Bi-anisotropic Metamaterials Effective Constitutive Parameters Extraction Using Oblique Incidence S-Parameters Method

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***Abstract*—The S-parameters method for bi-anisotropic metamaterials effective constitutive parameters extraction is extended to oblique incidence. The proposed method enables to extract all unknown parameters using S-parameters measured over a single metamaterial slab. The bi-anisotropic metamaterial is a pseudochiral omega medium and assumed to have diagonal permittivity and permeability tensors. The extraction process suggested involves both analytical extraction equations and numerical optimization. This method is utilized to show the limited validity of the assumption of absence of spatial dispersion on which the proposed approach is based. The extraction method is demonstrated over bulk and artificial media such as SRR metamaterial and the results are validated with satisfactory agreement published data in the literature.**

***Index Terms*—Artificial, bi-anisotropic, constitutive parameters extraction, metamaterials, oblique incidence, SRR.**

I. Introduction

THE EXTRACTION of metamaterials effective constitutive parameters has been a subject to extensive study in the last two decades. Many extraction methods have been reported throughout the years and perhaps one of the most common methods is the S-parameters extraction method, which has been investigated and reviewed thoroughly [1]–[16]. Other common extraction methods such as field homogenization [17]–[20] and Floquet modal decomposition [21], [22] utilize numerical electromagnetic simulation tools in order to obtain the microscopic fields within the metamaterial unit cell.

The S-parameters method for extraction of metamaterial's effective constitutive parameters was first introduced in [1]–[3] and focused on the retrieval of only one of the metamaterial's effective constitutive parameters, using S-parameters measured for a plane wave normal incidence upon a metamaterial slab. The S-parameters are used to extract the metamaterial's effective constitutive parameters by solving the inverse problem derived from known transmission line equations. This method has been later extended [4], [5], [9] to enable the extraction of all effective parameters of bi-anisotropic structures using multiple normal incidence measurements on six different structures. The SRR-based metamaterials, which are discussed in this paper, exhibit pseudochiral omega medium properties [23] as shown in [24]. For such metamaterials, the normal incidence S-parameters extraction method requires multiple measurements along all its principal axes, in order to extract all seven unknown parameters of its tensors $\bar{\bar{\varepsilon}}$, $\bar{\bar{\mu}}$, $\bar{\bar{\xi}}$, and $\bar{\bar{\zeta}}$. This procedure requires measurements on a total of six different metamaterial structures in order to extract all the unknown parameters explicitly. Constitutive parameters extraction methods utilizing oblique incidence S-parameters measurements have been published as well, however, all differ from the method presented in this paper. An approach focusing on the extraction of only one parameter characterizing a fishnet metamaterial structure is presented in [8]. In addition, a recent paper uses the oblique incidence S-parameters method to extract the constitutive parameters of a bulk homogeneous biaxial bi-anisotropic metamaterial with chiral properties [14] utilizing the state equations representation of Maxwell's equations. There, the extraction process is based on the numerical evaluation of the inverse of the transfer matrix and is demonstrated only on an analytical bulk homogeneous model. Another recent paper [15] suggests an extraction method based on measurements of two metamaterial slabs with different thicknesses. However, the authors state their method cannot be used with artificial metamaterials made of discrete unit cells such as the SRR metamaterial. The S-parameters method extraction equations suffer from ambiguity and require specific attention. One common method used to resolve the ambiguity is simply to add measurements over a slab with a different thickness, in order to compare the extracted parameters from both measurements and gain an explicit solution. Other methods dealing with this pitfall have been published in [10], [11]. Other extraction methods, which are based on numerical optimization approach [7], [16], offer to extract all unknown parameters of a more general bi-anisotropic media using different optimization algorithms.

In this paper, the S-parameters extraction method is extended to oblique incidences for bi-anisotropic metamaterials with pseudochiral omega medium properties. The proposed method enables to extract six out of seven unknown parameters explicitly using S-parameters measured over a single metamaterial slab for two orthogonal polarizations (TE and TM) at two oblique and one normal incidences. The unknown permittivity and permeability diagonal tensors are extracted analytically and the chirality parameter is evaluated using

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numerical optimization. This is in contrast to normal incidence S-parameters method for bi-anisotropic metamaterials, which requires measurements of two slabs with different thicknesses for each one of the metamaterial's unit cell principal axes. Furthermore, in contrast to oblique incidence S-parameters methods which use a numerical optimization approach [7], [16] for the extraction of all unknown parameters, in the proposed method, six out of seven unknown parameters are extracted analytically. In order to extract all unknown parameters, four complex value measurements of the reflection S_{11} and four complex value measurements of the transmission S_{21}, at one oblique and one normal incidences for two orthogonal polarizations (TE and TM), are used to obtain eight sets of equations in order to analytically extract six unknown parameters and two branch indices to obtain an explicit solution of the bi-anisotropic complex parameters. The remaining unknown chirality parameter is obtained from an additional oblique incidence measurement at both TE and TM polarizations using a numerical optimization procedure. In addition, the proposed method is utilized to show the limited validity of the assumption of absence of spatial dispersion on which the proposed approach is based. The validity limitations of nonlocal effective parameters have largely been discussed in the literature [25]–[27]. The extraction method is validated over a bulk medium and demonstrated over an edge-coupled SRR metamaterial.

In Section II-A, the scattering of obliquely incident electromagnetic plane waves from a bi-anisotropic slab is studied, and analytical expressions for the reflected and transmitted coefficients are derived. In Section II-B, the oblique incidence S-parameters extraction method is derived. In Section III, a numerical validation of the extraction process is given for a bulk bi-anisotropic medium and an edge-coupled SRR metamaterial.

