Appl Phys B (2008) 93: 339–343
DOI 10.1007/s00340-008-3168-2

Applied Physics B
Lasers and Optics

RAPID COMMUNICATION

Eigen electric moments and magnetic–dipolar vortices in quasi-2D ferrite disks

M. Sigalov · E.O. Kamenetskii · R. Shavit

Received: 5 March 2008 / Revised version: 27 July 2008 / Published online: 16 September 2008

Abstract In a quasi-2D ferrite disk with a dominating role of magnetic–dipolar (non-exchange-interaction) spectra, one can observe the vortex structures. The vortices are guaranteed by the chiral edge states of magnetic–dipolar modes which result in appearance of eigen electric moments oriented normally to the disk plane. Due to the eigen-electric-moment properties, a ferrite disk placed in a microwave cavity is strongly affected by the cavity RF electric field with a clear evidence for multi-resonance oscillations. For different cavity parameters, one may observe the resonance absorption and resonance repulsion behaviors.

PACS 76.50.+g · 68.65.-k · 73.22.Gk

Different wave processes occurring in ferromagnetic systems can be characterized by different scales of the fields. For example, one can distinguish the scales of the spin (exchange interaction) fields, the magnetostatic (dipole-dipole interaction) fields, and the electromagnetic fields. It is very interesting to note that these characteristic scales may define different magnetic vortex states. Nevertheless, in spite of the fact that vortices can appear in different kinds of magnetic phenomena, yet such "swirling" entities seem to elude an all-inclusive definition.

In a dot of a ferromagnetic material of micrometer or submicrometer size, a curling spin configuration—that is, a magnetization vortex—has been proposed. The vortex consists of an in-plane, flux-closure magnetization distribution, and a central core whose magnetization is perpendicular to the dot plane. It has been shown that under certain conditions a vortex structure will be stable because of competition between the exchange and dipole interactions [1, 2]. It appears that the characters of magnetic vortices in magnetically soft "small" (with the dipolar and exchange energy competition) and magnetically saturated "big" (when the exchange is neglected) ferrite disks are very different. A magnetization vortex in a magnetically soft sample cannot be characterized by some invariant, such as the flux of vorticity. So a vorticity thread may not be defined for the magnetization vortex [3]. At the same time, in magnetically saturated samples with magnetic–dipolar vortices, one can observe the flux of the pseudo-electric (gauge) fields [4, 5]. The vortices of magnetic–dipolar-mode (MDM) oscillations in a ferrite disk become apparent due to the symmetry breaking effects which result in appearance of eigen electric moments oriented normally to the disk plane [4–6].

Magnetostatic (MS) ferromagnetism has a character essentially different from exchange ferromagnetism. This statement finds strong confirmation in confinement phenomena of MDM oscillations. The dipole interaction provides us with a long-range mechanism of interaction, where a magnetic medium is considered as a continuum. A property associated with a vortex structure in a ferrite disk with a dominating role of magnetic–dipolar (non-exchange-interaction) spectra becomes evident from an analysis of the boundary volume problem for magnetic–dipolar oscillations. It has been shown [4, 5, 7] that for MDMs in a ferrite disk, one has evident quantum-like attributes. The spectrum is characterized by energy-eigenstate oscillations. It appears, however, that because of the boundary condition on a lateral surface of a ferrite disk, MS-potential eigen functions cannot be considered as single-valued functions. This fact raises a question about validity of the energy orthogonality relations for the MDMs. The most basic implication of the existence

M. Sigalov · E.O. Kamenetskii (✉) · R. Shavit
Ben-Gurion University of the Negev, Beer Sheva 84105, Israel
e-mail: kmntsk@ee.bgu.ac.il

of a phase factor in eigen functions is operative in the case on the border ring region. It follows that in order to cancel the "edge anomaly," the boundary excitation must be described by chiral states. Because of these chiral states of MDMs in a ferrite disk, the eigen electric moments appear. It was shown experimentally [6], for the first time, that such moments really exist. There are electric moments characterizing by special symmetry properties, the anapole-moment properties [4, 5]. The purpose of this letter is to analyze the spectral distribution of eigen electric (anapole) moments of a MDM ferrite disk. Our experimental studies of absorption spectra for a ferrite disk placed in a microwave cavity are aimed to investigate possible mechanisms of interaction of the disk eigen electric moments with an external RF electric field and to verify the proposed theoretical model for the MDM vortices and anapole moment oscillations.

