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Determination of the Pole-Singularity Order in Spectral MoM Formulations for Source-Free Planar Periodic Structures

Y. Kaganovsky and R. Shavit

***Abstract*—The order of the pole singularities encountered in spectral method of moments formulations for source-free periodic problems is investigated. The solution of the source-free problem is often obtained by searching for the zeros of the Z matrix determinant using an iterative algorithm. During this process, pole singularities of the determinant are encountered and may cause numerical instability. In order to cancel the poles, their order must be known. A rigorous proof of the pole singularity order in the Z matrix determinant is given. The proof is general and holds for any problem which is periodic in at least one of the spatial directions. This knowledge enables to cancel the poles by an appropriate fixed factor with a simple routine.**

***Index Terms*—Periodic structures, pole-singularity, spectral method of moments.**

I. INTRODUCTION

The spectral method of moments (MoM) formulations for planar periodic structures usually employ the spectral Green's function of the background structure. As a result, pole-singularities corresponding to the modal solutions of the background structure and branch-point singularities are introduced into the Z matrix and its determinant. In planar stratified structures, the spectral Green's function can be derived analytically with the use of a transmission line model and the locations of its poles can be found numerically by searching for the zeros of the denominator. The locations of the branch points of the spectral Green's function are well known.

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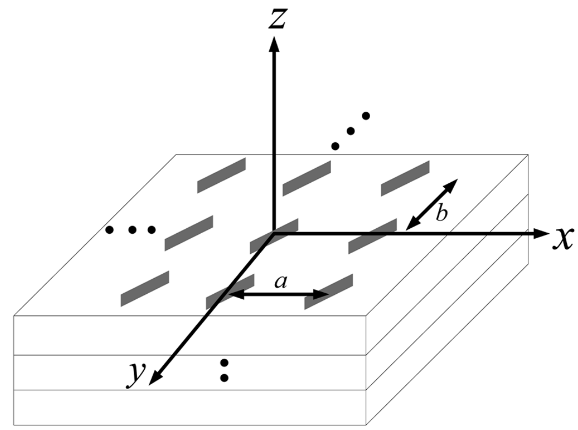

Fig. 1. Periodic structure on a general planar stratified structure.

A mapping of the spectral variables to the complex angle plane [1] can be used to cancel the branch point singularities of a specific space harmonic, thereby leaving only the pole singularities at some distance away from the rest of the branch-points.

During the search for the zeros of the Z matrix determinant, the pole singularities can be encountered, causing divergence of the zero search algorithms. Since the locations of the poles are known, they can be removed by multiplying the determinant by a factor which matches the pole's order. To this end, we investigate the order of the determinant's poles.

We present a rigorous proof, which establishes the degree of the pole singularities of the determinant. It is shown that a simple (first order) pole in the spectral Green's function for the background structure corresponds to a simple pole in the determinant.

II. STATEMENT OF THE PROBLEM

The general formulation outlined here pertains to a class of planar stratified structures with metallic patches periodic along the x and y axes, as shown in Fig. 1.

The electric field integral equation for this problem is

$$\underline{E}(x,y,z=0)=\int\int \underline{\underline{G}}(x,y)\underline{J}_s(x,y)dxdy=0. \qquad (1)$$

The unknown induced currents, $\underline{J}_s(x,y)$ on the metallic patches can be written using separation of variables as

$$\underline{J}_s(x,y)=\underline{S}(x)\underline{T}(y) \qquad (2)$$

Due to the periodicity of the structure, the current can be written in the form of a double summation of spatial harmonics, namely [2]

$$\underline{J}(x,y)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\underline{\widetilde{S}}(k_{xn})\underline{\widetilde{T}}(k_{ym})e^{-jk_{xn}x}e^{-jk_{ym}y} \qquad (3)$$

where

$$k_{xn}=k_x+\frac{2\pi n}{a} \quad ; \quad k_{ym}=k_y+\frac{2\pi m}{b} \qquad (4)$$