II. Theory

A. Scattering of Obliquely Incident Electromagnetic Plane Waves From a Bi-Anisotropic Slab

The problem considered is depicted in Fig. 1, wherein a monochromatic plane wave propagating from the left-hand side is obliquely incident upon a bi-anisotropic slab of thickness d. The anisotropic slab is infinite in the x- and y-directions and it is placed in vacuum. The incident electric field is $\mathbf{E}_i$, the angle of incidence is θ_1, and the reflected and transmitted electric fields are denoted by $\mathbf{E}_r$ and $\mathbf{E}_t$, respectively. The bi-anisotropic constitutive relations considered are

$$\begin{aligned}\mathbf{D} &= \bar{\bar{\varepsilon}} \cdot \mathbf{E} + \bar{\bar{\xi}} \cdot \mathbf{H}\\ \mathbf{B} &= \bar{\bar{\mu}} \cdot \mathbf{H} + \bar{\bar{\zeta}} \cdot \mathbf{E}\end{aligned} \tag{1}$$

where

$$\begin{aligned}\bar{\bar{\varepsilon}} &= \varepsilon_0 \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}, \quad \bar{\bar{\mu}} = \mu_0 \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix}\\ \bar{\bar{\xi}} &= \frac{1}{c_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -j\xi_0 & 0 \end{pmatrix}, \quad \bar{\bar{\zeta}} = \frac{1}{c_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & j\xi_0 \\ 0 & 0 & 0 \end{pmatrix}\end{aligned} \tag{2}$$

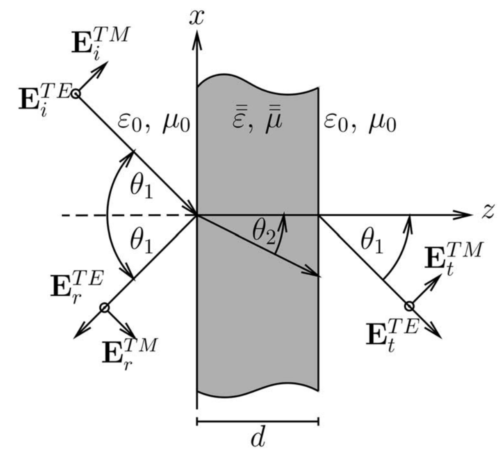

Fig. 1. Geometry of a bi-anisotropic slab, illuminated by an oblique incident electromagnetic field.

where ε_0 and μ_0 are the permittivity and permeability of free space and $c_0 = 1/\sqrt{\varepsilon_0\mu_0}$ is the speed of light in free space. The time convention used throughout this paper is $\exp(j\omega t)$ and the material constitutive parameters are defined as $\varepsilon = \varepsilon' - j\varepsilon''$, $\mu = \mu' - j\mu''$. These constitutive relations can be used to approximately describe the effective constitutive parameters of the edge-coupled SRR-based metamaterial medium [24], [28] under quasi-static analysis.

The S-parameters for the problem at hand are derived analytically using the state equation method [29]–[31], which has been extended to stratified bi-anisotropic media in [32]. Following the notation in [30], consider the incident plane wave

$$\begin{aligned}\mathbf{E} &= \mathbf{E}_0 \exp(-j\mathbf{k} \cdot \mathbf{r})\\ \mathbf{H} &= \frac{1}{\eta_0} \hat{k} \times \mathbf{E}\end{aligned} \tag{3}$$

where $\mathbf{k} = (k_x, k_y, k_z)$ is the wave vector, $\hat{k} = \mathbf{k}/|\mathbf{k}|$ and η_0 is the wave impedance of free space, the state equation defining the wave propagation within the bi-anisotropic medium is [32]

$$\frac{d}{dz} \begin{bmatrix} \mathbf{E}_s \\ \mathbf{H}_s \end{bmatrix} = \bar{\bar{\Gamma}} \begin{bmatrix} \mathbf{E}_s \\ \mathbf{H}_s \end{bmatrix} \tag{4}$$

where $\mathbf{E}_s = (E_x, E_y)$, $\mathbf{H}_s = (H_x, H_y)$ are the the transverse electromagnetic components and $\bar{\bar{\Gamma}}_{ij}$ are given in Appendix A. A matrix exponential is used to solve the state equation and obtain the state vectors at the slab's boundaries $z = 0,\ d$,

$$\bar{\psi}(0) = \bar{\bar{A}}\bar{\psi}(d) \tag{5}$$

where $\bar{\bar{A}} = \exp(-\bar{\bar{\Gamma}}d)$ and $\bar{\psi} = [\mathbf{E}_s, \mathbf{H}_s]$ is a column vector of the transverse electromagnetic components, which in free space satisfies $\mathbf{E}_s = \bar{\bar{Z}} \cdot \mathbf{H}_s$, where

$$\bar{\bar{Z}} = \begin{pmatrix} 0 & \eta_0 \cos\theta_1 \\ -\eta_0/\cos\theta_1 & 0 \end{pmatrix}. \tag{6}$$