The topological effects in the MDM ferrite disk are manifested through the generation of relative phases which accumulate on the boundary wave functions $\delta_\pm$ [4, 5]: $\delta_\pm \equiv f_\pm e^{-iq_\pm\theta}$. The quantities $q_\pm$ are equal to $\pm l\frac{1}{2}$, $l = 1, 3, 5, \ldots$. For amplitudes f, we have $f_+ = -f_-$ with normalization $|f_\pm| = 1$. To preserve the single-valued nature of the membrane functions of the MDM oscillations, functions $\delta_\pm$ must change its sign when a disk angle coordinate θ is rotated by 2π so that $e^{-iq_\mp 2\pi} = -1$. The sign of a full chiral rotation, $q_+\theta = \pi$ or $q_-\theta = -\pi$, should be correlated with the sign of the parameter $i\mu_a$, the off-diagonal component of the permeability tensor $\overleftrightarrow{\mu}$. This becomes evident from the fact that the sign of $i\mu_a$ is related to the precession direction of a magnetic moment $\vec{m}$. In a ferromagnetic resonance, the bias field sets up a preferential precession direction. This means that for a normally magnetized ferrite disk with a given direction of a DC bias magnetic field, there are two types of resonant oscillations, which we conventionally designate as the $(+)$ resonance and the $(-)$ resonance. For the $(+)$ resonance, the direction of an edge chiral rotation coincides with the precession magnetization direction, while for the $(-)$ resonance, the direction of an edge chiral rotation is opposite to the precession magnetization direction. For a ferrite disk with r and θ in-plane coordinates and normal-axis z coordinate, the total MS-potential function ψ is represented as a product of three functions: $\psi = \tilde{\eta}(r,\theta)\xi(z)\delta_\pm$, where $\tilde{\eta}(r,\theta)$ is a single-valued membrane function, $\xi(z)$ is the function characterizing z-distribution of the MS potential in a ferrite disk, and $\delta_\pm$ is a double-valued edge (spin-coordinate-like) function. We write $\psi = \tilde{\eta}(r,\theta)\xi(z)\delta_+$ and $\psi = \tilde{\eta}(r,\theta)\xi(z)\delta_-$ as two wave functions corresponding to the positional wave function $\tilde{\eta}(r,\theta)\xi(z)$, which is a solution of the Walker equation for a ferrite disk with the so-called essential boundary conditions [7]. For a ferrite disk of radius $\Re$, the circulation of the gradient $\vec{\nabla}_\theta\delta_\pm = -i\frac{q_\pm f_\pm}{\Re}e^{-iq_\pm\theta}\vec{e}_\theta$ along a disk border contour $C = 2\pi\Re$ gives a nonzero quantity when $q_\pm$ is a number divisible by $\frac{1}{2}$. In the topological effects of the generation of relative phases which accumulate on the boundary wave function $\delta_\pm$, the quantity $\nabla_\theta\delta_\pm$ can be considered as the velocity of an irrotational "border" flow: $(\vec{\mathrm{v}}_\theta)_\pm \equiv \vec{\nabla}_\theta\delta_\pm$. In such a sense, functions $\delta_\pm$ are the velocity potentials. The circulation of $(\vec{\mathrm{v}}_\theta)_\pm$ along a contour C is equal to $\oint_C (\vec{\mathrm{v}}_\theta)_\pm \cdot d\vec{C} = \Re\int_0^{2\pi}\nabla_\theta\delta_\pm\,d\theta = -2f_\pm$. In the case of a cylindrical ferrite disk, a single-valued membrane function is represented as $\tilde{\eta}(r,\theta) = R(r)\phi(\theta)$, where $R(r)$ is described by the Bessel functions and $\phi(\theta) \sim e^{-i\nu\theta}$, $\nu = \pm1, \pm2, \pm3, \ldots$. Taking into account the "orbital" function $\phi(\theta)$, we may consider the quantity $[\vec{\nabla}_\theta(\phi\delta_\pm)]_{r=\Re}$ as the total ("orbital" and "spin") velocity of an irrotational "border" flow: $(\vec{V}_\theta)_\pm \equiv [\vec{\nabla}_\theta(\phi\delta_\pm)]_{r=\Re}$. It is evident that $(\vec{V}_\theta)_\pm = -i\frac{(\nu+q_\pm)f_\pm}{\Re}e^{-i(\nu+q_\pm)\theta}\vec{e}_\theta$. For a given membrane function $\tilde{\eta}$ and given z-distribution of the MS potential, $\xi(z)$, we can define now the strength of a vortex of a whole disk as $s^e_\pm \equiv R_{r=\Re}\int_0^d \xi(z)\,dz\oint_C(\vec{V}_\theta)_\pm\cdot d\vec{C} = -2f_\pm R_{r=\Re}\int_0^d\xi(z)\,dz$, where d is a disk thickness. The quantity $(\vec{V}_\theta)_\pm$ has a clear physical meaning. In the spectral problem for MDM ferrite disks, non-singlevaluedness of the MS-potential wave function appears due to the border term which is defined as $-i\mu_a(H_\theta)_{r=\Re}$. This border term arises from the demand of conservation of the magnetic flux density [4, 5]. It is evident that an annual magnetic field on the border circle, $(H_\theta)_{r=\Re}$, is expressed as $((\vec{H}_\theta(z))_\pm)_{r=\Re} = -\xi(z)(\vec{V}_\theta)_\pm$. We define now the angular moment $a^e_\pm \equiv \int_0^d\oint_C[-i\mu_a(H_\theta)_{r=\Re}]\vec{e}_\theta\cdot d\vec{C}\,dz = i\mu_a s^e_\pm$. This angular moment can be formally represented as a result of a circulation of a quantity, which we call the density of an effective boundary magnetic current $\vec{i}^m$: $a^e_\pm = 4\pi\int_0^d\xi(z)\,dz\oint_C\vec{i}^m_\pm\cdot d\vec{C}$, where $\vec{i}^m_\pm \equiv \rho^m(\vec{V}_\theta)_\pm$ and $\rho^m \equiv i\frac{\mu_a}{4\pi}\xi R_{r=\Re}$. In our continuous-medium model, a character of the magnetization motion becomes apparent via the gyration parameter μ_a in the boundary term for the spectral problem. There is magnetization motion through a non-simply-connected region. On the edge region, the chiral symmetry of the magnetization precession is broken to form a flux-closure structure. The edge magnetic currents can be observable only via its circulation integrals, not pointwise. This results in the moment oriented along a disk normal. Such a moment has a response in an external microwave electric field [6]. This clarifies a physical meaning of a superscript "e" in designations of $s^e_\pm$ and $a^e_\pm$. In a ferrite disk particle, the vector $\vec{a}^e$ is an electric moment characterized by special symmetry properties.

