Direction of Arrival Estimation in the Presence of Noise Coupling in Antenna Arrays

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***Abstract*—The direction of arrival (DOA) estimation problem in the presence of signal and noise coupling in antenna arrays is addressed. In many applications, such as smart antenna, radar and navigation systems, the noise coupling between different antenna array elements is often neglected in the antenna modeling and thus, may significantly degrade the system performance. Utilizing the exact noise covariance matrix enables to achieve high-performance source localization by taking into account the colored properties of the array noise. The noise covariance matrix of the antenna array consists of both the external noise sources from sky, ground and interference, and the internal noise sources from amplifiers and loads. Computation of the internal noise covariance matrix is implemented using the theory of noisy linear networks combined with the method of moments (MoM). Based on this noise statistical analysis, a new four-port antenna element consisting of two orthogonal loops is proposed with enhanced source localization performance. The maximum likelihood (ML) estimator and the Cramer-Rao lower bound (CRLB) for DOA estimation in the presence of noise coupling is derived. Simulation results show that the noise coupling in antenna arrays may substantially alter the source localization performance. The performance of a mismatched ML estimator based on a model which ignores the noise coupling shows significant performance degradation due to noise coupling. These results demonstrate the importance of the noise coupling modeling in the DOA estimation algorithms.**

***Index Terms*—Antenna array, antenna noise, Cramer-Rao lower bound (CRLB), direction of arrival (DOA), maximum likelihood, noise coupling, sky noise, smart antennas, source localization, vector sensor.**

I. Introduction

THE noise statistics in a receiving system is important to determine its performance. The noise statistical information at the receiving ports of the sensor array is assumed to be known for implementation of optimal source localization methods [1]. In noise analysis of typical receiving arrays, it is often assumed that the port noises are statistically independent [2]–[4]. This implicit assumption is rarely satisfied due to mutual coupling and environmental noise. The effect of noise statistics mis-specification significantly degrades the performances of most high-resolution array processing algorithms [5], [6].

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The increase in demand for compact multi-antenna receiving arrays has yielded a large amount of array designs in the recent literature [4], [7]. Many of these array configurations exhibit a strong correlation between the port noises of the different elements in the array. This problem is especially significant in vector-sensor arrays where a strong mutual coupling exists between the various antenna array elements.

Several works [8], [9], have suggested approaches for the compensation of signal mutual coupling effects in antenna arrays. Other works have examined the impact on channel capacity due to signal mutual coupling [10], [11]. However, all these works assume statistically independent port noises. Recent works address the subject of noise coupling. In [12], the noise covariance matrix is computed to obtain the multiple input multiple output (MIMO) channel capacity in a communication system. The system noise coupling effect is considered through the scattering matrix of the transmit and receive arrays, which is assumed to be known. In [13], the noise power at each element is obtained by considering the noise coupling between the different elements for finite and infinite arrays. The noise power at each element was used to compute the signal-to-noise ratio (SNR) pattern at the output of a beamformer. In [14], the radio astronomy problem of optimal signal detection (in terms of signal-to-interference noise ratio) by a reflector with a focal plane array is considered. The noise contribution is from amplifiers, spillover and interference. The noise coupling effect is considered by assuming known scattering matrix. All these works assume a prior knowledge of the scattering matrix.

This paper addresses the problem of source localization in the presence of signal and noise coupling in the antenna array. Utilizing the complete noise covariance matrix enables to achieve high-performance source localization by taking into account the colored properties of the array noise. The noise covariance matrix is computed by taking into consideration the external noise from sky, ground and interference, in addition to the internal noise due to amplifiers and loads. The analysis of the antenna array noise covariance matrix is based on the well-established linear noisy networks theory [13] combined with the method of moments (MoM). The use of MoM enables to consider antenna arrays with complex geometries.

The paper is organized as follows. In Section II the signal and noise models are derived using the MoM. The noise model includes the external and internal noise contributions. Based on this formulation, the maximum-likelihood (ML) for direction-of-arrival (DOA) estimation and the Cramér-Rao lower bound (CRLB) for this problem are derived in Section III. Section IV presents numerical examples of a two-dipole antenna array and of a two-orthogonal-loop antenna array, which illustrate the application of the theory developed in this paper and its importance in antenna array design for source localization. Section V summarizes the main results of this study.

II. Signal and Noise Models in Antenna Arrays

In this section, the signal and noise models for a general array of coupled antennas are established. These models are required for implementation of mismatch-free source localization algorithms. In this paper, we consider a wire antenna array in the receive mode. The wire antenna array can be analyzed using the MoM by describing the antenna array as an L-port network [16].