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and $\underline{\widetilde{S}}(k_{xn}), \underline{\widetilde{T}}(k_{ym})$ are the Fourier transforms of $\underline{S}(x)$ and $\underline{T}(y)$, respectively. Substitution of (3) into (1) and discretization of the integral equation using the MoM, yields a $N \times N$ matrix, denoted as $\mathbf{Z}$, with the following structure:

$$\mathbf{Z}(\underline{k}) = \begin{pmatrix} \mathbf{Z}_{xx}(\underline{k}) & \mathbf{Z}_{xy}(\underline{k}) \\ \mathbf{Z}_{yx}(\underline{k}) & \mathbf{Z}_{yy}(\underline{k}) \end{pmatrix} \tag{5}$$

where $\underline{k} = k_x \underline{\widehat{x}} + k_y \underline{\widehat{y}}$ and $\mathbf{Z}_{uv}(\underline{k}) (u, v = x \text{ or } y)$ denotes an $N_u \times N_v$ matrix in which the elements are obtained in a similar fashion to [3] given by

$$Z_{uv,pr}(\underline{k}) = \sum_{n=-M_x}^{M_x} \sum_{m=-M_y}^{M_y} \widetilde{W_{u,p}}(-\underline{k}_{nm}) \widetilde{G}_{uv}(\underline{k}_{nm}) \widetilde{B_{v,r}}(\underline{k}_{nm}) \tag{6}$$

where $\underline{k}_{nm} = k_{xn} \underline{\widehat{x}} + k_{ym} \underline{\widehat{y}}, \widetilde{W_{u,p}}$ is the spectral pth weight function for the field in the u direction and $\widetilde{B_{v,r}}$ is the spectral rth basis function for the induced current in the v direction. M_x and M_y are the number of spatial harmonics in the x and y directions, respectively. $\widetilde{G}_{uv}(\underline{k}_{nm})$ in (6) is the spectral Green's function sampled at the wave-numbers of the spatial harmonics and can be written in the following form:

$$\widetilde{G}_{uv}(\underline{k}_{nm}) \triangleq \frac{g_{\mathrm{uv}}(\underline{k}_{nm})}{D(\underline{k}_{nm})} \tag{7}$$

where the zeros of $D(\underline{k}_{nm})$ correspond to homogenous solutions to the background structure.

Let $\underline{k}_P \triangleq (k_{xP} + (2\pi n_P)/(a)) \underline{\widehat{x}} + (k_{yP} + (2\pi m_P)/(b)) \underline{\widehat{y}}$ denote the Pth pole of (7) for the spatial harmonics $n = n_P$ and $m = m_P$ with $\underline{k} = k_{xP} \underline{\widehat{x}} + k_{yP} \underline{\widehat{y}}$. This pole corresponds to the zero of $D(\underline{k}_P)$.

Let $\mathbf{Z}^{(P)}(\underline{k})$ denote the matrix $\mathbf{Z}$ with the Pth singularity removed, obtained by multiplying each of the matrix elements by the scalar function $D(\underline{k}_P)$

$$\mathbf{Z}^{(P)}(\underline{k}) = \mathbf{Z}(\underline{k}) D(\underline{k}_P) \tag{8}$$

Substitution of (6) and (7) into (8) yields the elements of $\mathbf{Z}_{uv}^{(P)}(\underline{k})$ given in the following form:

$$\begin{aligned} Z_{uv,pr}^{(P)}(\underline{k}) = \sum \sum_{\substack{(n,m)\neq \\ (n_P,m_P)}} \Big[& \widetilde{W_{u,p}}(-\underline{k}_{nm}) \widetilde{G}_{uv}(\underline{k}_{nm}) \\ & \widetilde{B_{v,r}}(\underline{k}_{nm}) D(\underline{k}_P) \Big] + \widetilde{W_{u,p}}(-\underline{k}_P) g_{uv}(\underline{k}_P) \widetilde{B_{v,r}}(\underline{k}_P). \end{aligned} \tag{9}$$