The 4×4 matrix $\bar{\bar{A}}$ can be rewritten as a 2×2 block matrix,

$$\bar{\bar{A}} = \begin{pmatrix} \bar{\bar{Q}}_{11} & \bar{\bar{Q}}_{12} \\ \bar{\bar{Q}}_{21} & \bar{\bar{Q}}_{22} \end{pmatrix} \tag{7}$$

which is used to derive the reflection $\bar{\bar{R}}$ and transmittance $\bar{\bar{T}}$ matrices

$$\begin{aligned}\mathbf{E}_{rs}(0) &= \bar{\bar{R}} \cdot \mathbf{E}_{is}(0)\\ \mathbf{E}_{ts}(d) &= \bar{\bar{T}} \cdot \mathbf{E}_{is}(0)\end{aligned} \tag{8}$$

where

$$\begin{aligned}\bar{\bar{R}} &= \left(\bar{\bar{Z}}_a - \bar{\bar{Z}}_b\right)\left(\bar{\bar{Z}}_a + \bar{\bar{Z}}_b\right)^{-1}\\ \bar{\bar{T}} &= 2\bar{\bar{Z}}\left(\bar{\bar{Z}}_a + \bar{\bar{Z}}_b\right)^{-1}\end{aligned} \tag{9}$$

and

$$\begin{aligned}\bar{\bar{Z}}_a &= \left(\bar{\bar{Q}}_{11}\bar{\bar{Z}} + \bar{\bar{Q}}_{12}\right)\\ \bar{\bar{Z}}_b &= \bar{\bar{Z}}\left(\bar{\bar{Q}}_{21}\bar{\bar{Z}} + \bar{\bar{Q}}_{22}\right).\end{aligned} \tag{10}$$

The elements over the main diagonal of the reflection and transmittance matrices are the S-parameters due to incident TE or TM polarized plane waves, while the elements off the diagonal represent the scattering coefficients of TE polarized plane waves due to incident TM polarized plane wave and vice versa

$$\begin{aligned}\bar{\bar{R}} &= \begin{pmatrix} S_{11}^{TM} & S_{11}^{TE,TM}\\ S_{11}^{TM,TE} & S_{11}^{TE}\end{pmatrix}\\ \bar{\bar{T}} &= \begin{pmatrix} S_{21}^{TM} & S_{21}^{TE,TM}\\ S_{21}^{TM,TE} & S_{21}^{TE}\end{pmatrix}.\end{aligned} \tag{11}$$

In the special case of incident plane waves in the x–z plane of incidence ($k_y = 0$), the reflection and transmittance matrices can be evaluated analytically, whereas for any other case numerical computation is used due to the complexity of the analytical expressions. For the former case, plane waves must satisfy the following dispersion relations [33]

$$\varepsilon_y \mu_x \mu_z k_0^2 = \mu_x (k_x^{TE})^2 + \mu_z (k_z^{TE})^2 \tag{12a}$$

$$\left(\varepsilon_x \varepsilon_z \mu_y - \varepsilon_x \xi_0^2\right) k_0^2 = \varepsilon_x (k_x^{TM})^2 + \varepsilon_z (k_z^{TM})^2 \tag{12b}$$

where $k_x^{TE}, k_z^{TE}, k_x^{TM}, k_z^{TM}$ are the wave vector components of a plane wave propagating within the bi-anisotropic medium. Equation (12a) represents the dispersion relation of TE polarized plane waves and (12b) represents the dispersion relation of TM polarized plane waves. In addition, utilizing Maxwell's equations boundary conditions at the bi-anisotropic slab interfaces at $z = 0, d$, Snell's law is obtained for the bi-anisotropic media in the x–z plane of incidence. The bi-anisotropic refraction index for each polarization is derivable after some algebraic manipulation as a function of the angle of incidence θ_1 and polarization [34]. The wave number in the z direction for TE and TM polarizations can be written by

$$\begin{aligned}k_z^{TM} &= k_0 n^{TM} \cos\theta_2\\ k_z^{TE} &= k_0 n^{TE} \cos\theta_2\end{aligned} \tag{13}$$

where

$$\begin{aligned}n^{TM} &= \sqrt{\varepsilon_x \mu_y + (1 - \varepsilon_x/\varepsilon_z)\sin^2\theta_1 - (\varepsilon_x/\varepsilon_z)\xi_0^2}\\ n^{TE} &= \sqrt{\varepsilon_y \mu_x + (1 - \mu_x/\mu_z)\sin^2\theta_1}\end{aligned} \tag{14}$$

are the refraction indices for TM and TE polarizations, respectively. The reflection and transmittance matrices (11) are analytically evaluated to obtain

$$\begin{aligned}S_{11}^{TM} &= \frac{\Gamma^{TM}\left(1 - e^{-j2k_z^{TM}d}\right)}{1 - (\Gamma^{TM})^2 e^{-j2k_z^{TM}d}}\\ S_{11}^{TE} &= \frac{\Gamma^{TE}\left(1 - e^{-j2k_z^{TE}d}\right)}{1 - (\Gamma^{TE})^2 e^{-j2k_z^{TE}d}}\\ S_{21}^{TM} &= \frac{\left(1 - (\Gamma^{TM})^2\right) e^{-jk_z^{TM}d}}{1 - (\Gamma^{TM})^2 e^{-j2k_z^{TM}d}}\\ S_{21}^{TE} &= \frac{\left(1 - (\Gamma^{TE})^2\right) e^{-jk_z^{TE}d}}{1 - (\Gamma^{TE})^2 e^{-j2k_z^{TE}d}}\end{aligned} \tag{15}$$

and all cross-polarization terms are null, where

$$\begin{aligned}\Gamma^{TM} &= \frac{\eta^{TM}/\cos\theta_1 - 1}{\eta^{TM}/\cos\theta_1 + 1}\\ \Gamma^{TE} &= \frac{\eta^{TE}\cos\theta_1 - 1}{\eta^{TE}\cos\theta_1 + 1}\\ \eta^{TM} &= \sqrt{\left(\mu_y - (\sin^2\theta_1 + \xi_0^2)/\varepsilon_z\right)/\varepsilon_x}\\ \eta^{TE} &= \sqrt{\mu_x/\left(\varepsilon_y - \sin^2\theta_1/\mu_z\right)}.\end{aligned} \tag{16}$$

$\eta^{TE/TM}$ are the normalized wave impedances for the TE or TM polarization, respectively, such that $\hat{k} \times \mathbf{E}^{TE/TM} = \eta_0 \eta^{TE/TM} \mathbf{H}^{TE/TM}$ and θ_2 is the plane wave propagation angle in the bi-anisotropic medium with respect to the z-axis. It is instructive to observe that for isotropic medium where $\varepsilon = \varepsilon_i$, $\mu = \mu_i$, $i = x, y, z$, and $\xi_0 = 0$, the refraction indices and wave impedances simplifies to $n^{TE/TM} = \sqrt{\varepsilon\mu} = n$ and $\eta^{TE/TM} = \sqrt{\mu/\varepsilon} = \eta/\eta_0$.