A. Signal Model in Antenna Arrays

The induced current vector on the antenna array segments, $\mathbf{I}$, is related to the induced voltage vector by $\mathbf{Z}\cdot\mathbf{I}=\mathbf{V}$, where $\mathbf{Z}$ is computed using the MoM formulation. Using the *Galerkin pulse—pulse* method [16], the vector of voltages induced on the N segments of a wire antenna array by an incident wavefront arriving from elevation and azimuth angles, (θ,ϕ), can be expressed in the form [17]

$$\mathbf{V}=\mathbf{H}(\theta,\phi)\mathbf{E}, \tag{1}$$

in which $\mathbf{H}(\theta,\phi)$ denotes the antenna matrix effective height, and $\mathbf{E}=\begin{bmatrix}E_\theta\\E_\phi\end{bmatrix}$ is the incident electric field vector. In the typical MoM formulation, all the segments, except the input/output port segments, are short-circuited to yield a continuous wire antenna. The segments corresponding to the antenna ports are usually loaded by impedance loads. The port voltages vector, $\mathbf{V}_p$ and the port currents vector $\mathbf{I}_p$ may be defined as $\mathbf{V}_p=\mathbf{PV},\ \ \mathbf{I}_p=\mathbf{PI}$, where $\mathbf{P}$ is the indexing matrix of dimension $L\times N$ indicating the locations of each one of the ports in the MoM decomposition. Let $\mathbf{Z}_{in}$ denote a diagonal matrix of size $L\times L$ with the port load impedances on its diagonal, and $\mathbf{Y}'$ denote the *embedded* admittance matrix of the antenna array [27], defined as $\mathbf{Y}'_p=\mathbf{P}\left(\mathbf{Z}+\mathbf{P}^T\mathbf{Z}_{in}\mathbf{P}\right)^{-1}\mathbf{P}^T$, where $(\cdot)^T$ is the matrix transposition operator. The induced currents in the ports of the array can be described by $\mathbf{I}_p=\mathbf{PY'V}$, and using (1) it can be rewritten in the form

$$\mathbf{I}_p=\mathbf{U}(\theta,\phi)\mathbf{E} \tag{2}$$

where $\mathbf{U}(\theta,\phi)=\mathbf{PY'H}(\theta,\phi)$ is the *steering matrix* of dimension $L\times 2$.

B. Noise Model in Antenna Arrays

The problem of noise coupling in linear networks has been addressed in [18] and [19] in which the noise current covariance matrix is found to be linearly dependent on the network ambient temperature, T, and the real part of the network admittance matrix. This analysis can be applied for the calculation of the noise current covariance matrix in an antenna array loaded by passive impedances. In this case, the noise current covariance matrix of the array ports can be expressed by

$$\mathbf{N}_I\triangleq E\left\{\mathbf{I}_p\mathbf{I}_p^H\right\}=4KTB\Re e\left(\mathbf{Y}'_p\right). \tag{3}$$

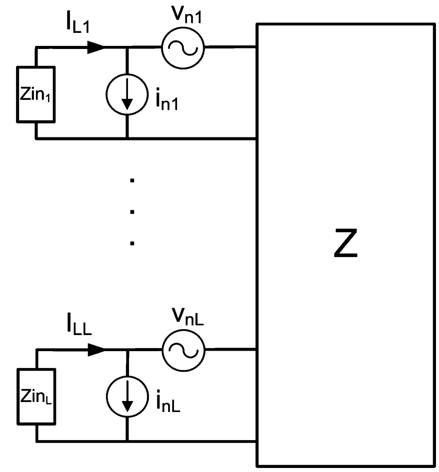

Fig. 1. L-port antenna array configuration loaded with LNAs.

where $(\cdot)^H$ is the conjugate transpose operator, K is Boltzman's constant, and B is the system bandwidth.

This formulation provides a simplified noise model, which considers mutual coupling in an antenna array, isolated from external noise and internal noise due to low noise amplifiers (LNAs). However, it is limited to particular cases of passive loading of the antenna array and isotropic external noise distribution. In the practical cases of active loading of the antenna array, a more complex noise model is required as described in Section II-B.1. Moreover, in the case of anisotropic external noise distribution, the noise covariance matrix cannot be characterized by a single scalar temperature T as implied in (3). Section II-B.2 analyzes the external noise covariance matrix for this general case.

The total noise contribution at the receiver input can be considered as a superposition of two uncorrelated noise sources: (1) the internal noise due to active (LNAs) and passive components, and (2) external noise due to environmental distributed sources. Accordingly, the total noise current covariance matrix can be expressed by

$$\mathbf{N}_I=\mathbf{N}_I^A+\mathbf{N}_I^E \tag{4}$$

where $\mathbf{N}_I^A$ is the internal noise current covariance matrix and $\mathbf{N}_I^E$ denotes the external noise current covariance matrix. In Section II-B.1, the noise current covariance matrix, $\mathbf{N}_I^A$ due to LNAs contribution is obtained. In Section II-B.2, the external noise current covariance matrix $\mathbf{N}_I^E$ is analyzed.