Based on (8), the determinant of $\mathbf{Z}(\underline{k})$ can be expressed by

$$\det(\mathbf{Z}(\underline{k})) = \frac{\det\left(\mathbf{Z}^{(P)}(\underline{k})\right)}{(D(\underline{k}_P))^N} \tag{10}$$

It is well known that the Green's function in (7) has poles of the first order [4], so $D(\underline{k}_P)$ is assumed to have only a zero of the first order at $\underline{k}_P$. All functions in (9) are assumed to be analytical at $\underline{k}_P$ (which is mostly the case in practical situations, see for example [3]) and therefore, which is obtained from a sum of products of the functions in (9), is also analytical at $\underline{k}_P$ and can have only a zero of an integer degree.

In the next section, it is rigorously proven that the determinant of $\mathbf{Z}^{(P)}(\underline{k})$ defined by (8) has a zero of the $(N-1)$th order at the Pth singularity point.

III. Proof

Definition: Let $\mathbf{M}_{\mathrm{RC}}^{(L)}(\mathbf{Z})$ denote a minor of the Lth order of the matrix $\mathbf{Z}$, defined as

$$\mathbf{M}_{\mathbf{RC}}^{(L)}(\mathbf{Z}) \triangleq \mathbf{M}_{R_L C_L}(\mathbf{M}_{R_{L-1} C_{L-1}}(\ldots \mathbf{M}_{R_1 C_1}(\mathbf{Z}))) \tag{11}$$

where $\mathbf{M}_{R_l C_l}(\mathbf{A})$ is a first order minor matrix obtained by removing the th row and the C_lth column of a given matrix $\mathbf{A}. R_l$ and C_l are the lth elements of the vectors $\mathbf{R}$ and $\mathbf{C}$ respectively. These vectors are of dimensions 1xL and contain all indexes of rows and columns, respectively, which were removed in each minor operation.

Corollary: $\det(\mathbf{M}_{\mathbf{RC}}^{(L)}(\mathbf{Z}^{(P)}(\underline{k}))) = 0$ at $\underline{k} = \underline{k}_P$ for any $0 \leq L \leq N-2 (N \geq 2)$ and for any choice of $\mathbf{R}$ and $\mathbf{C}$.

Proof: The elements of the Lth order minor of the matrix $\mathbf{Z}_{uv}^{(P)}(\underline{k})$, given by (9), can be written at $\underline{k} = \underline{k}_P$ (the Pth singularity point) as

$$\left[\mathbf{M}^{(L)}(\mathbf{Z}_{uv}^{(P)}(\underline{k}_P))\right]_{p,r} = \widetilde{W_{u,I_p}}(-\underline{k}_P) g_{uv}(\underline{k}_P) \widetilde{B_{v,J_r}}(\underline{k}_P) \tag{12}$$

where $\mathbf{I}$ and $\mathbf{J}$ are the vectors containing the indexes of the rows and columns of $\mathbf{Z}_{uv}^{(P)}(\underline{k})$ which are left after the Lth order minor. It is straight forward to verify that the Lth order minor of the matrix $\mathbf{Z}^{(P)}(\underline{k}_P)$ has the same structure of (12) and that the rows and columns of these matrixes are linearly dependent (for a matrix with dimensions greater than 1×1). The later can be visualized in (13), shown at the bottom of the page, where the dependence on $\underline{k}_P$ was omitted for simplicity. N_I and N_J are the number of rows and columns left after the Lth order minor. It can be seen in (13) that any two rows or columns differ only by a factor equal to the ratio between two different basis or weight functions. Therefore, the determinant of $\mathbf{M}^{(L)}(\mathbf{Z}_{uv}^{(P)}(\underline{k}_P))$ is zero. The matrix which results after the Lth order minor is of dimensions greater than 1×1 if the minor is of the order: $L \leq N-2$ (For $L = N-1$, we are left with a scalar for which the determinant is non zero).