B. Derivation of the Oblique Incidence S-Parameters Extraction Method

The oblique incidence S-parameters extraction method of metamaterials' effective constitutive parameters requires a total of 12 measurements over a single metamaterial slab in order to obtain explicitly all seven complex unknown parameters in (2). Eight complex measurements are performed in the x–z plane at normal and oblique incidence for TE and TM polarizations, and are used to extract $\varepsilon_x, \varepsilon_y, \varepsilon_z, \mu_x, \mu_z$ analytically. The remaining measurements are performed for $k_y \neq 0$ at a single oblique incidence at both TE and TM polarizations, and are used to extract the remaining unknown parameters μ_y, ξ_0 by a numerical optimization procedure.

The extraction equations for the first eight measurements in the x–z plane are derived analytically by inversion of (15) in the same manner as in the normal incidence S-parameters extraction method [1]. The bi-anisotropic wave impedance and refraction indices are evaluated from

$$\begin{aligned}\eta^{TE} &= \pm\sqrt{\frac{(1+S_{11}^{TE})^2-(S_{21}^{TE})^2}{(1-S_{11}^{TE})^2-(S_{21}^{TE})^2}}\sec\theta_1\\ \eta^{TM} &= \pm\sqrt{\frac{(1+S_{11}^{TM})^2-(S_{21}^{TM})^2}{(1-S_{11}^{TM})^2-(S_{21}^{TM})^2}}\cos\theta_1\\ n^{TE} &= \sqrt{\left[\frac{\log|\zeta^{TE}|+j(\angle(\zeta^{TE})+2\pi m)}{-jk_0 d}\right]^2+\sin^2\theta_1}\\ n^{TM} &= \sqrt{\left[\frac{\log|\zeta^{TM}|+j(\angle(\zeta^{TM})+2\pi \bar{m})}{-jk_0 d}\right]^2+\sin^2\theta_1}\end{aligned} \tag{17}$$

where $m, \bar{m}$ are independent integers and

$$\begin{aligned}\zeta^{TE} &= \frac{S_{21}^{TE}}{1-S_{11}^{TE}(\eta^{TE}\cos\theta_1-1)/(\eta^{TE}\cos\theta_1+1)}\\ \zeta^{TM} &= \frac{S_{21}^{TM}}{1-S_{11}^{TM}(\eta^{TM}/\cos\theta_1-1)/(\eta^{TM}/\cos\theta_1+1)}.\end{aligned} \tag{18}$$

Note that all equations are dependent on the incident angle and at normal incidence $\theta_1 = 0$, they reduce to the normal incidence S-parameters extraction method equations. The choice of the correct branch cut in (17) is taken such that the material is a passive medium, which satisfies the boundary conditions at infinity

$$\begin{aligned}(\eta^{TE/TM})' &\geq 0\\ (n^{TE/TM})'' &\leq 0\end{aligned} \tag{19}$$

where $(\cdot)'$ and $(\cdot)''$ denote the real and imaginary parts operators, respectively. The choice of the correct branch indices $m, \bar{m}$ is addressed in the following paragraph. Out of measurements for normal incidence at TE and TM polarizations in the x–z plane, the following parameters are extracted from (14) and (16)

$$\begin{aligned}&\varepsilon_y = n^{TE}/\eta^{TE}\big|_{\theta_1=0}, \quad \mu_x = n^{TE}\eta^{TE}\big|_{\theta_1=0}\\ &\varepsilon_x = n^{TM}/\eta^{TM}\big|_{\theta_1=0}, \quad \mu_y - \xi_0^2/\varepsilon_z = n^{TM}\eta^{TM}\big|_{\theta_1=0}.\end{aligned} \tag{20}$$

Note that the extracted parameters above are ambiguous, since the branch indexes $m, \bar{m}$ of the refraction indices $n^{TE/TM}$ are yet unknown (17). The branch indexes and the longitudinal constitutive parameters ε_z, μ_z are extracted by utilizing the oblique incidence measurements in the x–z plane. The longitudinal constitutive parameters are extracted out of two independent equations, the normalized wave impedances equations (16) and the refraction indices equations (14). Using the extracted parameters in (20), the following sets of equations as a function of the branch indices $m, \bar{m}$ are derived

$$\begin{aligned}\varepsilon_z^{(1)} &= \varepsilon_x\sin^2\theta_1/\left(\sin^2\theta_1-(n^{TM})^2+(n^{TM})^2\big|_{\theta_1=0}\right)\\ \mu_z^{(1)} &= \sin^2\theta_1/((n^{TE}/\eta^{TE})\big|_{\theta_1=0}-\mu_x/(\eta^{TE})^2)\\ \varepsilon_z^{(2)} &= \sin^2\theta_1/\left((n^{TM}\eta^{TM})\big|_{\theta_1=0}-\varepsilon_x(\eta^{TM})^2\right)\\ \mu_z^{(2)} &= \mu_x\sin^2\theta_1/\left(\sin^2\theta_1-(n^{TE})^2+(n^{TE})^2\big|_{\theta_1=0}\right).\end{aligned} \tag{21}$$