1) Noise in Antenna Arrays With Amplifiers: In a practical receiving system, the antenna is connected to LNAs and therefore, the previous model should be extended to consider the noise contribution of the LNAs. Fig. 1 describes the configuration for the noise model for an L-port antenna array with LNAs. This problem was investigated in [13]. In the following, the covariance matrix of the LNA noise current will be developed using the model presented in [20], in which it is shown that the effect of internal noise sources of LNAs can be represented by a noiseless circuit with external noise sources, placed at its input terminal.

The amplifier at the ith port is characterized by two correlated noise voltage and current sources, V_{ni} and I_{ni}, respectively. The variances of these sources and their correlation are given by

$$E\left\{|V_{ni}|^2\right\} = 4KTBR_{ni} \tag{5}$$
$$E\left\{|I_{ni}|^2\right\} = 4KTBG_{ni} \tag{6}$$
$$E\left\{V_{ni}I_{ni}^*\right\} = 4KTBY_{\gamma_{ni}}^* R_{ni} \tag{7}$$

where R_{ni}, G_{ni}, and $Y_{\gamma_{ni}}$ are the noise resistance, conductance, and admittance of the LNA, respectively.

The vector of noise currents flowing through the input impedances $\mathbf{Z}_{in} = \mathrm{diag}(Z_{in1}, \ldots, Z_{inL})$ is given by

$$\mathbf{I}_L = \mathbf{Y}'_p\left(\mathbf{V}_n + \mathbf{Z}_p\mathbf{I}_n\right) \tag{8}$$

where $\mathbf{V}_n = [V_{n1}, \ldots, V_{nL}]^T$, $\mathbf{I}_n = [I_{n1}, \ldots, I_{nL}]^T$. The port impedance matrix, $\mathbf{Z}_p$, can be computed by standard numerical methods. Using the MoM, $\mathbf{Z}_p$ can be expressed by $\mathbf{Z}_p = \left(\mathbf{P}\mathbf{Z}^{-1}\mathbf{P}^T\right)^{-1}$, and by simple matrix manipulations, $\mathbf{Y}'_p = \mathbf{P}\left(\mathbf{Z} + \mathbf{P}^T\mathbf{Z}_{in}\mathbf{P}\right)^{-1}\mathbf{P}^T$, defined in Section II-A, can be simplified to $\mathbf{Y}'_p = \left(\mathbf{Z}_p + \mathbf{Z}_{in}\right)^{-1}$. Accordingly, the LNAs input noise current covariance matrix can be described by

$$\begin{aligned}\mathbf{N}_I = E\left\{\mathbf{I}_L\mathbf{I}_L^H\right\} &= \mathbf{Y}'_p E\left\{\mathbf{V}_n{\mathbf{V}_n}^H\right\}{\mathbf{Y}'_p}^H \\ &+ \mathbf{Y}'_p E\left\{\mathbf{V}_n{\mathbf{I}_n}^H\right\}\mathbf{Z}_p^H{\mathbf{Y}'_p}^H \\ &+ \mathbf{Y}'_p\mathbf{Z}_p E\left\{\mathbf{I}_n{\mathbf{V}_n}^H\right\}{\mathbf{Y}'_p}^H \\ &+ \mathbf{Y}'_p\mathbf{Z}_p E\left\{\mathbf{I}_n{\mathbf{I}_n}^H\right\}\mathbf{Z}_p^H{\mathbf{Y}'_p}^H.\end{aligned} \tag{9}$$

The noise voltage and current sources of the different LNAs are uncorrelated and therefore, the covariance matrices $E\left\{\mathbf{V}_n{\mathbf{V}_n}^H\right\}$, $E\left\{\mathbf{I}_n{\mathbf{I}_n}^H\right\}$, and the cross covariance matrix $E\left\{\mathbf{V}_n{\mathbf{I}_n}^H\right\}$ are diagonal matrices whose ith elements are given by (5)–(7).

2) External Noise Model: The noise model described above, can be extended to include external/environmental noise received by the antenna array. In general, the external noise sources are spatially distributed, like sky and ground noise. The external noise can be modeled as an ensemble of TEM plane noise waves in both polarizations incident from all azimuth and elevation angles. Furthermore, the noise waves are assumed to be spatially uncorrelated.