An electric moment $a^e_\pm$ is characterized by the anapole-moment properties [4, 5]. This is a certain-type toroidal moment. Some important notes should be given here to characterize properties of the moment $\vec{a}^e$. From classical consideration it follows that for a given electric current $\vec{i}^e$, a magnetic dipole moment is described as $\vec{m} = \frac{1}{2c}\int\vec{r}\times\vec{i}^e\,dv$, while the toroidal dipole moment is described as $\vec{t} = \frac{1}{3c}\int\vec{r}\,\times$

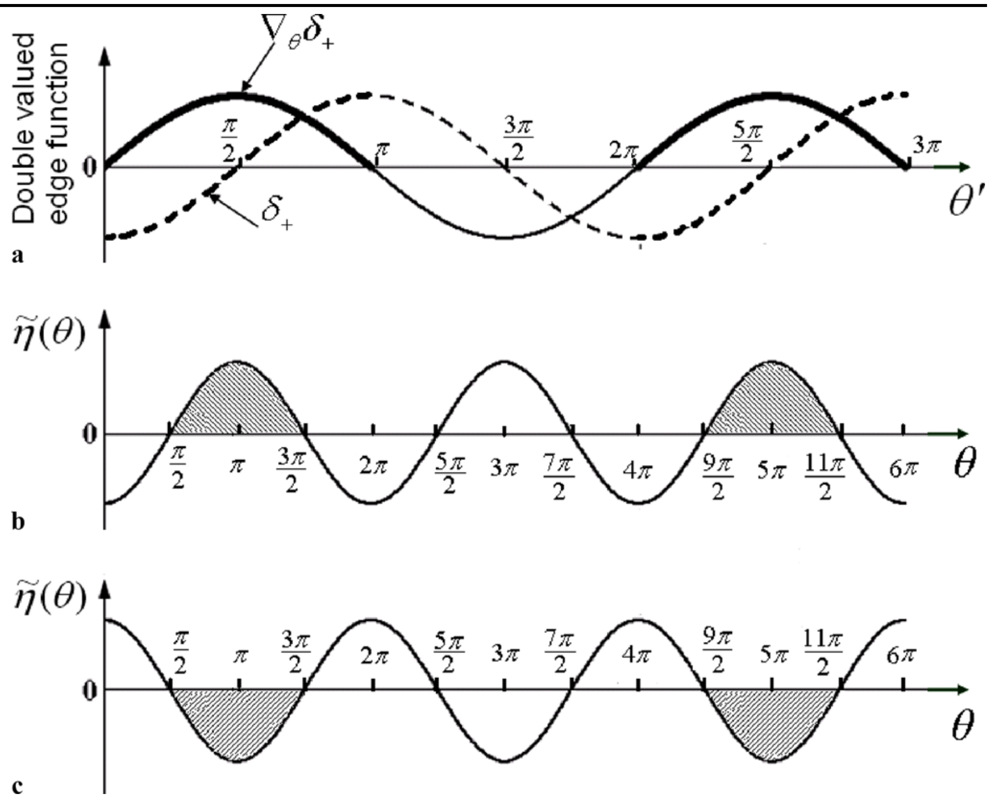

Fig. 1 A model explaining the mechanism of an interaction between the double-valued and single-valued functions. **a** The double-valued-function MDM edge state for the $(+)$ resonance; **b** the single-valued-function MDM membrane oscillation corresponding to the resonance absorption; **c** the single-valued-function MDM membrane oscillation corresponding to the resonance repulsion

$(\vec{r} \times \vec{i}^e)\,dv$. When we introduce the notion of an elementary magnet, $\vec{M} \equiv \vec{r} \times \vec{i}^e$, we can represent the toroidal dipole moment as a linear integral around a loop, $\vec{t} = \frac{1}{3c}\int \vec{r} \times \vec{M}\,dl$. It is considered as a ring of elementary magnets $\vec{M}$. In this formulation, it is clear that a toroidal moment is parity odd and time reversal odd. In the case where $\vec{M}$ is time varying, one has a magnetic current $\vec{i}^m \sim \frac{\partial \vec{M}}{\partial t}$, and a linear integral of this current around a loop defines a moment which is parity odd and time reversal even. This is the case of an anapole moment $\vec{a}^e$ which has the symmetry of an electric dipole. From a classical point of view such a definition presumes no azimuth variations of the loop magnetic current i^m. In our case, however, for oscillating MDMs, one has the azimuth varying ring magnetic current. The magnetic current i^m is described by the double-valued functions. This results in appearance of an anapole moment $\vec{a}^e$.

