

Electric self-inductance of quasi-two-dimensional magnetic-dipolar-mode ferrite disks

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An electric current flowing around a loop produces a magnetic field and hence a magnetic flux through the loop. The ratio of the magnetic flux to the electric current is called the (magnetic) self-inductance. Can there be a dual situation with a magnetic current flowing around a loop and producing an electric field and hence an electric flux through the loop? Following the classical electrodynamics laws an answer to this question should be negative. Nevertheless, special spectral properties of magnetic-dipolar modes in a quasi-two-dimensional ferrite disk show that there are the double-valued-function loop magnetic currents which may produce eigenelectric fields and hence eigenelectric fluxes through the loop. In this case one can definitely introduce a notion of an electric self-inductance as the ratio of the electric flux to the magnetic current. In this paper we show experimentally that in the magnetic-dipolar-mode ferrite disks there exist eigenelectric fluxes. These fluxes are very sensitive to the permittivity parameters of materials abutting to a ferrite disk. Dielectric samples above a ferrite disk with a higher permittivity than air confine the electric field closely outside the ferrite, thereby changing the loop magnetic currents and thus transforming the magnetic-dipolar-mode oscillating spectrum. © 2008 American Institute of Physics.

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I. INTRODUCTION

Effective interactions of electric and magnetic dipoles in solid state structures are often quite different from the fundamental laws of electrodynamics. This gives rise to unusual phenomena interesting both for fundamental physical studies and potential technological applications. A fascinating effect of coupling between electric and magnetic dipoles is observed in multiferroic materials.¹ A control of dielectric properties by magnetism is possible because electric polarization in these materials is induced by magnetic ordering. The symmetry arguments of magnetic inhomogeneity are important factors in such magnetoelectric (ME) effects.² A role of topology of magnetic ordering in the ME effect can become apparent not only in bulk multiferroic structures. It appears very surprisingly that there exist microwave ME effects, which one can observe due to special symmetry properties of magnetic ordering in thin ferrite disks with magnetostatic (MS) [or magnetic-dipolar-mode (MDM)] oscillations.

MS ferromagnetism has a character essentially different from exchange ferromagnetism.^{3,4} This statement finds strong confirmation in confinement phenomena of MDM oscillations. The magnetic dipole interaction provides us with a long-range mechanism of interaction, where a magnetic medium is considered as a continuum. In a general case, both short-range exchange and long-range dipole-dipole (MS) interactions contribute to eigenfrequencies of the collective spin excitation. The importance of MS energy increases gradually as the particle size increases. MDMs are not eigenmodes of magnetization. Contrary to exchange spin waves, in magnetic-dipolar waves the local fluctuations of magneti-

zation do not propagate due to interaction between the neighboring spins. When field differences across the sample become comparable to the bulk demagnetizing fields, the local-oscillator approximation is no longer valid and there should be certain propagating fields—the MS fields—which cause and govern propagation of magnetization fluctuations. In other words, space-time magnetization fluctuations are corollary of the propagating MS fields, but there are no magnetization waves. The boundary conditions should be imposed on the MS field and not on the rf magnetization.

Ferrite MDM samples are well localized in space, their extension is assumed to be much smaller than the variation length of the electromagnetic (EM) field. It is well known that in a general case of small (compared to the free-space EM-wave wavelength) samples made of media with strong temporal dispersion, the role of displacement currents in Maxwell equations can be negligibly small, so oscillating fields are the quasistationary fields. In small samples with strong temporal dispersion of the permeability tensor: $\vec{\mu} = \vec{\mu}(\omega)$, variation of the electric energy is negligibly small compared to variation of the magnetic energy and so one can neglect the electric displacement current in Maxwell equations.⁵ These magnetic samples can exhibit the MS resonance behavior in microwaves.^{6–9} For such resonance MDMs, the quasi-MS fields are described by a scalar MS potential ψ . This potential does not have the same physical meaning as in a situation of pure magnetostatics. In fact, there is the MS-potential wave function $\psi(\vec{r}, t)$ describing the resonant behavior in a small magnetic object.

Can the MS-potential wave function be used for comprehensive description of MDM oscillations in small magnetic samples? In attempts to use the notion of scalar wave function $\psi(\vec{r}, t)$ for the spectral analysis one becomes faced with

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evident contradictions with the dynamical Maxwell equations. This fact can be perceived, in particular, from the remarks made by McDonald.¹⁰ For MS resonances in small magnetic objects one neglects the electric displacement current: $\partial \vec{D}/\partial t = 0$. From the Maxwell–Faraday equation, $\nabla \times \vec{E} = -(1/c)(\partial \vec{B}/\partial t)$, one obtains $\nabla \times (\partial \vec{E}/\partial t) = -(1/c) \times (\partial^2 \vec{B}/\partial t^2)$. If a sample does not possess any dielectric anisotropy, we have $\partial^2 \vec{B}/\partial t^2 = 0$. It follows that the magnetic field in small resonant magnetic objects varies linearly with time. This leads, however, to arbitrary large fields at early and late times, and is excluded on physical grounds. An evident conclusion suggests itself at once: the magnetic (for MS resonances) fields are constant quantities. This contradicts to the fact of temporally dispersive media and any resonant conditions. One becomes faced with a fact that in a case of MS resonances a physical meaning of the Maxwell–Faraday equation is not clearly determined. Concerning the MS-wave propagation effects, it was disputed also in Refs. 11–13 that from a classical electrodynamics point of view one does not have a physical mechanism describing the effect of transformation of the curl electric field to the potential magnetic field. Also the gauge transformation in this derivation does not fall under the known gauge transformations, neither the Lorentz gauge nor the Coulomb gauge, and cannot formally lead to the wave equation.

MS waves can propagate only due to ferrite-medium confinement phenomena.^{14,15} So the power flow density of MS waves should be considered via an analysis of the mode spectral problem in a certain waveguide structure, but not based on an analysis of the wave propagation in a boundless magnetic medium. It means that the MS-wave power flow density should have the only physical meaning as a norm of a certain propagating mode in a magnetic waveguide structure. As a necessary consequence, this leads to the question of orthogonality and completeness of MDMs in such a waveguide. In this problem MS-potential function ψ acquires a special physical meaning as a scalar wave function in a Hilbert functional space. In an analysis of the MDM oscillating spectra, a ferrite disk particle is considered as a section of an axially magnetized ferrite rod. For a flat ferrite disk, having a diameter much bigger than a disk thickness, one can successfully use separation of variables for the MS-potential wave function.^{16,17} A similar way of separation variables is used, for example, in solving the EM-wave spectral problem in dielectric disks.¹⁸ It was shown^{12,13,16,17} that for MDMs in a ferrite disk one has evident quantumlike attributes. The spectrum is characterized by *energy eigenstate* and *eigenelectric fluxes* for oscillating MDMs.