Based on (2), the total noise vector induced in the antenna ports, is a superposition of all external noise waves. Thus, the port noise current induced by the external noise sources can be expressed by

$$\mathbf{I}_p^E = \int_\Omega \mathbf{U}(\theta,\phi)\mathbf{E}(\theta,\phi)d\Omega \tag{10}$$

where $\mathbf{E}(\theta,\phi)$ is the electric field of the external noise wave incident from direction (θ,ϕ), and Ω is the steradian angle for which $d\Omega = \sin\theta d\theta d\phi$. Using this expression, and under zero-mean noise assumption, the covariance matrix of the induced noise current is

$$\begin{aligned}\mathbf{N}_I^E &= E\left(\mathbf{I}_p^E{\mathbf{I}_p^E}^H\right) \\ &= \int_\Omega\int_{\Omega'} \mathbf{U}(\theta,\phi)E\left(\mathbf{E}(\theta,\phi)\mathbf{E}^H(\theta',\phi')\right) \\ &\quad\times \mathbf{U}^H(\theta',\phi')d\Omega d\Omega'.\end{aligned} \tag{11}$$

Under the assumption of uncorrelated white noise in space and polarization, the expectation in (11) can be expressed in terms of the spatial power density, $S(\theta,\phi)$, using the following relation

$$\frac{E\left(\mathbf{E}(\theta,\phi)\mathbf{E}^H(\theta',\phi')\right)}{\eta} = S(\theta,\phi)\mathbf{I}_2\delta(\Omega-\Omega') \tag{12}$$

where $\eta = 120\pi$ ohm is the free space characteristic impedance and $\mathbf{I}_2$ is an identity matrix of size 2. In a narrowband system, satisfying $B \ll c/\lambda$ (c is light velocity and λ is the carrier wavelength), the received noise power from an incremental solid angle, $d\Omega$, can be described by

$$S(\theta,\phi)\frac{\lambda^2}{4\pi}D(\theta,\phi)d\Omega = KBT(\theta,\phi)D(\theta,\phi)\frac{d\Omega}{4\pi} \tag{13}$$

in which $D(\theta,\phi)$ is the antenna directivity, and $T(\theta,\phi)$ is the environmental noise temperature. Measured values of $T(\theta,\phi)$ in various frequencies can be found in the literature [17]. Substitution of (12) and (13) into (11), and evaluating the integration with respect to Ω', results in

$$\mathbf{N}_I^E = \frac{KB\eta}{\lambda^2}\int_\Omega T(\theta,\phi)\mathbf{U}(\theta,\phi)\mathbf{U}^H(\theta,\phi)d\Omega\,. \tag{14}$$

This external noise current covariance matrix can be used for the computation of the total noise current covariance matrix, $\mathbf{N}_I$, described in (4). A similar approach taking into account the noise correlation due to environmental noise temperature, can be found in [21].

III. Source Localization Using Coupled Antenna Arrays

Source localization is an important and common application of antenna arrays. Mutual coupling in an antenna array may have a significant impact on its source localization performance. In this section, the mathematical tools [1], [22], [23] needed to evaluate the source localization performance of a general antenna array are established by taking into account the signal and noise mutual coupling in the array. The noise analysis performed in Section II, is used in this section, to describe the noise measured at the ports of coupled antenna arrays.

The model described in the previous section refers to a three-dimensional scenario, providing azimuth and elevation information for source localization. For simplicity, in the following a two-dimensional scenario will be considered in which θ is assumed to be known and set to 90°. For brevity, $\mathbf{U}(\phi)$ will be used to denote $\mathbf{U}(\theta = 90°, \phi)$, defined in (2).

After establishing the signal and noise models, the complete data model of a receiving antenna array, may be defined as

$$\mathbf{y} = \mathbf{U}(\phi)\mathbf{E} + \mathbf{n} \tag{15}$$

where $\mathbf{y}$ is the vector of the measured port currents and the noise vector $\mathbf{n}$ is of size L representing the port noise currents. We assume that the noise current is zero-mean, circular complex Gaussian distributed:

$$\mathbf{n} \sim CN(\mathbf{0}, \mathbf{N}_I) \tag{16}$$

where $\mathbf{N}_I$ is derived in Section II-B.