Note that η^{TE}, η^{TM} are known explicitly and n^{TE}, n^{TM}, $\varepsilon_x, \varepsilon_y, \mu_x, \mu_y$ are ambiguous. The two sets of the extracted longitudinal constitutive parameters (21) are equated in order to obtain the correct branch indices $m, \bar{m}$ that hold

$$\begin{aligned}\varepsilon_z^{(1)}(\bar{m}) &= \varepsilon_z^{(2)}(\bar{m})\\ \mu_z^{(1)}(m) &= \mu_z^{(2)}(m).\end{aligned} \tag{22}$$

Once the correct branch indices $m, \bar{m}$ are obtained, the ambiguous extracted parameters can be evaluated explicitly from equations (20) and (21), thus obtain $\varepsilon_x, \varepsilon_y, \varepsilon_z, \mu_x, \mu_z$ explicitly. In addition, the remaining unknown parameters μ_y, ξ_0 are related analytically based on equation (20), $\mu_y - \xi_0^2/\varepsilon_z = n^{TM}\eta^{TM}|_{\theta_1=0}$. In order to fully extract the remaining unknown parameters, additional measurements must be conducted to gain an additional set of equations. Since the parabolic relation between μ_y, ξ_0 holds for any incidence angle θ_1 in the x–z plane, it is impossible to extract these unknown parameters by an additional oblique incidence in the x–z plane. Therefore, oblique measurements outside the x–z plane of incidence $k_y \neq 0$, $\phi_1 \neq 0$ must be considered as well.

The extraction process of the remaining unknown parameters μ_y, ξ_0 is obtained using numerical tools due to the complexity of the analytical expressions involved. From the state equation (4) derived for bulk bi-anisotropic slab in the previous section, the reflection and transmittance matrices $\bar{\bar{R}}$ and $\bar{\bar{T}}$ (8) can be evaluated for any given constitutive parameters with respect to (1). At this point, all effective constitutive parameters of the metamaterial measured are assumed known, except for μ_y and ξ_0, which are related by a parabolic analytical expression (20) and (21). Therefore, given ξ_0 the reflection and transmittance matrices $\bar{\bar{R}}$ and $\bar{\bar{T}}$ can be evaluated at any angle of incidence of interest following the derivation in the previous section. This is used to extract the remaining unknown parameters using a numerical optimization over ξ_0, where the cost function is the norm of the difference between the measured and numerically evaluated reflection and transmittance matrices at any oblique incidence for $k_y \neq 0$. The initial condition for the chirality parameter ξ_0 is set to 0 for $f \to 0$ [23].

$$\begin{aligned}&\min_{\xi_0\in\mathbb{C}} \left|\bar{\bar{R}}^{meas}-\bar{\bar{R}}^{comp}\right| + \left|\bar{\bar{T}}^{meas}-\bar{\bar{T}}^{comp}\right|\\ &\text{s.t.}\\ &\xi_o|_{f\to 0} = 0\end{aligned} \tag{23}$$

where superscripts $meas$ and $comp$ denote the measured and numerically evaluated reflection and transmittance matrices, respectively. Once ξ_0 is obtained, μ_y is derived from equation (20) $\mu_y = \xi_0^2/\varepsilon_z + n^{TM}\eta^{TM}|_{\theta_1=0}$. Note that cross-polarization may occur due to the bi-anisotropic properties of the medium, thus cross-polarization elements of the reflection and transmittance matrices (11) must be measured as well.

III. Numerical Results

A. Bulk Homogeneous Bi-Anisotropic Medium

In order to examine the oblique incidence extraction method, it is first practiced over a bulk homogeneous dispersive bi-anisotropic medium with constitutive relations as in (1). The

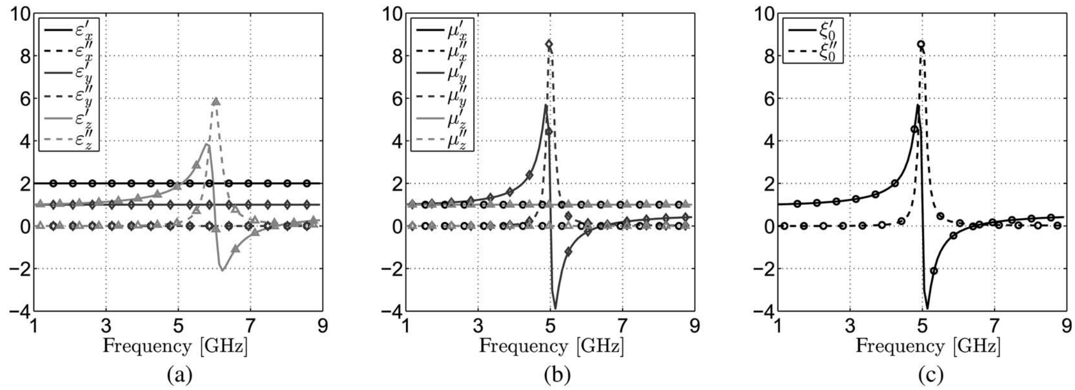

Fig. 2. Extracted constitutive parameters using the proposed oblique incidence extraction method (curve markers) of a bulk homogeneous dispersive bi-anisotropic medium with constitutive relations given in (24) (solid and dashed lines). The measurement angle of incidence used in both the x–z and y–z planes was $\theta_1 = 20°$ and the slab thickness was $d = 15.75$ (mm). (a) Permeability. (b) Permittivity. (c) Chirality parameter.