Special spectral properties of magnetic-dipolar modes in a quasi-two-dimensional (2D) ferrite disk reveal unique experimental characteristics of the microwave ME effect. In this paper we give an experimental evidence for eigenelectric fluxes of MDM oscillations. We show strong sensitivity of the MDM spectral characteristics to the dielectric-load permittivity quantity. From experimental results it follows that the oscillating MDMs can be characterized by certain electric self-inductance parameters.

We start with a brief description of such nontrivial no-

tions as energy eigenstates, eigenelectric fluxes, persistent magnetic currents, and anapole moments in MDM ferrite disks. These notions are partly known from the literature. Nevertheless, an initial classification of unique topological effects in the MDM ferrite disks could be very useful for proper characterization of our experimental results.

II. ENERGY EIGENSTATES FOR MDM OSCILLATIONS IN QUASI-TWO-DIMENSIONAL FERRITE DISKS

The statement that confinement phenomena for MS oscillations in a normally magnetized ferrite disk demonstrate typical atomiclike properties of discrete energy levels can be illustrated by an analysis of experimental absorption spectra. The main feature of multiresonance line spectra observed in experiments^{8,9} is the fact that high-order peaks correspond to lower quantities of the bias dc magnetic field. Physically, the situation looks as follows. Let $H_0^{(A)}$, $H_0^{(B)}$ and be, respectively, the upper and lower values of a bias magnetic field corresponding to the borders of a region. We can estimate a total depth of a “potential well” as: $\Delta U = 4\pi M_0(H_0^{(A)} - H_0^{(B)})$, where M_0 is the saturation magnetization. Let $H_0^{(1)}$ be a bias magnetic field, corresponding to the main absorption peak in the experimental spectrum ($H_0^{(B)} < H_0^{(1)} < H_0^{(A)}$). When we put a ferrite sample into this field, we supply it with the energy: $4\pi M_0 H_0^{(1)}$. To some extent, this is a pumping-up energy. Starting from this level, we can excite the entire spectrum from the main mode to the high-order modes. As a value of a bias magnetic field decreases, the “particle” obtains the higher levels of negative energy. One can estimate the negative energies necessary for transitions from the main level to upper levels. For example, to have a transition from the first level $H_0^{(1)}$ to the second level $H_0^{(2)}$ ($H_0^{(B)} < H_0^{(2)} < H_0^{(1)} < H_0^{(A)}$) we need the density energy surplus: $\Delta U_{12} = 4\pi M_0(H_0^{(1)} - H_0^{(2)})$. The situation resembles increasing a negative energy of the hole in semiconductors when it “moves” from the top of a valence band. In a classical theory, negative-energy solutions are rejected because they cannot be reached by a continuous loss of energy. However, in quantum theory, a system can jump from one energy level to a discretely lower one; so the negative-energy solutions cannot be rejected, out of hand. When one continuously varies the quantity of the dc field H_0 , for a given quantity of frequency ω , one sees a discrete set of absorption peaks. It means that one has the discrete-set levels of a potential energy. The line spectra appear due to the quantumlike transitions between energy levels of a ferrite disk-form particle.

The energy eigenvalue problem is defined by the differential equation

$$\hat{F}_\perp \tilde{\varphi}_p = E_p \tilde{\varphi}_p, \quad (1)$$

where $\tilde{\varphi}_p$ is a dimensionless membrane MS-potential wave function and E_p is the normalized average (on the rf period) density of accumulated magnetic energy of mode p . A 2D (“in-plane”) differential operator $\hat{F}_\perp$ in Eq. (1) is defined as

$$\hat{F}_\perp = \frac{g_p}{16\pi} \mu \nabla_\perp^2, \quad (2)$$

where $\nabla_\perp^2$ is the 2D (with respect to cross-sectional coordinates) Laplace operator, μ is a diagonal component of the permeability tensor, and g_p is a dimensional normalization coefficient for mode p . Operator $\hat{F}_\perp$ is a positive definite differential operator for negative quantity μ . The energy density of mode p is determined as

$$E_p = \frac{g_p}{16\pi} (\beta_{z_p})^2, \quad (3)$$

where β_{z_p} is the propagation constant of mode p along disk axis z . For MDMs in a ferrite disk at a constant frequency one has the energy orthonormality condition

$$(E_p - E_{p'}) \int_S \tilde{\varphi}_p \tilde{\varphi}_{p'}^* dS = 0, \quad (4)$$

where S is a cylindrical cross section of an open disk. For a constant frequency, there are different energies of MDMs at different quantities of a bias magnetic field. From the principle of superposition of states, it follows that wave functions $\tilde{\varphi}_p$ ($p=1, 2, \dots$), describing our “quantum” system, are “vectors” in an abstract space of an infinite number of dimensions—the Hilbert space. In quantum mechanics, this is the case of so-called energetic representation, when the system energy runs through a discrete sequence of values.^{19,20}

It was shown, however, that because of the boundary condition on a lateral surface of a ferrite disk, membrane functions $\tilde{\varphi}_p$ cannot be considered as single-valued functions.^{12,13} There is a certain phase factor in $\tilde{\varphi}$ on the border ring region. This fact raises a question about validity of the energy orthogonality relation for the MDMs. In order to cancel the “edge anomaly,” the boundary excitation must be described by chiral states. Because of such chiral states, the double-valued-function loop magnetic currents occur. These currents may produce eigenelectric fields and hence eigenelectric fluxes through the loop.^{12,13} In this case one can definitely introduce the notion of the electric self-inductance as the ratio of the electric flux to the magnetic current.

III. EIGENELECTRIC FLUXES, PERSISTENT MAGNETIC CURRENTS, AND ANAPOLE MOMENTS IN MDM FERRITE DISKS

The topological effects in the MDM ferrite disk are manifested through the generation of relative phases which accumulate on the *boundary* wave functions $\delta_\pm$.^{12,13}

$$\delta_\pm = f_\pm e^{-iq_\pm \theta}, \quad (5)$$

where θ is an azimuth coordinate in a cylindrical coordinate system. The quantities $q_\pm$ are equal to $\pm l/2$, $l=1, 3, 5, \dots$. For amplitudes f we have $f_+ = -f_-$ with normalization $|f_\pm| = 1$. To preserve the single-valued nature of the membrane functions of the MDM oscillations, functions $\delta_\pm$ must change its sign when a disk angle coordinate θ is rotated by 2π so that $e^{-iq_\pm 2\pi} = -1$. A sign of a full chiral rotation, $q_+ \theta = \pi$ or $q_- \theta = -\pi$, should be correlated with a sign of the pa-

rameter $i\mu_a$ —the off-diagonal component of the permeability tensor $\tilde{\mu}$. This becomes evident from the fact that a sign of $i\mu_a$ is related to a precession direction of a magnetic moment $\vec{m}$. In a ferromagnetic resonance, the bias field sets up a preferential precession direction. It means that for a normally magnetized ferrite disk with a given direction of a dc bias magnetic field, there are two types of resonant oscillations, which we conventionally designate as the (+) resonance and the (−) resonance. For the (+) resonance, a direction of an edge chiral rotation coincides with the precession magnetization direction, while for the (−) resonance, a direction of an edge chiral rotation is opposite to the precession magnetization direction.