A. The Maximum-Likelihood Estimator

After establishing the data model, we wish to derive the ML estimator for the problem of DOA estimation of a signal with arbitrary polarization. The vector of unknown parameters to be estimated is

$$\boldsymbol{\eta} = \begin{bmatrix} \phi \\ \Re e\{E_\theta\} \\ \Im m\{E_\theta\} \\ \Re e\{E_\phi\} \\ \Im m\{E_\phi\} \end{bmatrix}. \tag{17}$$

Equation (15) and (16) imply that the data model distribution can be described by

$$\mathbf{y} \sim CN(\mathbf{U}(\phi)\mathbf{E}, \mathbf{N}_I). \tag{18}$$

The ML estimator of ϕ for the model given in (18) is given by [22], [23]

$$\hat{\phi} = \arg\max_{\phi} L(\phi) \tag{19}$$

where $L(\phi)$ is the log-likelihood function given by

$$L(\phi) = \left\| \mathbf{P}_{\tilde{\mathbf{U}}(\phi)} \tilde{\mathbf{y}} \right\|^2 \tag{20}$$

in which $\tilde{\mathbf{y}} = \mathbf{N}_I^{-1/2}\mathbf{y}$ is the whitened signal vector and

$$\mathbf{P}_{\tilde{\mathbf{U}}(\phi)} = \tilde{\mathbf{U}}(\phi)\left(\tilde{\mathbf{U}}^H(\phi)\tilde{\mathbf{U}}(\phi)\right)^{-1}\tilde{\mathbf{U}}^H(\phi) \tag{21}$$

is the projection matrix into the subspace spanned by the columns of $\tilde{\mathbf{U}} = \mathbf{N}_I^{-1/2}\mathbf{U}$.

The covariance matrix $\mathbf{N}_I$ is Hermitian and positive-definite. Thus, by expanding (20), the ML estimator may be expressed as

$$\begin{aligned}\hat{\phi} &= \arg\max_{\phi}\left\{\mathbf{y}^H\mathbf{N}_I^{-1}\mathbf{U}(\phi)\left(\mathbf{U}^H(\phi)\mathbf{N}_I^{-1}\mathbf{U}(\phi)\right)^{-1} \right.\\ &\qquad \left.\times \mathbf{U}^H(\phi)\mathbf{N}_I^{-1}\mathbf{y}\right\} \\ &= \arg\max_{\phi}\left\|\left(\mathbf{U}^H(\phi)\mathbf{N}_I^{-1}\mathbf{U}(\phi)\right)^{-1/2}\mathbf{U}^H(\phi)\mathbf{N}_I^{-1}\mathbf{y}\right\|^2.\end{aligned} \tag{22}$$

B. The Cramér-Rao Lower Bound

The CRLB [22] provides an analytic lower bound for the estimation error variance of any unbiased estimator, and it is commonly used as a lower reference for examining the expected estimation performance of unbiased estimators. The CRLB can be used in the feasibility study stage of an estimation problem using a data model. Accordingly, the CRLB can serve as an important tool in the antenna design process.

In the problem at hand, we wish to establish the CRLB for DOA estimation using a wire antenna, and compare the performance of different estimators to the CRLB. The Fisher information matrix (FIM), $\mathbf{J}$ for the estimation of the vector of unknown parameters $\boldsymbol{\eta}$, defined in (17), from the data vector $\mathbf{y}$ modeled in (18), is given by [22], [23]

$$\mathbf{J} = 2\Re e\left\{\left(\frac{\partial(\mathbf{U}(\phi)\mathbf{E})}{\partial\boldsymbol{\eta}}\right)^H \mathbf{N}_I^{-1}\left(\frac{\partial(\mathbf{U}(\phi)\mathbf{E})}{\partial\boldsymbol{\eta}}\right)\right\}. \tag{23}$$

Using the FIM, the CRLB for estimation of $\boldsymbol{\eta}$ is given by

$$\mathrm{cov}(\hat{\boldsymbol{\eta}}) \geq \mathbf{C} \triangleq \mathbf{J}^{-1}. \tag{24}$$

The CRLB on DOA estimation error variance may now be extracted from the matrix $\mathbf{C}$

$$\mathrm{var}(\hat{\phi}) \geq [\mathbf{C}]_{1,1}. \tag{25}$$

Thus, if $[\mathbf{C}]_{1,1} \to \infty$, then the antenna array is not capable of estimating ϕ for a signal characterized by the parameter vector $\boldsymbol{\eta}$. This fact may be used to analytically explore the limitations of a given antenna array for source localization.

IV. Numerical Examples

In this section, the theory developed in the previous sections is applied to analyze the effects of noise coupling on two different antenna configurations. First, the use of the mathematical framework developed for the calculation of the noise current covariance matrix, is demonstrated using a simple two-dipole array. The behavior of the noise current covariance matrix is examined for various dipole distances. Next, a new simple antenna array design named "Two-Loop Vector" is presented and analyzed. It is shown to yield surprising estimation capabilities, by exploiting the strong noise coupling between its array ports.