Lemma 1:

$$\frac{\partial^m}{\partial k_x^{m_x} \partial k_y^{m-m_x}} \left[\mathbf{adj}\left(\mathbf{M}_{\mathbf{RC}}^{(L)}(\mathbf{Z}^{(P)}(\underline{k}))\right)\right]$$

$$\mathbf{M}^{(L)}\left(\mathbf{Z}^{(P)}(\underline{k}_P)\right) = \begin{pmatrix} \widetilde{W}_{I_1} g \widetilde{B}_{J_1} & \widetilde{W}_{I_1} g \widetilde{B}_{J_2} & \cdots\cdots\cdots & \widetilde{W}_{I_1} g \widetilde{B}_{J_{N_J}} \\ \widetilde{W}_{I_2} g \widetilde{B}_{J_1} & \widetilde{W}_{I_2} g \widetilde{B}_{J_2} & \cdots\cdots\cdots & \widetilde{W}_{I_2} g \widetilde{B}_{J_{N_J}} \\ & & \vdots & \\ \widetilde{W}_{I_{N_I}} g \widetilde{B}_{J_1} & \widetilde{W}_{I_{N_I}} g \widetilde{B}_{J_2} & \cdots\cdots\cdots & \widetilde{W}_{I_{N_I}} g \widetilde{B}_{J_{N_J}} \end{pmatrix} \tag{13}$$

is the zero matrix at $\underline{k} = \underline{k}_P$ for any non-negative integers m, m_x and L satisfying: $m + L \leq N - 3 (N \geq 3)$ and $m_x \leq m$, where m is the order of the derivative and m_x is the order of the derivative with respect to k_x.

Proof: We prove by induction

Step 1: We check for $\mathrm{m} = 0$. Making use of the relation [5]

$$[\mathbf{adj}(\mathbf{Z})]_{pr} = (-1)^{p+r} \det(\mathbf{M}_{rp}(\mathbf{Z})) \tag{14}$$

we have

$$\left[\mathbf{adj}\left(\mathbf{M}_{\mathbf{RC}}^{(L)}\left(\mathbf{Z}^{(P)}(\underline{k})\right)\right)\right]_{pr} = (-1)^{p+r} \det\left(\mathbf{M}_{\mathbf{R_2C_2}}^{(L+1)}\left(\mathbf{Z}^{(P)}(\underline{k})\right)\right) \tag{15}$$

where

$$\mathbf{R_2} = [\mathbf{R}\, r]$$

and

$$\mathbf{C_2} = [\mathbf{C}\, p]. \tag{16}$$

From the corollary, we can deduce that the determinant in (15) is zero at $\underline{k} = \underline{k}_P$ for every L satisfying: $L \leq N - 3$. Thus the lemma holds for $\mathrm{m} = 0$.

Step 2 -Hypothesis: We assume the lemma holds for any m, m_x and L satisfying $0 \leq m \leq M_0, M_0 + L \leq N - 3$ and $m_x \leq m$. Without loss of generality, we also assume that $N \geq 4$.

Step 3: We now have to prove that the lemma holds for any m, m_x and L satisfying $0 \leq m \leq M_0 + 1, M_0 + L \leq N - 4 (N \geq 4)$ and $m_x \leq m$. We start by increasing by 1 the order of the derivative with respect to k_x and using (15)

$$\begin{aligned}\frac{\partial^{M_0+1}}{\partial k_x^{m_x+1} \partial k_y^{M_0-m_x}} &\left[\mathbf{adj}\left(\mathbf{M}_{RC}^{(L)}\left(\mathbf{Z}^{(P)}(\underline{k})\right)\right)\right]_{pr} \\ &= (-1)^{p+r} \frac{\partial^{M_0}}{\partial k_x^{m_x} \partial k_y^{M_0-m_x}} \frac{\partial}{\partial k_x} \det\left(\mathbf{M}_{\mathbf{R_2C_2}}^{(L+1)}\left(\mathbf{Z}^{(P)}(\underline{k})\right)\right)\end{aligned} \tag{17}$$

we use the following relation [6]:

$$\frac{\partial}{\partial k} \det(\mathbf{Z}(k)) = \operatorname{tr}\left(\mathbf{adj}(\mathbf{Z}(k)) \frac{\partial}{\partial k} \mathbf{Z}(k)\right) \tag{18}$$

which holds for any matrix $\mathbf{Z}$ and $k = k_x, k_y$.