For a ferrite disk with r and θ in-plane coordinates, the total MS-potential membrane function $\tilde{\varphi}$ is represented as a product of two functions:^{12,13}

$$\tilde{\varphi} = \tilde{\eta}(r, \theta) \delta_\pm, \quad (6)$$

where $\tilde{\eta}(r, \theta)$ is a single-valued membrane function and $\delta_\pm$ is a double-valued edge (spin-coordinate-like) function. We may introduce a “spin variable” σ , representing the orientation of the “spin moment” and two double-valued wave functions, $\delta_+(\sigma)$ and $\delta_-(\sigma)$. The two wave functions are normalized and mutually orthogonal, so that they satisfy the equations $\int \delta_+^2(\sigma) d\sigma = 1$, $\int \delta_-^2(\sigma) d\sigma = 1$, and $\int \delta_+(\sigma) \delta_-(\sigma) d\sigma = 0$. A membrane wave function $\tilde{\varphi}$ is then a function of three coordinates, two positional coordinates such as r , θ , and the “spin coordinate” σ . For the positional wave function $\tilde{\eta}(r, \theta)$, which is a solution of the Walker equation for a ferrite disk with the so-called essential boundary conditions,¹⁷ there could be two equiprobable solutions for the membrane wave functions: $\tilde{\varphi}_+ = \tilde{\eta}(r, \theta) \delta_+(\sigma)$ and $\tilde{\varphi}_- = \tilde{\eta}(r, \theta) \delta_-(\sigma)$.

The geometrical phase factor is not single-valued under continuation around a circuit and can be correlated with the vector potential. The vector potential is considered to be non-observable in Maxwellian electromagnetism. At the same time, the vector potential can be observable in the Aharonov–Bohm²¹ or Aharonov–Casher²² effects, but only via its line integral, not pointwise. From an analysis in^{12,13} it follows that restoration of single valuedness (and, therefore, hermicity) of the MDM spectral problem in a ferrite disk is due to a line integral $\oint_C (i\vec{\nabla}_\theta \delta_\pm)(\delta_\pm^*)^* dC$, where $C = 2\pi\Re$ is a contour surrounding a cylindrical ferrite core, $\Re$ is a ferrite disk radius, and $\delta_\pm^0$ are the boundary wave functions for the conjugate problem. Such a line integral is an observable quantity which can be represented as an integral of a certain vector potential:

$$i\Re \int_0^{2\pi} [(\vec{\nabla}_\theta \delta_\pm)(\delta_\pm^*)^*] d\theta_{r=\Re} \equiv \oint_C (\vec{A}_\theta^m)_\pm \cdot d\vec{C} = 2\pi q_\pm. \quad (7)$$

One can see that in the problem under consideration the Berry’s phase²³ is generated from the broken dynamical symmetry. To compensate for sign ambiguities and thus to make wave function single valued we added a vector-potential-type term to the MS-potential Hamiltonian. A circulation of vector $\vec{A}_\theta^m$ should enclose a certain flux. The corresponding

flux of pseudoelectric field $\vec{\Xi}$ (the gauge field) through a circle of radius $\mathfrak{R}$ is obtained as

$$\int_S (\vec{\Xi})_{\pm} \cdot d\vec{S} = \oint_C (\vec{A}_{\theta}^m)_{\pm} \cdot d\vec{C} = (\Xi^e)_{\pm}, \quad (8)$$

where $(\Xi^e)_{\pm}$ is the dimensionless flux of pseudoelectric field. There should be the positive and negative fluxes. These different-sign fluxes should be inequivalent to avoid the cancellation. Superscript m in the vector-potential term means that there is the physical quantity associated with the magnetization motion. A magnetic moment moving along contour C in the gauge field feels no force and undergoes the Aharonov–Bohm-type interference effect.

For every MDM, the total solution of the MS-potential wave function in a cylindrical ferrite disk is represented as

$$\psi(r, \theta, z) = \xi(z) \tilde{\varphi}(r, \theta), \quad (9)$$

while a single-valued membrane function is represented as

$$\tilde{\eta}(r, \theta) = R(r) \phi(\theta). \quad (10)$$

In these equations $\xi(z)$ describes an axial distribution of the MS-potential wave function, $R(r)$ is described by the Bessel functions and $\phi(\theta) \sim e^{-i\nu\theta}$, $\nu = \pm 1, \pm 2, \pm 3, \dots$

Based on Eqs. (7)–(10) and taking into account an analysis of self-adjointness of the MDM differential operators,^{12,13} we can represent the total flux of pseudoelectric field for every MDM in a ferrite disk as

$$\Xi_{\text{total}}^e = 2\pi q_{\pm} R_{r=\mathfrak{R}}^2 \int_0^d \xi(z) dz, \quad (11)$$

where d is a disk thickness.

In the topological effects of the generation of relative phases which accumulate on the boundary wave function $\delta_{\pm}$, the quantity $\nabla_{\theta} \delta_{\pm}$ can be considered as the velocity of an irrotational “border” flow:

$$(\vec{v}_{\theta})_{\pm} \equiv \vec{\nabla}_{\theta} \delta_{\pm}. \quad (12)$$

In such a sense, functions $\delta_{\pm}$ are the velocity potentials. Circulation of $(\vec{v}_{\theta})_{\pm}$ along a contour C is equal to $\oint_C (\vec{v}_{\theta})_{\pm} \cdot d\vec{C} = \mathfrak{R} \int_0^{2\pi} \nabla_{\theta} \delta_{\pm} d\theta = -2f_{\pm}$. Taking into account the “orbital” function $\phi(\theta)$, we may consider the quantity $[\vec{\nabla}_{\theta}(\phi \delta_{\pm})]_{r=\mathfrak{R}}$ as the total (orbital and “spin”) velocity of an irrotational border flow:

$$(\vec{V}_{\theta})_{\pm} \equiv [\vec{\nabla}_{\theta}(\phi \delta_{\pm})]_{r=\mathfrak{R}}. \quad (13)$$

It is evident that

$$(\vec{V}_{\theta})_{\pm} = -i \frac{(v + q_{\pm}) f_{\pm}}{\mathfrak{R}} e^{-i(v+q_{\pm})\theta} \vec{e}_{\theta}. \quad (14)$$

For a given membrane function $\tilde{\eta}$ and given z distribution of the MS potential, $\xi(z)$, we can define now the strength of a vortex of a whole disk as

$$\begin{aligned} s_{\pm}^e &\equiv R_{r=\mathfrak{R}} \int_0^d \xi(z) dz \oint_C (\vec{V}_{\theta})_{\pm} \cdot d\vec{C} \\ &= \mathfrak{R} R_{r=\mathfrak{R}} \int_0^d \xi(z) dz \int_0^{2\pi} (\vec{V}_{\theta})_{\pm} \cdot \vec{e}_{\theta} d\theta \\ &= -2f_{\pm} R_{r=\mathfrak{R}} \int_0^d \xi(z) dz. \end{aligned} \quad (15)$$

