular aperture placed behind a 4-layer spherical radome (figure 6). The first and last layers are assumed to be thin enough to use global coefficients. The radiation patterns are represented in the antenna reference, centered on the boresight direction. The comparison of the radiation patterns obtained from the gaussian beams (GB) approach and the conventional PWS-PO method shows a very good agreement up to –50dB, for both co- and cross-polarisation (figures 7 and 8), as well as for the evaluation of the boresight error (0.35°). The ratio between both computation times is here better than 20.

CONCLUSION

The use of gaussian beam expansion associated with an asymptotic steepest descent path evaluation of the physical optics spectral integrals seems to be an efficient solution to treat radomes as well as concerning accuracy and computation time. The global transmission and reflection coefficients for thin layers and the improved process to take into account multiple reflections allow to treat rapidly multilayer radomes. We are now looking for solutions to improve the efficiency of our method for grazing incidence and high curvature interfaces.

REFERENCES