constitutive parameters model chosen approximately describes an SRR metamaterial structure around its resonance frequency [24], [28]

$$\begin{aligned} &\varepsilon_x = C_1,\, \varepsilon_y = C_2 \\ &\varepsilon_z = 1 - F_e f^2 / \left(f^2 - f_e^2 + j\gamma_e f\right) \\ &\mu_x = \mu_z = C_2 \\ &\mu_y = 1 - F_m f^2 / \left(f^2 - f_m^2 + j\gamma_m f\right) \\ &\xi_0 = 1 - F_\xi f^2 / \left(f^2 - f_\xi^2 + j\gamma_\xi f\right) \end{aligned} \tag{24}$$

where $C_1 = 2, C_2 = 1, F_e = F_m = F_\xi = 0.4, f_e = 6$ GHz, $f_m = f_\xi = 5$ GHz, $\gamma_e = 0.4$ GHz, $\gamma_m = \gamma_\xi = 0.2$ GHz. In this example, the oblique incidence measurements were held for several angles of incidence θ_1 in the x–z and y–z planes ($k_x = 0$) of incidence. All S-parameters used in the extraction process were evaluated analytically by (11) and the optimization algorithm used to solve (23) was the simplex search method [35]. The extracted constitutive parameters coincides with the analytical model used in this example (24) for various angles of incidence. In Fig. 2, the results were obtained using $\theta_1 = 20°$ for both the x–z and y–z planes and the slab thickness was $d = 15.75$ (mm).

B. SRR-Based Metamaterial

Next, the proposed oblique incidence extraction method was tested for a lossless edge coupled SRR periodic structure with a unit cell shown in Fig. 3. The edge-coupled SRR-based metamaterial effective constitutive parameters can be approximately described around its resonance frequency as in (1) under quasi static analysis [24]. The slab thickness consists of three unit cells and the oblique incidence measurements were held for $\theta_1 = 20°$, $40°$, $60°$ both in the x–z and y–z planes of incidence. The metal strips are perfect conductors embedded in vacuum. All of the S-parameters needed for the extraction process were simulated using Ansys HFSS commercial software and the optimization algorithm used to solve (23) was the simplex search method [35]. The extracted constitutive parameters are presented in Fig. 4 for $\theta_1 = 20°$ along with those extracted using the normal incidence S-parameters method described in

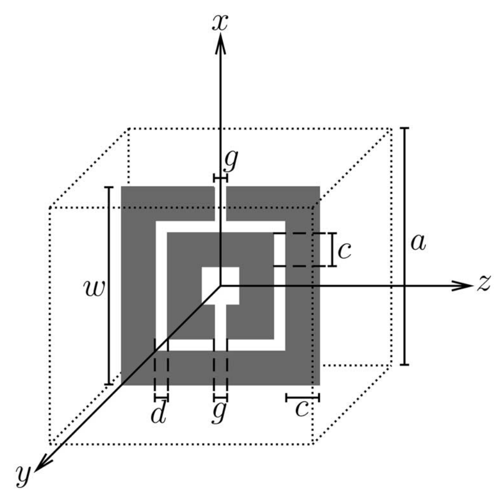

Fig. 3. Geometry of an edge-coupled SRR metamaterial unit cell [28]. The SRR parameters used for the oblique incidence extraction method simulation example are: $w = 0.8a, g = d = 0.04a, c = 0.08a$, where $a = 5.25$ (mm) is the unit cell length.

[4] and [5]. The results for $\theta_1 = 40°$, $60°$ coincide with those presented in Fig. 4 and therefore have been omitted. In addition, the extracted constitutive parameters ε_y, μ_x and, μ_z are unity and therefore are not presented as well.

The extracted effective constitutive parameters in this example and in artificial structures in general must be handled with care as they may violate physical laws [36]–[39] and cause artifact in the results, which may be incorrectly interpreted. In the current example, the shaded regions in results shown in Fig. 4 indicate frequency bands where the extracted constitutive parameters are not physical. In the nonphysical regions, the solution does not satisfy certain criteria and assumptions upon which the proposed method is based.

First, it is assumed that the SRR metamaterial structure can be described by diagonal permittivity and permeability tensors and a chiral element without spatial dispersion as described in (2), e.g., $\bar{\bar{\varepsilon}}(\omega, \mathbf{k}) = \bar{\bar{\varepsilon}}(\omega)$ [24], [28]. Second, following Poynting's theorem and the thermodynamic laws, it is assumed that the imaginary parts of the permittivity and permeability tensors must be positive $\varepsilon_i'' > 0, \mu_i'' > 0$ and their