The quantity $(\vec{V}_{\theta})_{\pm}$ has a clear physical meaning. In the spectral problem for MDM ferrite disks, non-single-valuedness of the MS-potential wave function appears due to a border term which arises from the demand of conservation of the magnetic flux density on a lateral surface of a disk.^{12,13} This border term is defined as $-i\mu_a(H_{\theta})_{r=\mathfrak{R}}$, where μ_a is the off-diagonal component of the permeability tensor and $(H_{\theta})_{r=\mathfrak{R}}$ is an annular magnetic field on the border circle. It is evident that

$$((\vec{H}_{\theta}(z))_{\pm})_{r=\mathfrak{R}} = -\xi(z)(\vec{V}_{\theta})_{\pm}. \quad (16)$$

We define now an angular moment: $\vec{a}_{\pm}^e$

$$a_{\pm}^e \equiv \int_0^d \oint_C [-i\mu_a(H_{\theta})_{r=\mathfrak{R}}] \vec{e}_{\theta} \cdot d\vec{C} dz = i\mu_a s_{\pm}^e. \quad (17)$$

This angular moment can be formally represented as a result of a circulation of a quantity, which we call a density of an effective boundary magnetic current $\vec{i}^m$:

$$a_{\pm}^e = 4\pi \int_0^d \xi(z) dz \oint_C \vec{i}_{\pm}^m \cdot d\vec{C}, \quad (18)$$

where

$$\vec{i}_{\pm}^m \equiv \rho^m (\vec{V}_{\theta})_{\pm} \quad (19)$$

and

$$\rho^m \equiv i \frac{\mu_a}{4\pi} R_{r=\mathfrak{R}}. \quad (20)$$

In our continuous-medium model, a character of the magnetization motion becomes apparent via the gyration parameter μ_a in the boundary term for the spectral problem. There is the magnetization motion through a nonsimply connected region. On the edge region, the chiral symmetry of the magnetization precession is broken to form a flux-closure structure. The edge magnetic currents can be observable only via its circulation integrals, not pointwise. This results in the moment oriented along a disk normal. It was shown experimentally²⁴ that such a moment has a response in an external rf electric field. This clarifies a physical meaning of a superscript e in designations of $s_{\pm}^e$ and $a_{\pm}^e$.

An electric moment $a_{\pm}^e$ is characterized by the anapole-moment properties. This is a certain-type toroidal moment. Some important notes should be given here to describe the properties of moment $\vec{a}^e$. From a classical consideration it follows that for a given electric current $\vec{i}^e$, a magnetic dipole moment is described as $\vec{m} = 1/2c \int \vec{r} \times \vec{i}^e dv$, while the toroidal dipole moment is described as $\vec{\tau} = 1/3c \int \vec{r} \times (\vec{r} \times \vec{i}^e) dv$ (see

e.g., Ref. 25). When we introduce the notion of an elementary magnet: $\vec{M} \equiv \vec{r} \times \vec{i}^e$, we can represent the toroidal dipole moment as a linear integral around a loop: $\vec{i} = 1/3c \oint \vec{r} \times \vec{M} d\ell$. It is considered as a ring of elementary magnets $\vec{M}$. In this formulation, it is clear that a toroidal moment is parity odd and time reversal odd. In a case when $\vec{M}$ is time varying, one has a magnetic current $\vec{i}^m \sim (\partial \vec{M} / \partial t)$ and a linear integral of this current around a loop defines a moment which is parity odd and time reversal even. This is the case of an anapole moment $\vec{a}^e$, which has the symmetry of an electric dipole. From a classical point of view such a definition presumes no azimuth variations of loop magnetic current i^m . In our case, however, for oscillating MDMs one has the azimuth varying ring magnetic current. The magnetic current i^m is described by the double-valued functions. This results in appearance of an anapole moment $\vec{a}^e$.

IV. CHARACTERIZATION OF INTERACTION OF MDMS WITH THE EXTERNAL ELECTROMAGNETIC FIELDS

Microwave experiments^{6,8,9} give evidence for the multi-resonance MDM oscillations in ferrite disks. The absorption peak positions corresponding to these MDMs depend on the ferrite material parameters and the disk geometry, but not on the type of a cavity. This statement was confirmed more in details in an experimental paper.²⁴

The mechanism of excitation of the multi-resonance MDM spectra by the cavity EM fields is, however, not so obvious. Following the theory in Refs. 11–13 one sees that the main factor of an interaction of MDMS with the cavity EM fields is caused by the presence of topological singularities on a lateral surface of a ferrite disk. These topological singularities are described by the double-valued functions and the main aspects concern the question how one can describe effective resonance interactions between the double-valued-function edge states and single-valued-function cavity EM fields. The mechanism of such an interaction we illustrate here by the following qualitative models.

Suppose that we have the (+) resonance which is characterized by azimuth number $q = +\frac{1}{2}$. Figure 1(a) shows the double-valued functions $\delta_+(\theta')$ and $\nabla_{\theta'} \delta_+(\theta')$. Here we use designation θ' to distinguish the spin angular coordinate from a regular angle coordinate θ . Because of the edge-function chiral rotation,^{12,13} for the (+) resonance one has to select only positive derivatives: $(\partial \delta_+ / \partial \theta') > 0$. The corresponding parts of the graphs are distinguished in Fig. 1(a) by bold lines. Figures 1(b) and 1(c) give two cases of the single-valued-function cavity field $F(\theta)$ which may lead to resonance interactions with the double-valued-function edge state. It is evident that in a case of Fig. 1(b), positive halves of function $\nabla_{\theta'} \delta_+(\theta')$ are phased for a resonance interaction with some positive halves of function $F(\theta)$, while in a case of Fig. 1(c) positive halves of function $\nabla_{\theta'} \delta_+(\theta')$ are phased for a resonance interaction with some negative halves of function $F(\theta)$. An interaction with the cavity field shown in Fig. 1(b) can be characterized as the “resonance absorption” while an interaction with the cavity field shown in Fig. 1(c)—as the “resonance repulsion.” Both types of interac-

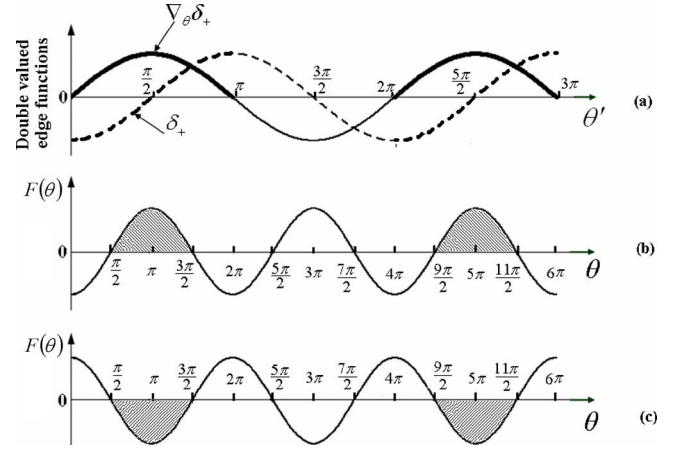


FIG. 1. (Color online) Interaction of the double-valued-function edge state with the single-valued-function cavity EM fields in a case of the (+) resonance. (a) Double-valued edge function; (b) the cavity field at the resonance absorption interaction; (c) the cavity field at the resonance repulsion interaction.

tions are equiprobable and can be exhibited separately. One may also expect that in a certain situation, transition between these two resonance behaviors can be demonstrated. A similar model can be used for illustration of a possible mechanism of interactions in a case of the (−) resonance. For $q = -\frac{1}{2}$, the double-valued functions $\delta_-(\theta')$ and $\nabla_{\theta'} \delta_-(\theta')$ are shown in Fig. 2(a). Here we have to select only negative derivatives: $(\partial \delta_- / \partial \theta') < 0$. In this case, an interaction with the cavity field shown in Fig. 2(b) can be characterized as the resonance absorption and an interaction with the cavity field shown in Fig. 2(c)—as the resonance repulsion.