Substitution of (18) into (17) yields

$$\begin{aligned}\frac{\partial^{M_0+1}}{\partial k_x^{m_x+1} \partial k_y^{M_0-m_x}} &\left[\mathbf{adj}\left(\mathbf{M}_{\mathrm{RC}}^{(L)}\left(\mathbf{Z}^{(P)}(\underline{k})\right)\right)\right]_{pr} \\ &= (-1)^{p+r} \frac{\partial^{M_0}}{\partial k_x^{m_x} \partial k_y^{M_0-m_x}} \operatorname{tr} \\ &\times \left(\mathbf{adj}\left(\mathbf{M}_{\mathbf{R_2C_2}}^{(L+1)}\left(\mathbf{Z}^{(P)}(\underline{k})\right)\right) \right.\\ &\times \left. \frac{\partial}{\partial k_x}\left(\mathbf{M}_{\mathbf{R_2C_2}}^{(L+1)}\left(\mathbf{Z}^{(P)}(\underline{k})\right)\right)\right).\end{aligned} \tag{19}$$

We now interchange the minor and derivative operations inside the main parentheses of (19) on the right hand side. We also interchange the trace operation and the M_0th derivative in (19). Using the extended rule for the derivative of a product we have

$$\begin{aligned}&= (-1)^{p+r} \sum_{\alpha=0}^{m_x} \binom{m_x}{\alpha} \sum_{\beta=0}^{M_0-m_x} \binom{M_0 - m_x}{\beta} \\ &\operatorname{tr}\left(\frac{\partial^{\alpha+\beta}}{\partial k_x^{\alpha} \partial k_y^{\beta}} \left[\mathbf{adj}\left(\mathbf{M}_{\mathbf{R_2C_2}}^{(L+1)}\left(\mathbf{Z}^{(P)}(\underline{k})\right)\right)\right] \right.\\ &\left. \frac{\partial^{(M_0-\alpha-\beta)}}{\partial k_x^{(m_x-\alpha)} \partial k_y^{(M_0-m_x-\beta)}} \left[\mathbf{M}_{\mathbf{R_2C_2}}^{(L+1)}\left(\frac{\partial \mathbf{Z}^{(P)}}{\partial k_x}(\underline{k})\right)\right]\right).\end{aligned} \tag{20}$$

From the given condition: $L + M_0 \leq N - 4$ it follows that for any: $0 \leq (\alpha + \beta) \leq M_0$ we have: $(\alpha + \beta) + (L + 1) \leq N - 3$ which is exactly the condition of the induction hypothesis (substitute $L + 1$ instead of L as the order of the minor and $\alpha + \beta$ instead of m as the derivative order in the assumption of step 2). It follows from the induction hypothesis that all the derivatives of the adjugate matrix in (20) are zero matrixes. In addition, the derivatives of the $\mathbf{Z}^{(P)}$ matrix in (20) can have no singularities since it's elements are assumed to be analytical functions at $\underline{k} = \underline{k}_P$. Thus, the trace is zero. If we increase in (17) the order of the derivative with respect to k_y instead of k_x, we obtain the expression in (20) with $\partial\mathbf{Z}^{(P)}/\partial k_x$ replaced by $\partial\mathbf{Z}^{(P)}/\partial k_y$ and the same result is obtained. In conclusion, it was shown that the lemma holds for $m = 0$ and that assuming the lemma holds for any $m \leq M_0$, it also holds for: $m = M_0 + 1$. Based on the induction, the lemma is thus proved.