One can suppose a classical mechanism of interaction between the eigen electric moment of a ferrite disk and the cavity E-field. But the question is why this interaction gives the multiresonance spectral pictures similar to the pictures shown in the well-known experiments with a ferrite disk placed in an external microwave magnetic field [8]. The main aspects concern the question how one may obtain effective resonance interactions between the edge double-valued functions (described by "spin" angular coordinates) and the membrane single-valued functions (described by "orbital" angle coordinates). The edge double-valued functions determine the anapole-moment properties, while the membrane single-valued functions determine the energy eigen states. To understand this mechanism, we suggest here the following qualitative model. Figure 1a shows, as a particular case, the double-valued functions $\delta_+(\theta')$ and $\nabla_{\theta'}\delta_+(\theta')$ for $q = +\frac{1}{2}$. Here we use designation θ' to distinguish a "spin" angular coordinate from a regular ("orbital") angle coordinate. Because of the edge-function chiral rotation [4, 5], one has to select only positive derivatives: $\frac{\partial \delta_+}{\partial \theta'} > 0$. The corresponding parts of the graphs are distinguished in Fig. 1a by bold lines. Figures 1b and 1c give two cases of the membrane single-valued functions $\tilde{\eta}(\theta)$ which may lead to resonance "spin-orbital" interactions. It is evident that in the case of Fig. 1b, a positive half of function $\delta_+(\theta')$ is phased for a resonance interaction with the positive halves of function $\tilde{\eta}(\theta)$, while in the case of Fig. 1c, a positive half of function $\delta_+(\theta')$ is phased for a resonance interaction with the negative halves of function $\tilde{\eta}(\theta)$. This shows how the double-valued-function spins precessing on the edge region may interact with the single-valued-function spins precessing in the core region. On the other hand, this model can explain the mechanism of interaction between the disk eigen electric moment and the cavity E-field. The case of Fig. 1b may correspond to the resonance absorption, while the case of Fig. 1c to the "resonance repulsion." Both types of interactions are equiprobable. One may expect that for different cavity parameters, two the above cases, the resonance absorption and resonance repulsion, can be exhibited separately. One may also expect that in a certain situation,

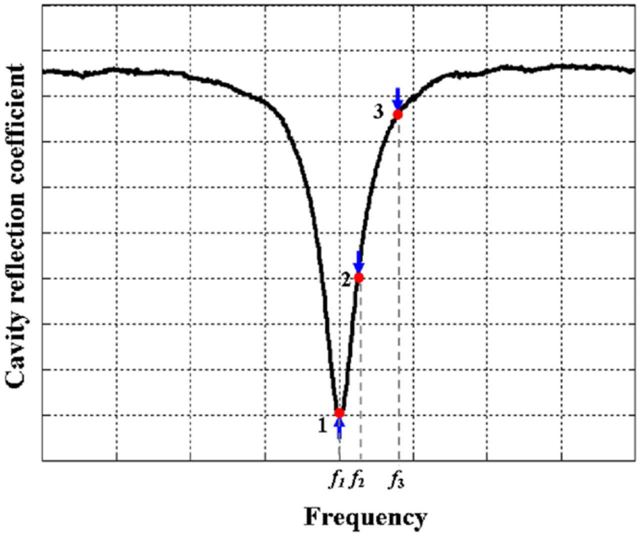

Fig. 2 Frequency dependence of the cavity reflection coefficient

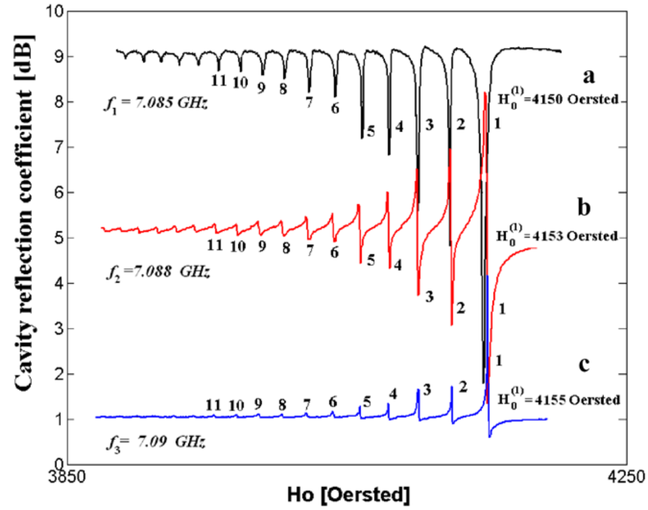

Fig. 4 Spectral pictures of reflection coefficients obtained at different frequencies