An interaction of the anapole moment of a ferrite disk with the cavity rf electric field was clearly demonstrated in an experimental paper.²⁴ Following the model shown in Figs. 1 and 2 one can describe this interaction by the average quantity $1/2 \pi \int_0^{2\pi} (\vec{a}^e \cdot \vec{E}) d\theta$, where $\vec{E}$ is the cavity electric field. The positive quantity of this integral corresponds to the

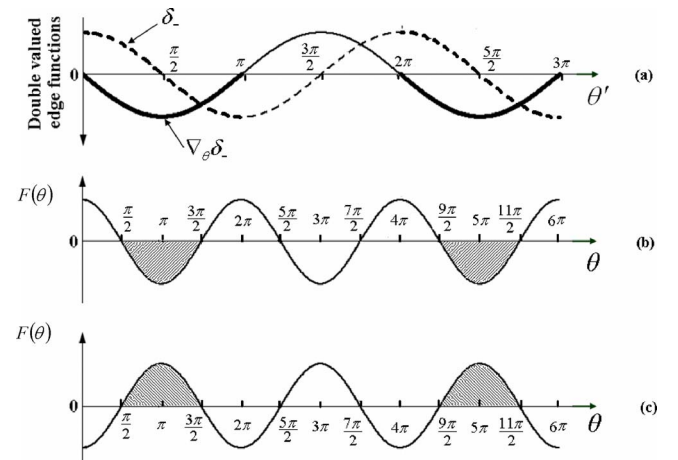


FIG. 2. (Color online) Interaction of the double-valued-function edge state with the single-valued-function cavity EM fields in a case of the (−) resonance. (a) Double-valued edge function; (b) the cavity field at the resonance absorption interaction; (c) the cavity field at the resonance repulsion interaction.

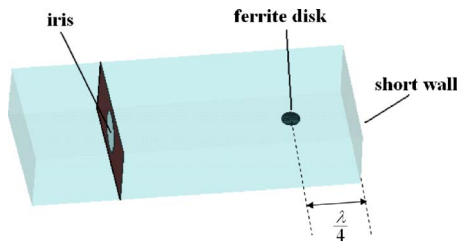


FIG. 3. (Color online) Rectangular cavity with an inserted ferrite disk. A bias magnetic field is normal to a disk.

resonance absorption while for a negative integral one has the resonance repulsion. These two behaviors we now demonstrate in the following experiments.

In experiments, we used a disk sample of a diameter $2R=3$ mm made of the yttrium iron garnet (YIG) film on the gadolinium gallium garnet (GGG) substrate (the YIG film thickness $d=49.6$ μm , saturation magnetization $4\pi M_0=1880$ G, and linewidth $\Delta H=0.8$ Oe; the GGG substrate thickness is 0.5 mm). A normally magnetized ferrite disk sample was placed in a rectangular waveguide cavity with the TE_{102} resonant mode. The disk axis was oriented along the waveguide E-field (see Fig. 3). We analyzed the MDM spectra for the dc magnetic field variation at certain constant frequencies. To investigate behaviors of the resonance repulsion and resonance absorption and transition between these behaviors we analyzed the ferrite disk spectra measured at different frequencies. The multiresonance spectral pictures are shown in Fig. 4(a). These frequencies, f_1 , f_2 , and f_3 , correspond to different positions on the resonance curve of the cavity [see Fig. 4(b)]. The spectral peaks of the MDM oscillations one obtains at certain permeability tensor parameters.^{13,17} Since the permeability tensor parameters are dependent both on frequency and a bias magnetic field, we are able to match the peak position by small variations of a bias field. The bias magnetic fields corresponding to the first peaks are adduced in Fig. 4(a). The digits characterize the MDM numbers.

In Fig. 4(a), the spectrum corresponding to f_1 represents the resonance repulsion behavior. At the same time, the spectrum corresponding to f_3 , clearly demonstrates the resonance absorption behavior. It becomes evident that the spectrum corresponding to f_2 , shows the transitions between the resonance repulsion and resonance absorption. A qualitative explanation of the observed three cases could be the following. Since at frequency f_1 the cavity is “viewed” by the incoming signal as an active load, one can clearly observe the resonance repulsion due to a ferrite disk. Contrary, at frequency f_3 the cavity is characterized mainly as a reactive load. In this case one observes the resonance absorption behavior. At frequency f_2 both cases are mixed and a transitional behavior takes place.

The main conclusion following from the above consideration is that there exists a real mechanism of the effective resonance interactions between the double-valued-function edge states of MDMs and single-valued-function EM fields. The character of the MDM resonance responses may depend on the EM field structure but the resonance peak positions

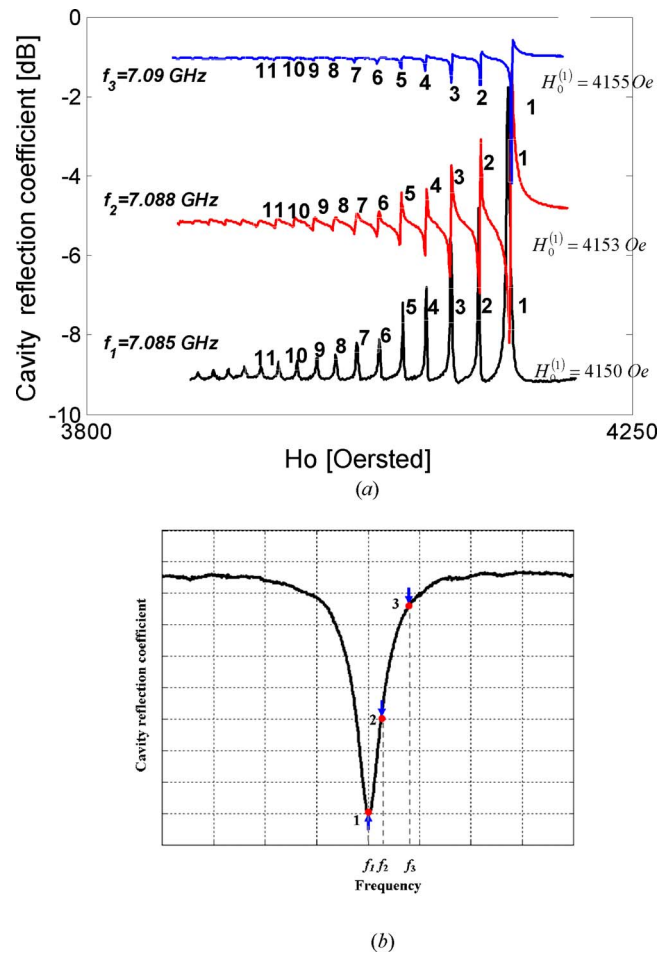


FIG. 4. (Color online) Experimental evidence for the resonance repulsion, resonance absorption, and transitional behaviors. (a) The multiresonance spectral pictures for the resonance repulsion (at frequency f_1), resonance absorption (at frequency f_3), and transitional behavior (at frequency f_2); (b) frequencies f_1 , f_2 , and f_3 on the resonance curve of the cavity.