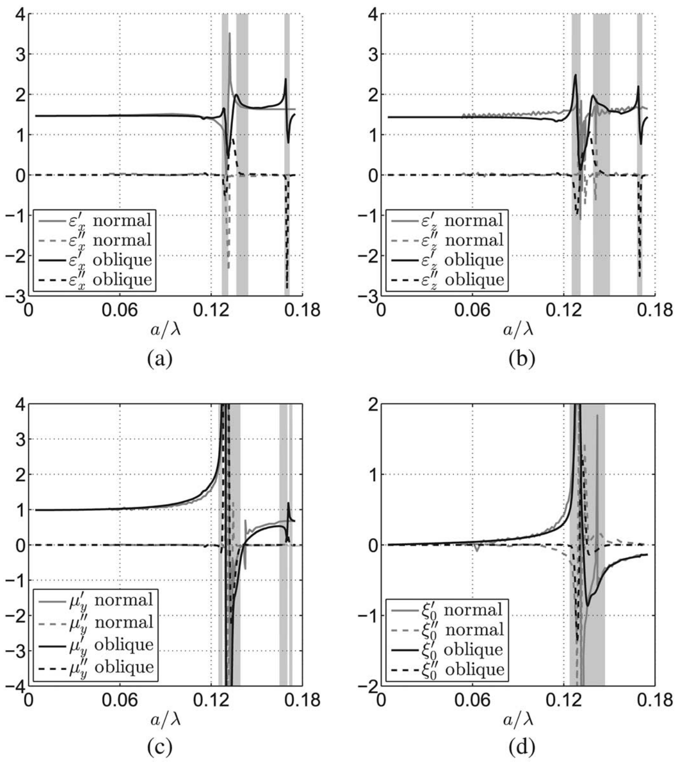

Fig. 4. Extracted effective constitutive parameters for the edge-coupled SRR-based metamaterial medium using the proposed oblique incidence extraction method. The shaded regions indicate frequency bands where the extracted effective constitutive parameters are not physical and therefore unreliable. The SRR unit cell is defined in Fig. 3. The oblique incidence S-parameters are evaluated at $\theta_1 = 20°$ for both y–z and x–z planes of incidence. (a) ε_x. (b) ε_z. (c) μ_y. (d) ξ_0.

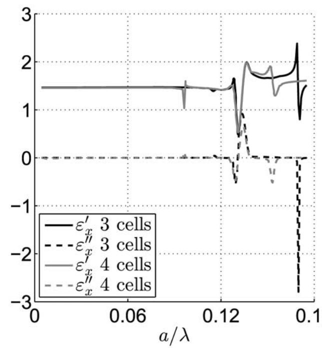

Fig. 5. Extracted effective constitutive parameter ε_x for a 3 and 4 unit cells edge-coupled SRR based metamaterial slab. The SRR unit cell is defined in Fig. 3. The S-parameters are evaluated at $\theta_1 = 20°$ for both y-z and x-z planes of incidence.

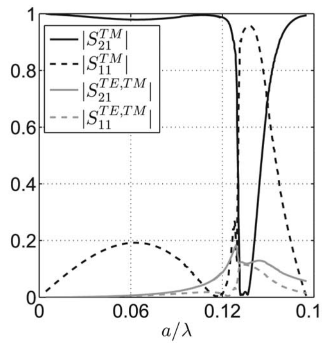

Fig. 6. $S_{11}^{TM}, S_{21}^{TM}, S_{11}^{TE,TM}, S_{21}^{TE,TM}$ for incident plane waves in the y–z plane of incidence over an edge-coupled SRR-based metamaterial slab. The SRR unit cell is defined in Fig. 3. The S-parameters are evaluated at $\theta_1 = 20°$.

real parts must obey $\partial(\omega\varepsilon_i')/\partial\omega > 0, \partial(\omega\mu_i')/\partial\omega > 0$, where $i = x, y, z$ as described in [36], [39]. Furthermore, since we are dealing with lossless reciprocal media, it is assumed that the chirality parameter ξ_0 must be real except in the close vicinity of resonances and it vanishes if the frequency tends to zero [23].

Moreover, another problem that needs to be carefully considered and is common to all S-parameters methods is the failure to extract the structure's effective constitutive parameters over frequencies in which the reflection coefficient is close to zero and the transmission coefficient is close to 1, as seen in (17). This occurs at a discrete set of frequencies whenever $d = p\lambda_g^{TE/TM}/2$, where $\lambda_g^{TE/TM}$ is the effective longitudinal (z-direction) wavelength of the TE or TM polarized incident plane wave in the metamaterial slab and p is an integer. This singularity affects the accuracy of the extracted constitutive parameters in narrow frequency bands around this set of frequencies. This failure in the solution can be readily solved, at least partially by adding measurements for a different slab thickness d, thus change the discrete set of frequencies in which these singularities are present. This is demonstrated in the extraction of ε_x presented in Fig. 5 using measurements over a three and four unit cells slabs.

The reflection and transmission coefficients may be used in order to support the existence of the nonphysical results, if they are due to spatial dispersion not taken into account or due to near zero reflection coefficients. Fig. 6 shows the $S_{21}^{TM}, S_{11}^{TM}, S_{21}^{TE,TM}, S_{11}^{TE,TM}$ parameters for incident plane waves in the y–z plane of incidence. They can be used for comparison with the shaded nonphysical regions in the extracted constitutive parameters in Fig. 4. One can observe that, at $d = p\lambda_g^{TE/TM}/2$ where $d = 3a$, the reflection coefficient S_{11}^{TM} is close to zero and the transmission coefficient S_{21}^{TM} is close to one, while for $0.125 < a/\lambda < 0.14$, where the bi-anisotropic resonance effect of the SRR metamaterial is strongest, the S-parameters are not close to zero or one such that the nonphysical results are due to spatial dispersion. Note that in the bulk homogeneous bi-anisotropic medium example Section III-A, there are no gray nonphysical regions since the analytical model used in that example is lossy and has no spatial dispersion.

The results obtained in this example were verified in several ways. First, the extracted constitutive parameters were compared to those obtained in [4] and [5] using the normal incidence S-parameters method as shown in Fig. 4. If we do not include in the comparison the shaded nonphysical regions, the agreement is very good. Second, computed results obtained from data at different incidence angles were compared. Excluding the shaded nonphysical areas, the agreement is very good as shown in Fig. 7. The divergence of the traces for $a/\lambda > 0.24$ in parameter ε_z is an indication on the limitation of the bi-anisotropic model used for large SRR unit cells. This is significant information when designing metamaterial structures for applications where oblique incident angles are of

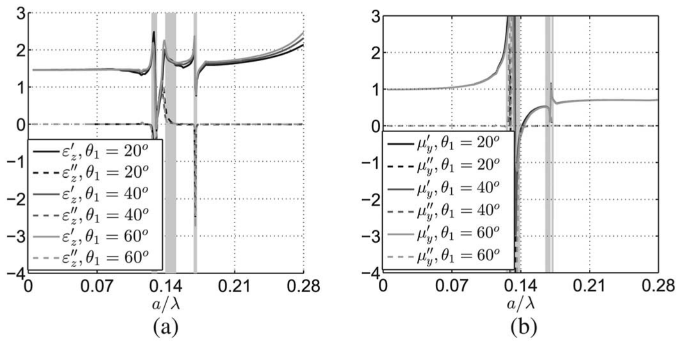

Fig. 7. Extracted effective constitutive parameters for the edge-coupled SRR-based metamaterial medium using the proposed oblique incidence extraction method. The SRR unit cell is defined in Fig. 3. The S-parameters are evaluated at $\theta_1 = 20^\circ, 40^\circ, 60^\circ$ for both y–z and x–z planes of incidence. (a) ε_z. (b) μ_y.