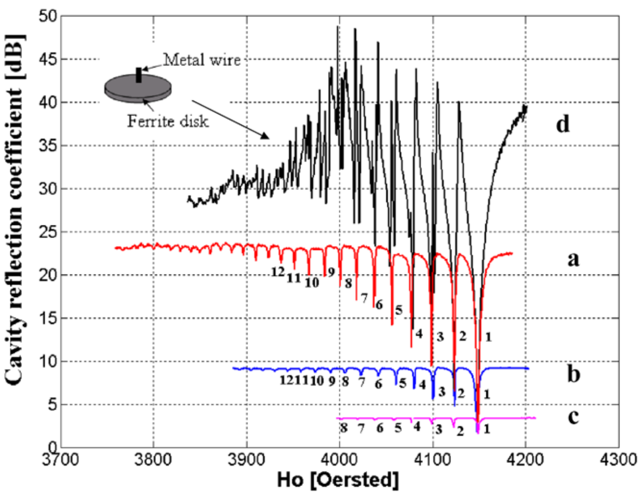

Fig. 3 Spectral pictures of reflection coefficients for different cavity structures: **a** Critically coupled cavity; **b** Noncritically coupled cavity; **c** Cavity with an inserted loss material; **d** Critically coupled cavity with inserted small metallic wire above a ferrite disk

transitions between these two resonance behaviors can be demonstrated. Our experiments clearly verify these effects of resonance interactions.

An indirect observation of MDM vortices is possible if eigen-electric moments interact with the cavity electromagnetic fields. In experiments, we used a disk sample of a diameter $2\Re = 3$ mm made of the yttrium iron garnet (YIG) film on the gadolinium gallium garnet (GGG) substrate (the YIG film thickness $d = 50$ mkm, saturation magnetization $4\pi M_0 = 1880$ G, linewidth $\Delta H = 0.8$ Oe) and a short-wall rectangular-waveguide cavity with an entering iris. A normally magnetized ferrite disk was placed in a cavity in a maximal RF electric field of the TE_{102} mode and was oriented normally to the E-field. In Fig. 2, showing the frequency dependence of the cavity reflection coefficient (CRC), we point out three characteristic frequencies used in experiments. Figure 3a shows a typical multiresonance spectrum of an interaction of a MDM ferrite disk with a microwave electric field of a cavity. This is dependence of the absolute value of the CRC on a bias magnetic field obtained for a critically coupled cavity at the resonance frequency $f_1 = 7.085$ GHz. The digits are the MDM numbers. For the cavities with reduced Q-factors and resonant at the same frequency (to preserve the resonance frequency, we used small tuning elements), the character of the ferrite-disk spectrum remains the same (see Figs. 3b, c). Following the above analysis, one can conclude that the spectra in Figs. 3a, b, and c correspond to the resonance repulsion. When we put a small piece of a metallic wire (made of copper) above a ferrite disk and parallel to the cavity E-field, we obtained a very strong interaction between the disk and cavity Fig. 3d. In this case, we can discern two fundamental aspects. First of all, the fact that an additional small capacitive coupling (due to a piece of a nonmagnetic wire) strongly affects on magnetic oscillation proves, once again, the presence of the electric-dipole moments of the MDMs in a quasi-2D ferrite disk. Secondly, we see very unique features in the spectral picture. There are sharp jumps of the CRCs in the regions of the resonance peaks. It can be definitely supposed that these jumps are caused by sharp phase transitions between two 2π-behaviors shown in Figs. 1b and 1c.

To investigate in more details transitions between behaviors of the resonance repulsion and resonance absorption, we analyzed the ferrite disk spectra measured at different frequencies. These frequencies, f_1, f_2, and f_3, correspond to different positions on the resonance curve of the cavity (see Fig. 2). The multiresonance spectral pictures for these frequencies are shown in Fig. 4. Since the permeability tensor parameters are dependent both on frequency and a bias magnetic field, we were able to match the peak position by small variations of a bias field. The fields corresponding to the first peaks are adduced in the figure.

The spectrum in Fig. 4a corresponding to f_1 (and being the same as the spectrum in Fig. 3b) represents the resonance repulsion behavior. At the same time, the spectrum in Fig. 4c corresponding to f_3 clearly demonstrates the resonance absorption behavior. It becomes evident that the spectrum in Fig. 4b corresponding to f_2 shows the transitions between the resonance repulsion and resonance absorption. A qualitative explanation of the observed three cases could be the following. Since at frequency f_1 the cavity is "viewed" by the incoming signal as an active load, one can clearly observe the resonance repulsion due to a ferrite disk. Contrary, at frequency f_3, the cavity is characterized mainly as a reactive load. In this case, one observes the resonance absorption behavior. At frequency f_2, both cases are mixed, and a transitional behavior takes place. It is worth noting that for transitional behaviors shown in Figs. 3d and 4b, the between-peak derivatives of CRCs with respect to the bias field are of different signs.