Lemma 2: $(\partial^m)/(\partial k_x^{m_x} \partial k_y^{m-m_x}) \det(\mathbf{M}_{\mathbf{RC}}^{(L)}(\mathbf{Z}^{(P)}(\underline{k}))) = 0$ at $\underline{k} = \underline{k}_P$ for any non-negative integers: m, m_x and L satisfying $m + L \leq N - 2 (N \geq 2)$ and $m_x \leq m$, for any choice of $\mathbf{R}$ and $\mathbf{C}$.

Proof: We use (18) and interchange the order of the trace and derivatives to obtain

$$\begin{aligned}\frac{\partial^m}{\partial k_x^{m_x} \partial k_y^{m-m_x}} &\det\left(\mathbf{M}_{\mathbf{RC}}^{(L)}\left(\mathbf{Z}^{(P)}(\underline{k})\right)\right) \\ &= tr\left\{\frac{\partial^{m-1}}{\partial k_x^{m_x-1} \partial k_y^{m-m_x}} \right.\\ &\times \left.\left(\mathbf{adj}\left(\mathbf{M}_{\mathbf{RC}}^{(L)}\left(\mathbf{Z}^{(P)}(\underline{k})\right)\right) \mathbf{M}_{\mathbf{RC}}^{(L)}\left(\frac{\partial}{\partial k_x} \mathbf{Z}^{(P)}(\underline{k})\right)\right)\right\}.\end{aligned} \tag{21}$$

Based on Lemma 1, the expression in (21) is zero at $\underline{k} = \underline{k}_P$ for every: m, m_x and L satisfying $m_x \leq m$ and $(m - 1) + L \leq N - 3$ or $m + L \leq N - 2$.

It follows from Lemma 2 that for L=0 we have:

$$\frac{\partial^m}{\partial k_x^{m_x} \partial k_y^{m-m_x}} \det\left(Z^{(P)}(\underline{k})\right) = 0$$

at $\underline{k} = \underline{k}_P$ for any m and m_x satisfying $0 \leq m \leq N-2$ and $m_x \leq m$.

The above conclusion states that $\det(\mathbf{Z}^{(P)}(\underline{k}))$ (which as stated before is an analytical function) has a zero of at least the $(N - 1)$th order at $\underline{k} = \underline{k}_P$. From (10) it follows that if $\det(\mathbf{Z}(\underline{k}))$ has a singularity point at $\underline{k} = \underline{k}_P$, it is a *pole* singularity of the *first order*. Numerical calculations show that $\det(\mathbf{Z}(\underline{k}))$ has singularity points at $\underline{k} = \underline{k}_P$ and this leads to the conclusion that $\det(\mathbf{Z}(\underline{k}))$ has a pole of the first order at $\underline{k} = \underline{k}_P$ (for any P).

IV. Conclusion

The pole singularity order in the determinant of the Z matrix in MoM formulations for planar stratified periodic structures was investigated. It has been proven rigorously that the determinant can only have a simple pole singularity at a simple pole of the spectral Green's function used to construct the Z matrix. Numerical observations were made in order to establish the fact that the determinant has indeed singularities at the poles of the Green's function. The knowledge of the singularity order enables to cancel automatically the singularities whenever a zero search is conducted in their proximity. The proof given in this work is general and can be applied to any planar structure which is periodic in one of the spatial directions.