A. Noise Analysis of Two Parallel Dipoles

Consider a simple antenna array consisting of two parallel z-directed dipole antennas. The dipoles are of length $\lambda/2$ and spaced d apart, where λ is the wavelength of the incident wave. Using the MoM notation, each dipole is divided into 63 segments of length $\lambda/125$, and contains one port in its central segment. The diameter of each dipole is taken to be $\lambda/500$.

Thus, in this case, the number of ports and the number of MoM segments were set to $L = 2$ and $N = 126$, respectively. The port segments of the two dipoles (segments 32 and 95) are loaded by impedances Z_{in1} and Z_{in2}, respectively. All other segments are short-circuited, to yield a continuous dipole.

Equation (3) provides an expression for the noise currents covariance matrix of the antenna array with passive loading. The matrix $\mathbf{Y}'_p$ for the case at hand can be computed using the MoM to yield

$$\mathbf{Y}'_p = \begin{pmatrix} [\mathbf{Y}']_{32,32} & [\mathbf{Y}']_{32,95} \\ [\mathbf{Y}']_{95,32} & [\mathbf{Y}']_{95,95} \end{pmatrix} \tag{26}$$

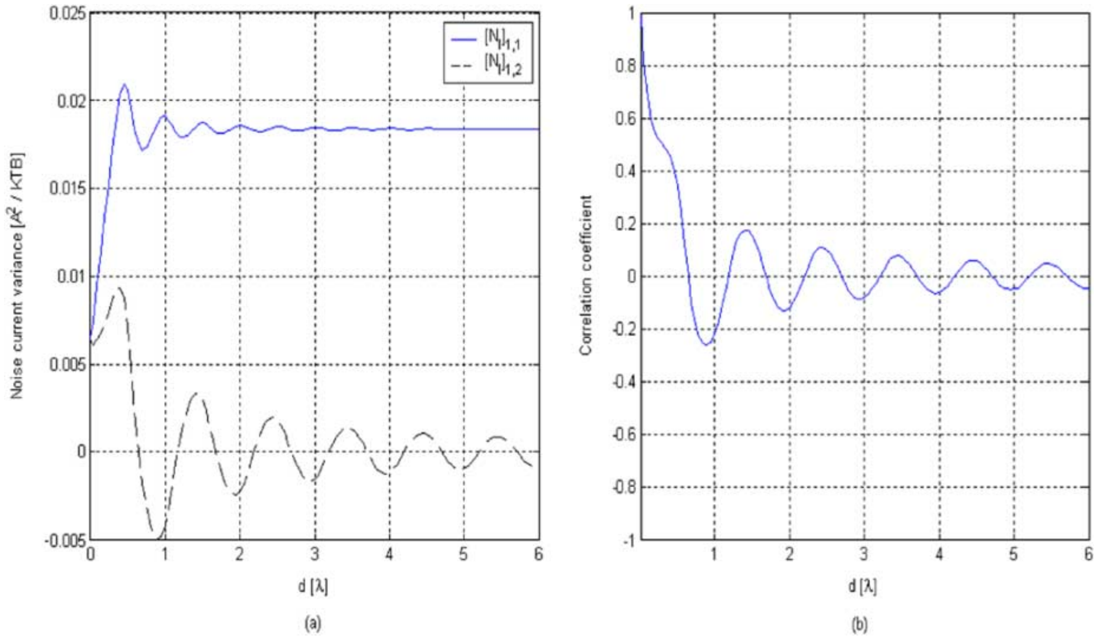

Fig. 2. Noise coupling between two unloaded $\lambda/2$ dipole antennas versus the inter-element spacing, d. (a) Self noise current variance, $[\mathbf{N}_I]_{1,1}$ and mutual noise current covariance, $[\mathbf{N}_I]_{1,2}$. (b) Noise current correlation coefficient, $[\mathbf{C}_I]_{1,2}$.

This configuration of a twin dipole array was simulated for various dipole spacings. The simulated dipoles are short-circuited ($\mathbf{Z}_{in} = \mathbf{0}$). The MoM impedance matrix was calculated using Galerkin's method with pulse basis and weighting functions. Fig. 2(a) shows $[\mathbf{N}_I]_{1,1}$ and $[\mathbf{N}_I]_{1,2}$ for two parallel dipoles versus their spacing, d. One can observe that as d increases, the *self noise current variance* $[\mathbf{N}_I]_{1,1} \rightarrow \text{const}$ while the *mutual noise current covariance* $[\mathbf{N}_I]_{1,2} \rightarrow 0$, as expected. As d decreases, the *self noise current variance* decreases, since the dipoles are affected by the near scatterer, while the *mutual noise current covariance* increases due to the dipoles proximity. At the limit of zero spacing, both values become equal, as both antennas (and ports) unite. However, these results break down as $d \rightarrow 0$ due to unmodeled azimuthal variation of the true currents on the finite diameter wires, which are not considered in this analysis.