For the above disk parameters used in experiments, we calculated amplitudes of eigen electric moments $\vec{a}^{e}_{\pm}$ for "orbital" azimuth number $\nu = 1$. The calculation results of the eigen-electric-moment amplitudes are shown in Fig. 5. To compare the calculation results with the experimental ones, we took the measured relative peak amplitudes. We "tied" together the calculated and measured amplitudes of the first-mode peaks and normalized them to unit. Evidently that since the mode peak amplitudes were measured with respect to a bias magnetic field at the constant frequency, we had negligibly small from-mode-to-mode variations of the cavity E-field amplitudes. So the measured relative peak amplitudes should correspond to experimental MDM distributions of the eigen electric moments. Figure 5 shows relatively good agreement between the experimental and calculation results. Some disagreement can be explained by certain inaccuracy in precise experimental characterization of amplitudes of very sharp resonant peaks.

The eigen electric moments of a ferrite disk arise not from the classical curl electric fields of magnetostatic oscillations. On the other hand, any induced electric polarization effects in YIG or GGG materials are beyond the frames of the experimentally observed multiresonance spectra. We discussed a model which gives a possible picture of interaction of the MDM oscillations with external RF electric fields. Our experimental results, shown in this letter, convincingly confirmed the proposed model of the anapole moment oscillations caused by edge chiral rotations in a MDM ferrite disk. We calculated the spectral distribution of the eigen electric (anapole) moments of an MDM ferrite disk. There is good correlation between the calculated and experimental results.

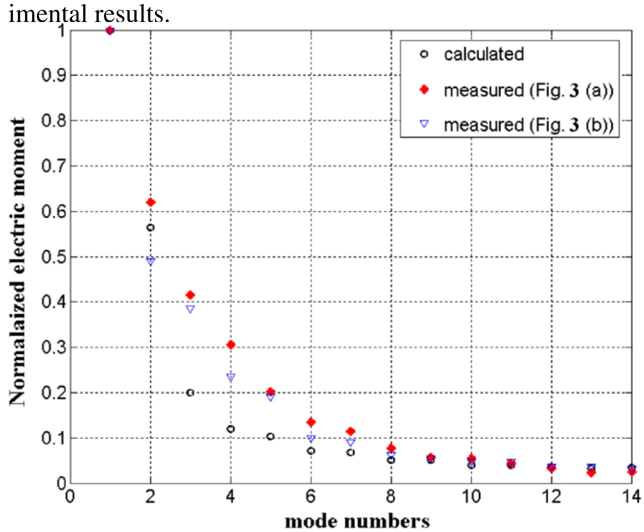

Fig. 5 Calculated and measured the anapole moment amplitudes versus the MDM numbers

The major advancement of the presented investigations presented is an experimental evidence for special interactions between MDM oscillations in a ferrite disk and electromagnetic fields of a microwave cavity. The experiments clearly verify the proposed mechanism of interaction between the double-valued-function MDM edge states and the single-valued-function MDM membrane oscillation. The questions raised in this paper are very important in view of the present strong interest in electromagnetic artificial materials with magnetoelectric properties. The eigen-electric-moment properties of MDM oscillations shown in this paper give new insights into a problem of local microwave ME composites.

References

1. T. Shinjo, T. Okuno, R. Hassdorf, K. Shigeto, T. Ono, Science **289**, 930 (2000)
2. K.Yu. Guslienko, V. Novosad, Y. Otami, H. Shima, K. Fukamichi, Phys. Rev. B **65**, 024414 (2001)
3. J. Miltat, A. Thiaville, Science **298**, 555 (2002)
4. E.O. Kamenetskii, Phys. Rev. E **73**, 016602 (2006)
5. E.O. Kamenetskii, J. Phys. A: Math. Theor. **40**, 6539 (2007)
6. E.O. Kamenetskii, A.K. Saha, I. Awai, Phys. Lett. A **332**, 303 (2004)
7. E.O. Kamenetskii, M. Sigalov, R. Shavit, J. Phys.: Condens. Matter **17**, 2211 (2005)
8. J.F. Dillon Jr., J. Appl. Phys. **31**, 1605 (1960)