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The Reverberating Chamber as a Line-of-Sight Wireless Channel Emulator

Antonio Sorrentino, Giuseppe Ferrara, and Maurizio Migliaccio

***Abstract*—The reverberating chamber (RC) is employed to physically emulate line-of-sight (LOS) propagation channels and to test the quality of a digital transmission. Use of different absorber configurations is able to generate various LOS propagation channels. The LOS channels are objectively characterized by the Rician K factor and results show that K is not generally dependent only on the number of absorbers but also on their configuration. Experiments are accomplished at the electrically large mode-stirred RC of the Università di Napoli Parthenope, formerly Istituto Universitario Navale (IUN) and a Global System for Mobile Communications (GSM) digital signal is used.**

***Index Terms*—Line-of-sight, reverberating chamber, wireless.**

I. INTRODUCTION

In recent years wireless digital communications had an enormous impact in daily life which required and still requires the development of new devices. Therefore the need of effective and economical test facilities to evaluate the functionality of the devices, in various wireless channels, is a key issue. In particular it is important to evaluate the transmission quality and robustness of the digital signals, employed by these wireless devices, in non-line-of-sight (NLOS) channels, where only scattered waves arrive to the receiver, and in line-of-sight (LOS) channels, in which there is also a direct link between the transmitting and the receiving antenna. In the first case, the field structure is modelled by a Rayleigh field model; in the second one, the field structure is modelled by a Rice field model. From a physical point of view, the Rayleigh distribution is well justified, as the received signal is assumed to be a superposition of a large number of randomly phased waves. More important are the LOS channels where the received signal is the combination of a dominant component, due to the direct link between the transmitting and the receiving antenna, and multiple scattered waves. In this last case, i.e., in a LOS channel, the fading depth on the received signal can be measured by means of the Rice factor, K, as explained in the following. Each LOS channel is characterized by its own K value. A NLOS channel is a particular case of LOS channel obtained when $K = 0$.

Usually, the natural norm to characterize the quality of the digital transmission in NLOS and LOS propagation channels is the bit-error-rate (*BER*). The *BER* values are related to signal-to-noise ratio (SNR) [1]. In a NLOS channel, the *BER* values are always less sensitive to SNR than in a LOS channel [2].

Many numerical simulation tools have been developed in these last years to simulate fading channels and to analyze the transmission quality and robustness of digital received signals transmitted through wireless channels.

This work proposes instead a physical, simple, controllable and economical convenient way to emulate LOS channels by means of the reverberating chamber (RC). The RC is a shielded, usually electrically large, cavity in which it is possible to generate a stochastic field by means of mechanical, platform, frequency or polarization stirring. Although the RC was first applied as an electromagnetic compatibility (EMC) test facility [3], [4] and to expose biological samples to a time variant stochastic field [5], [6], in recent years its use as propagation channel emulator has attracted the interest of some researches. In fact, the statistical behaviour of the field in a RC [7]–[11] suggests employing the chamber as an emulator of wireless NLOS and LOS channels. In fact, in [12] the RC is employed to analyze the robustness of a Global System for Mobile Communications (GSM) signal when it is transmitted through a NLOS propagation channel. Indeed, in [13]–[15] the RC is employed to generate a controllable NLOS channel to measure the receiver sensitivity [13], the radiation efficiency and the free space impedance of antennas of mobile phones [14] and to test the performances of a multiput-input, multiput-output (MIMO) systems [15]. Two methods are proposed in [13] to measure the receiver sensitivity of mobile phones in a small RC. In both of them the receiver sensitivity is quantified in terms of the input signal level at a certain *BER* level. In the first method the RC is mode-tuned operated and the receiver sensitivity is measured by the total isotropic sensitivity (TIS). In the other method, the small RC is mode-stirred operated and the receiver sensitivity is measured by the average fading sensitivity (AFS). The comparison between the results obtained by the two different working configurations shows that the mode-stirred is the most suitable and faster working configuration of the RC [13].

Above mentioned papers regard NLOS channels. In this paper we extend the use of a RC to emulate LOS propagating channels making benefit of the ground papers [12], [16]. In fact in [12] it is shown that the RC can be employed as an NLOS propagation channel emulator and in [16] that the Rice K factor can be controlled in a RC by placing some absorbers in the chamber.

In this paper, the large mode-stirred RC of the Università di Napoli Parthenope, formerly Istituto Universitario Navale (IUN), is employed

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