Fig. 2(b) shows the *noise current correlation coefficient*, $[\mathbf{C}_I]_{12}$ where

$$[\mathbf{C}_I]_{ij} = \frac{[\mathbf{N}_I]_{ij}}{\left([\mathbf{N}_I]_{ii}\,[\mathbf{N}_I]_{jj}\right)^{1/2}}.$$

This quantity describes the amount of coupling between the two dipole antennas, with $|[\mathbf{C}_I]_{ij}| \leq 1$. As d tends to zero, the magnitude of the noise current correlation coefficient tends to one indicating that the noise measured in the antennas ports become equal. Under this condition, the antenna array behaves as a single dipole antenna.

Many analyses of mutual coupling [10], [24] define a quantity named *antenna correlation*, which is obtained using an EM integral performed on the antennas patterns. This antenna correlation refers to signal only.

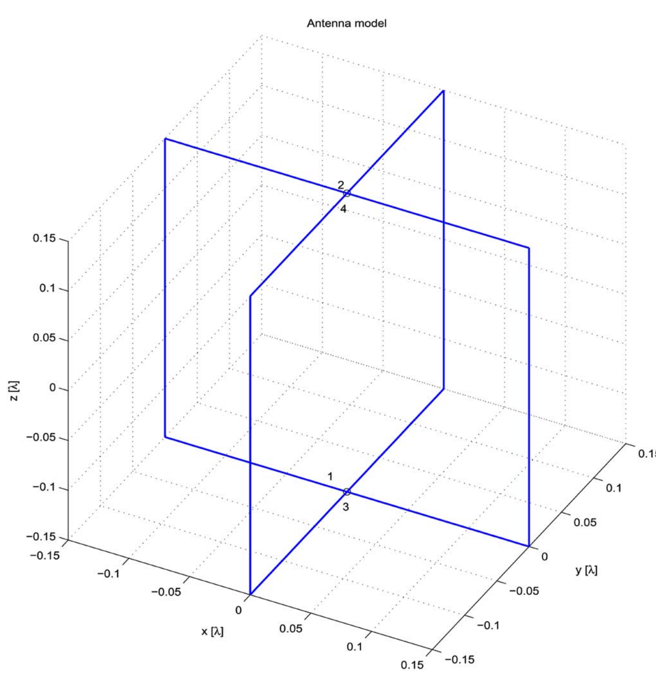

Fig. 3. TLV wire-antenna array with port locations.

B. Noise Analysis of Two Rectangular Loop Antennas

In this subsection, a vector sensor antenna array named "Two-Loop Vector" (TLV) is presented. The concept of two-collocated orthogonal thin-wire loops with two ports is presented in [25]. A variation of this concept was used to propose a four-port quadrature vector sensor, including two-orthogonal collocated thin-wire loops and two-orthogonal collocated dipoles [7].

The antenna shown in Fig. 3, is comprised of two wire rectangular loop antennas with their central axes pointing toward the

x-axis and the y-axis, respectively. Each antenna has two ports, creating in total a four ports antenna array. Similar arrays have been proposed in the literature for DOA estimation and polarization diversity. However, these antennas contain one port per loop. The array proposed here uses two ports per antenna and thus, as will be shown, it utilizes the noise coupling between the ports for enhanced performance in DOA and polarization estimation.

The ports of the TLV antenna are connected to four identical LNAs with input impedances of 50 Ω. For the antenna noise analysis, we consider the contributions of both external and internal noise sources, and the coupling between the antenna elements.

Using the MoM decomposition notation, each loop is segmented into 148 segments of length $\lambda/123$, giving a total length of 1.2λ for each loop. The diameter of the wire is taken to be $\lambda/500$. Ports 1 and 2 are located on x-directed wires of the first loop, and ports 3 and 4 are located on the y-directed wires of the second loop. Note that the locations of the two pairs of ports (1,3) and (2,4) overlap. The impedance matrix, $\mathbf{Z}_p$ for this array was calculated and the result is shown in (27) at the bottom of the page.

For the external noise current covariance matrix, the following environmental noise temperature distribution is considered

$$T(\theta,\phi) = \begin{cases} 100^\circ\mathrm{K}, & 0^\circ \le \theta < 90^\circ \\ 290^\circ\mathrm{K}, & 90^\circ \le \theta \le 180^\circ. \end{cases} \tag{28}$$

Using the above environmental noise temperature and (14), the external noise current covariance matrix is computed as in (29) at the bottom of the page.