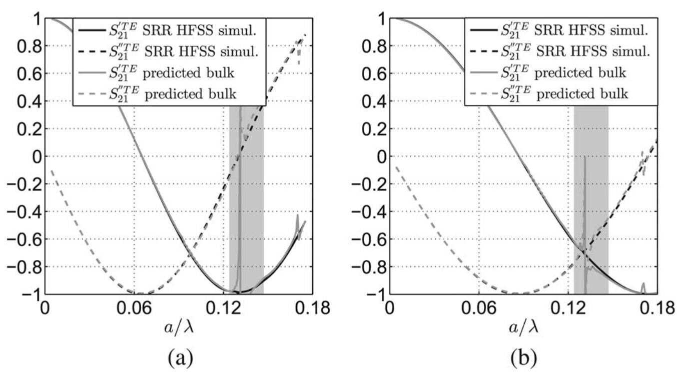

Fig. 8. Scattering parameter $S^{TE}_{21} = S'^{TE}_{21} + jS''^{TE}_{21}$ in the y–z plane of incidence. (a) Predicted S-parameter for bulk four unit cells slab at $\theta_1 = 20^\circ$ from extracted parameters from a three unit cells slab at $\theta_1 = 20^\circ$ compared to SRR four unit cells HFSS results. (b) Predicted S-parameter for bulk three unit cells slab at $\theta_1 = 20^\circ$ from extracted parameters from a three unit cells slab at $\theta_1 = 40^\circ$ compared to SRR three unit cells HFSS results.

interest. Finally, the extracted effective constitutive parameters are used to predict the S-parameters from a different metamaterial slab. Two cases are examined, the extracted parameters from a three unit cells slab at $\theta_1 = 20^\circ$ were used to evaluate the S-parameters of an equivalent four unit cells bulk homogeneous slab at $\theta_1 = 20^\circ$. Also, the extracted parameters from a three unit cells slab at $\theta_1 = 40^\circ$ were used to evaluate the S-parameters of an equivalent three unit cells bulk homogeneous slab at $\theta_1 = 20^\circ$. The predicted S-parameters for both cases are compared with HFSS simulation results of the relevant SRR metamaterial slab. In Fig. 8, one can observe a very good agreement between the two sets of S^{TE}_{21} parameters measured in the y–z plane of incidence except in the nonphysical regions, copied from Fig. 4(d), which reaffirms the validity of the extraction procedure. In Fig. 8, it is clear that within the shaded region the HFSS simulated and bulk homogeneous predicted results S^{TE}_{21} differ, as well as near $d = \lambda_g^{TE/TM}/2$ (corresponds to $a/\lambda = 0.17$), while elsewhere there is a very good match.

IV. Conclusion

A novel method for bi-anisotropic metamaterials constitutive parameter extraction is proposed. The proposed method enables to extract all unknown parameters using S-parameters measured over a single metamaterial slab. The proposed oblique incidence S-parameters method was examined for a homogeneous bi-anisotropic medium and an edge-coupled SRR metamaterial structure using numerical simulation software. In addition, the extracted effective constitutive parameters were validated by different measures: over several different angles of incidence, by reevaluation of the measured S-parameters from the extracted effective constitutive parameters and by comparing with other published extraction methods. The reliability of all extracted effective constitutive parameters was tested and detailed in the paper. The proposed method is also utilized to show the limited validity of the assumption of absence of spatial dispersion on which the proposed approach is based.

Appendix A

The explicit parameters of $\bar{\bar{\Gamma}}$ defined in (4) are

$$
\begin{aligned}
\Gamma_{13} &= \frac{k_y}{\varepsilon_0\varepsilon_z}\left(\frac{\xi_0}{c_0} - \frac{jk_x}{\omega}\right)\\
\Gamma_{14} &= -j\omega\mu_0\mu_y + \frac{j\omega}{\varepsilon_0\varepsilon_z}\left(\frac{\xi_0}{c_0}\right)^2 + \frac{jk_x^2}{\varepsilon_0\varepsilon_z\omega}\\
\Gamma_{23} &= j\omega\mu_0\mu_x - \frac{jk_y^2}{\omega\varepsilon_0\varepsilon_z}\\
\Gamma_{24} &= \frac{k_y}{\varepsilon_0\varepsilon_z}\left(\frac{\xi_0}{c_0} + \frac{jk_x}{\omega}\right)\\
\Gamma_{31} &= -\frac{jk_xk_y}{\mu_0\mu_z\omega}\\
\Gamma_{32} &= j\omega\varepsilon_0\varepsilon_y - \frac{jk_x^2}{\mu_0\mu_z\omega}\\
\Gamma_{41} &= -j\omega\varepsilon_0\varepsilon_x - \frac{jk_y^2}{\omega\mu_0\mu_z}\\
\Gamma_{42} &= -\frac{jk_xk_y}{\mu_0\mu_z\omega}\\
\Gamma_{ij} &= 0, \quad \text{otherwise.}
\end{aligned}
\tag{25}
$$

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