are fixed for a given ferrite disk. These results will allow us to analyze the electric self-inductance properties of MDM ferrite disks.

V. EXPERIMENTAL EVIDENCE FOR THE ELECTRIC SELF-INDUCTANCE PROPERTIES OF MDM FERRITE DISKS

In the above experiments, we analyzed the MDM excitation by the cavity fields as a result of interaction of the eigenelectric (anapole) moments with the external rf electric fields. An anapole moment is created by a loop magnetic current. It can be supposed that an equivalent alternative mechanism of excitation which describes an induction of the loop magnetic current by the time varying electric-field flux can also be used. A dual effect of induction of an electric displacement current in a dielectric-disk resonator due to the Faraday law was analyzed in Ref. 26. The physics of this effect, however, is completely different from our case.

First of all, let us define the notion of the electric self-inductance for MDMs. For every MDM in a ferrite disk, we define the electric self-inductance as the ratio of the electric flux to the loop magnetic current. Based on Eq. (11) we express the eigenelectric flux for a given mode p as

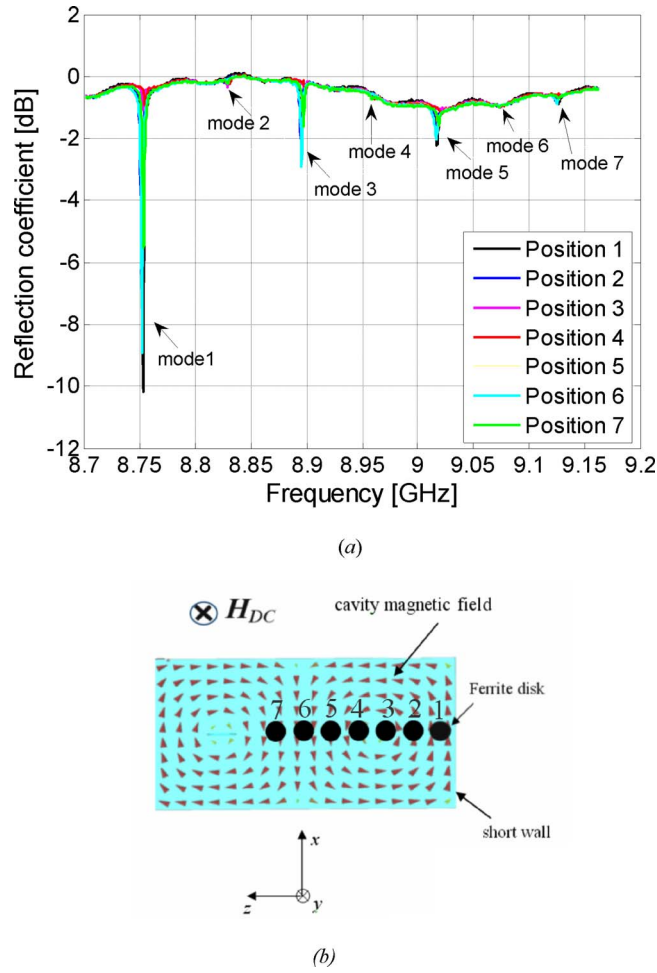


FIG. 5. (Color online) The spectra of a ferrite disk without dielectric loading. (a) The MDM spectra for different disk positions; (b) disk positions in a waveguide.

$$(\Xi_{\text{total}}^e)_p = 2\pi q_{\pm} (R_{r=\mathfrak{R}}^2)_p \int_0^d \xi_p dz. \quad (21)$$

Following Fig. 1 or Fig. 2 and based on Eqs. (19) and (20), the time average magnetic current for mode p is expressed as

$$(i_{\text{average}}^m)_p = i \frac{(\mu_a)_p}{4\pi} (R_{r=\mathfrak{R}})_p \frac{1}{\pi} \int_0^\pi \nabla_{\theta'} \delta d \theta' = i \frac{(\mu_a)_p}{2\pi^2} (R_{r=\mathfrak{R}})_p. \quad (22)$$

We define the electric self-inductance for a given mode p as

$$(L^e)_p = \frac{(\Xi_{\text{total}}^e)_p}{(i_{\text{average}}^m)_p} = \frac{4\pi^3}{(\mu_a)_p} (R_{r=\mathfrak{R}})_p q_{\pm} \int_0^d \xi_p dz. \quad (23)$$

One may presuppose that this is a constant quantity for every MDM in a ferrite disk.

For experimental evidence of the electric self-inductance properties of MDM ferrite disks we suggest the following measurements. Suppose that we put a dielectric sample above a ferrite disk. Because of the electric flux Ξ^e originated from a ferrite disk, the electric charges will be induced on surfaces of a dielectric sample. These surface charges create their own electric flux which will pierce a ferrite disk in an opposite direction with respect to the electric flux Ξ^e .

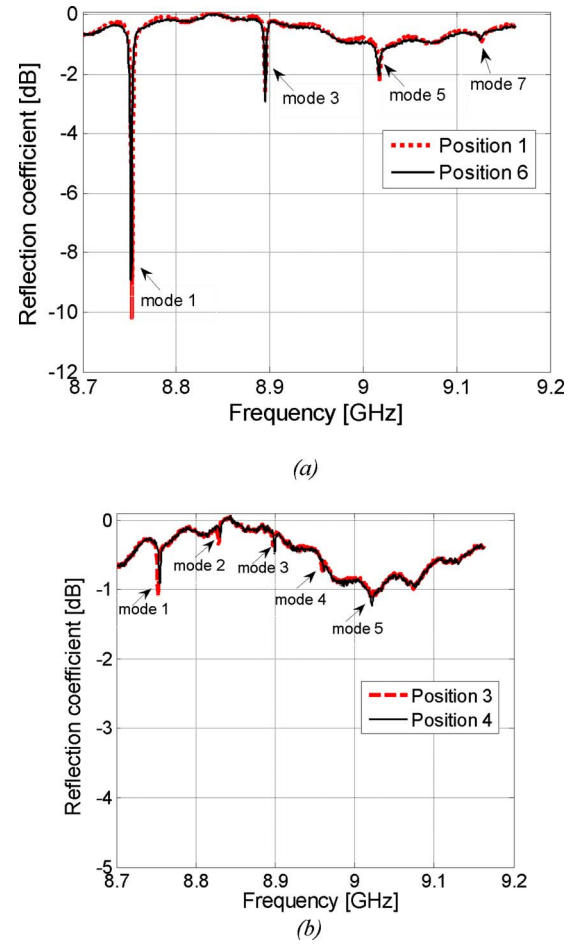


FIG. 6. (Color online) The spectra of a ferrite disk without dielectric loading (detailed spectral pictures). (a) The MDM spectra for positions numbered 1 and 6; (b) the MDM spectra for positions numbered 3 and 4.

As a result of the reduction in the total electric flux, the loop magnetic current in a ferrite disk will be reduced as well, preserving a constant quantity of L^e for a given mode p :

$$(L^e)_p = \frac{(\Xi_{\text{total}}^e)_p'}{(i_{\text{average}}^m)_p'}. \quad (24)$$

Here $(\Xi^e)_p'$ and $(i_{\text{average}}^m)_p'$ are, respectively, the total electric flux piercing a ferrite disk and the loop magnetic current corresponding to mode p when a dielectric sample is put on. For a given mode structure, the requirement of constancy of quantity L^e will lead to a shift of the MDM spectra to lower quantities of a modulus of μ_a [see Eq. (22)]. In experiments, this should give an evidence for shift of the MDM position in the spectrum to a lower bias field (at a constant frequency) or to a higher frequency (at a constant bias field). Obviously, this shift will be dependent on the permittivity parameter of a dielectric sample. For larger permittivity parameter ϵ_r of a dielectric sample, the shift should be larger as well. Moreover, since for a given geometry of a dielectric sample, the flux $(\Xi^e)_p'$ will abate linearly as ϵ_r increases and the current $(i_{\text{average}}^m)_p'$ decreases linearly as a modulus of μ_a decreases, there should be a linear dependence between ϵ_r and a modulus of μ_a for preserving constant quantity $(L^e)_p$. In our experiments, we clearly observe this effect for enough small quantities ϵ_r .

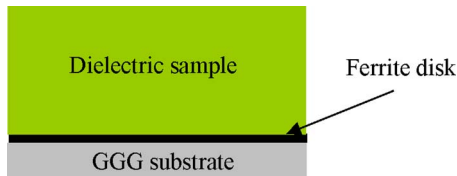


FIG. 7. (Color online) A ferrite disk on a GGG substrate with a dielectric sample.

Contrary to the cavity structure shown in Fig. 3, in these experiments we will use a short-wall rectangular waveguide without an entering iris. In this case we can analyze the MDM spectra with respect to the frequency at a constant dc magnetic field. We work at a bias magnetic field $H_0 = 4900$ Oe. We start with an analysis of the spectra of a ferrite disk without any dielectric loading at different disk positions in a waveguide. The forms of the MDM resonance peaks may be dependent on the EM field structure and, certainly, the effectiveness of excitation of the modes (the MDM amplitudes) can be different for different disk positions in a waveguide. At the same time, as we discussed above, the resonance peak positions should be fixed for given parameters of a ferrite disk. Figure 5(a) shows the MDM spectra for different disk positions in a waveguide. These positions are clarified in Fig. 5(b). In Figs. 6(a) and 6(b) one can see more detailed spectral pictures at the positions numbered 1, 6 and 3, 4, respectively. A shift of the disk may slightly perturb the cavity field structure. This perturbation, however, does not influence on the resonance peak positions. In fact, one can see that the peak positions in the MDM spectrum are independent on the EM field structure in a waveguide.

A ferrite disk is placed on a GGG substrate which has the dielectric permittivity parameter of $\epsilon_r = 15$. Now we put additional dielectric samples above a ferrite disk (see Fig. 7). There are dielectric disks of a diameter 3 mm and thickness 2 mm with different dielectric permittivity parameters. We used a set of disks of commercial microwave dielectric (non-magnetic) materials with the dielectric permittivity parameters of $\epsilon_r = 3.4$ (RO4003; Rogers Corporation), $\epsilon_r = 6.15$ (RO3006; Rogers Corporation), $\epsilon_r = 15$ (K-15; TCI Ceramics

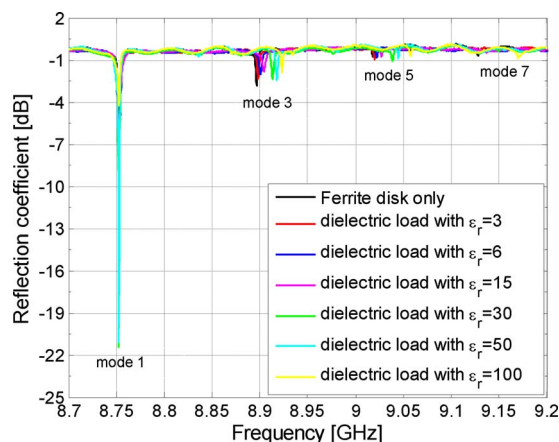


FIG. 8. (Color online) The frequency shift of the mode peak positions for a ferrite disk with dielectric loading. The first peaks in the spectra are matched by proper correlations of bias magnetic fields.

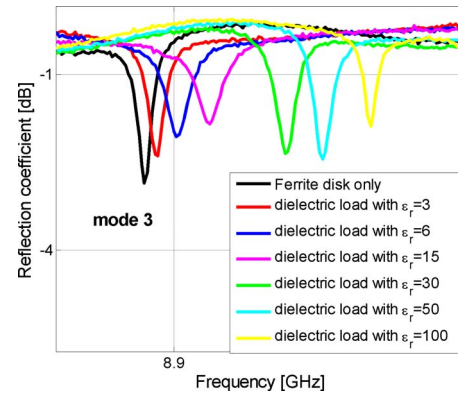


FIG. 9. (Color online) The frequency shift of the peak positions of mode 3 when a ferrite disk is loaded by dielectric samples.

Inc.), $\epsilon_r = 30$ (K-30; TCI Ceramics Inc.), $\epsilon_r = 50$ (K-50; TCI Ceramics Inc.), and $\epsilon_r = 100$ (K-100; TCI Ceramics Inc.). It can be supposed that an insertion of dielectric samples changes the cavity field structure but, following the results shown in Fig. 5, this field variation should not influence on the absorption peak position. We, however, observed strong variations of the spectral pictures when dielectric disks were placed above a ferrite disk. These transformations of the spectra become most evident when we match (by proper small shifts of the bias magnetic fields) positions of the first peaks in the spectra. From Fig. 8 one can see that as the dielectric permittivity parameter of a dielectric sample increases, the frequency shift of the mode peak position increases as well.

More detailed pictures are shown in Figs. 9–11 for modes 3, 5, and 7, respectively. It is evident that for a given mode (with the mode number more than one) the peak shifts are small compared to the frequency distances between the peak of this mode and the first mode in the oscillating spectra. In a presupposition that the peak shifts are very small one can use the perturbation method assuming that the mode spectral portrait does not change under the perturbation. In this case we can use Eq. (23) for the self-inductance characterization.

To analyze the electric self-inductance properties we have to investigate relations between parameters ϵ_r and μ_a for given modes. Figure 12 shows dependences between ϵ_r and a modulus of μ_a for modes 3, 5, and 7. The quantities μ_a

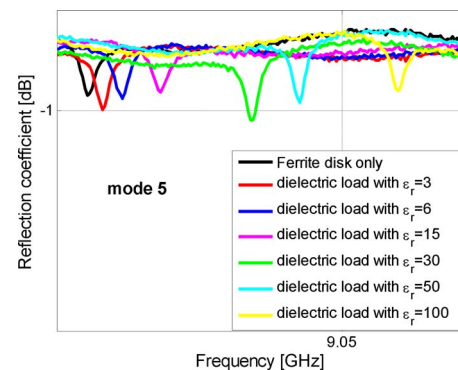


FIG. 10. (Color online) The frequency shift of the peak positions of mode 5 when a ferrite disk is loaded by dielectric samples.

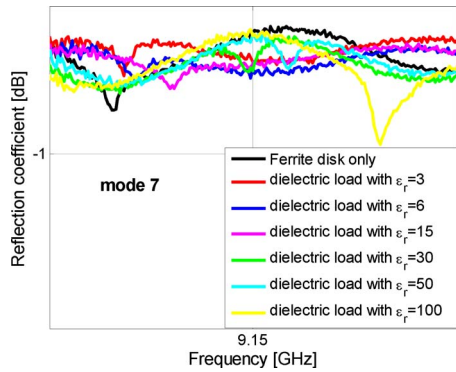


FIG. 11. (Color online) The frequency shift of the peak positions of mode 7 when a ferrite disk is loaded by dielectric samples.

we calculated for given parameters of a ferrite material, a given bias magnetic field, and for frequencies corresponding to the mode peak positions in Figs. 9–11. It is worth noting that for enough small quantities ϵ_r of a dielectric sample (for the permittivity less than 30 in our studies) we have almost linear character of dependence between ϵ_r and μ_a . For such small permittivity parameters, the Eq. (24) can be applicable.

VI. CONCLUSION

In this paper we give an experimental evidence for strong coupling between the magnetization characteristics of a ferrite disk and the polarization properties of loading di-

electric samples. Due to special symmetry properties of magnetic ordering in thin ferrite disks with MDM oscillations there exist eigenelectric fluxes. These fluxes are very sensitive to the permittivity parameters of materials abutting to the ferrite disk. Dielectric samples above a ferrite disk with a higher permittivity than air confine the electric field closely outside the ferrite, thereby changing the loop magnetic currents and thus transforming the MDM oscillating spectrum.

Our results show that external rf electric fields can induce loop magnetic currents in MDM ferrite disks. To a certain extent, one can consider this as a dual case with respect to an induction of loop electric currents in dielectric-disk resonators. The physics of these two effects, however, is completely different. In this paper we analyzed the mechanism of interaction of MDMs in a quasi-2D ferrite disk with the external rf fields. This is a special mechanism of interaction between the double-valued and single-valued functions. The double-valued functions describe the MDM geometrical phase factor on the lateral boundary of a ferrite disk, while the single-valued functions describe the cavity EM oscillations. We showed that due to this mechanism one has an evidence for the electric self-inductance properties of MDM ferrite disks.

It becomes evident that in the MDM oscillating problem, the permittivity parameters of a dielectric layer abutting to the ferrite are the material parameters that influence on the spectral characteristics of MDM oscillations. At the same

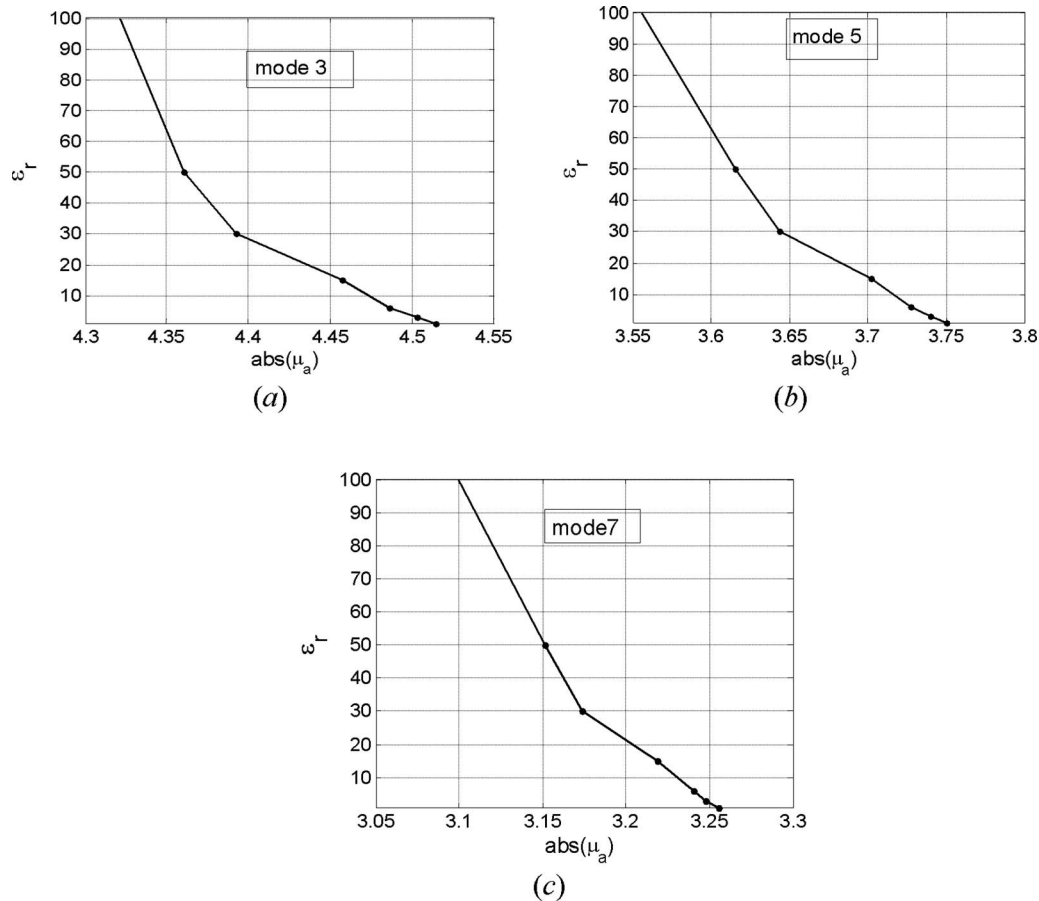


FIG. 12. Dependences between ϵ_r and a modulus of μ_a for modes 3, 5 and 7.

time, an additional dielectric loading is a small perturbation parameter for the original spectral characteristics.

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