For the internal noise current covariance matrix, we consider typical noise parameters for the identical LNAs: $R_{ni} = 5\ \Omega$, $G_{ni} = 0.2\ \mathrm{mS}$, and $Y_{\gamma ni} = (1 + j3)\ \mathrm{mS}$, $i = 1, \ldots, 4$. For the case of matched amplifiers, this is equivalent to a noise figure of 0.33 dB. Using these parameters and (9), the internal noise current covariance matrix was computed:

$$\mathbf{N}_I^A = KB \begin{pmatrix} 0.33 & 0.07 & 0 & 0 \\ 0.07 & 0.33 & 0 & 0 \\ 0 & 0 & 0.33 & 0.07 \\ 0 & 0 & 0.07 & 0.33 \end{pmatrix}. \tag{30}$$

Finally, the total noise current covariance matrix for this array is a superposition of the external and internal covariance matrices.

Note that the noise at the two ports of each loop antenna are correlated because of the high mutual admittance between the ports. This fact improves the ability of each loop antenna to detect an incident wave. This phenomenon can be interpreted as the supergain effect [26] obtained also in receive antenna arrays, which exploit the noise spatial coupling of closely spaced antenna elements for channel capacity enhancement. One can observe that the external noise correlations at ports 1,2 and ports 3,4 are higher than the corresponding LNA noise correlations. Therefore the LNA noise regularizes the total noise covariance matrix, resulting in a lower supergain effect. Obviously, the supergain effect fades when the LNA noise power increases.

The TLV antenna array was simulated for different SNRs using two estimators:

1. A matched ML estimator—the estimator described in (22), which utilizes the noise coupling for DOA estimation;
2. A mismatched ML estimator—the ML estimator described in (22), which assumes white measurement noise, i.e., $\mathbf{N}_I = \sigma^2 \mathbf{I}_L$, where $\mathbf{I}_L$ is the identity matrix of dimension L, and σ^2 is the ports noise current variance.

In the simulations for the configuration described above, a circular polarized incident signal from various azimuth angles, ϕ, was considered. The dependence of the estimation performance on ϕ was found to be minor. Thus, $\phi = 0^\circ$ is used in all simulations in this subsection.

Fig. 4 shows the root-mean-square error (RMSE) for DOA estimation versus SNR for the above two estimators. The SNR in the simulations is defined as

$$SNR = 10 \log \left(\frac{1}{\sigma^2} \left\| \mathbf{U}(\phi)\mathbf{E} \right\|^2 \right). \tag{31}$$

The CRLB is also shown for reference. This figure shows that the mismatch in the spatial noise statistics degrades the overall estimation performance of the ML estimator in comparison to the matched ML estimator where the asymptotic performance degradation is about 1.8 dB, which means that the required SNR to obtain a given DOA estimation RMSE is higher by 1.8 dB. The results demonstrate the capability of the proposed antenna to estimate the source DOA, ϕ. At low SNRs, the ML curves are lower than the CRLB curve, because the DOA search range

$$\mathbf{Z}_p = \begin{pmatrix} 114 - 542j & -35 + 580j & 0 & 0 \\ -35 + 580j & 114 - 542j & 0 & 0 \\ 0 & 0 & 114 - 542j & -35 + 580j \\ 0 & 0 & -35 + 580j & 114 - 542j \end{pmatrix}. \tag{27}$$

$$\mathbf{N}_I^E = KB \begin{pmatrix} 1.79 & 1.62 + 0.04j & 0 & 0 \\ 1.62 - 0.04j & 1.61 & 0 & 0 \\ 0 & 0 & 1.79 & 1.62 + 0.04j \\ 0 & 0 & 1.62 - 0.04j & 1.61 \end{pmatrix}. \tag{29}$$

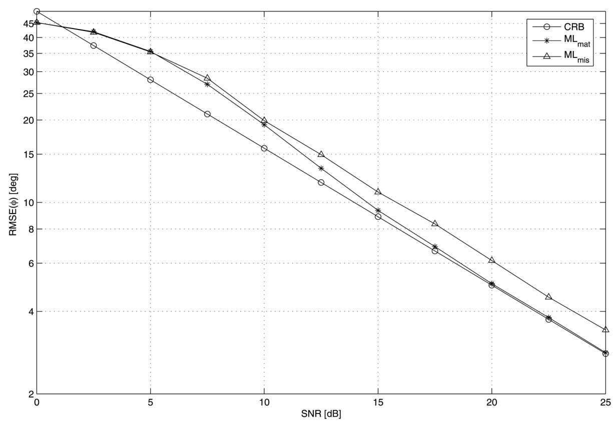

Fig. 4. Source localization performance using the TLV antenna array.

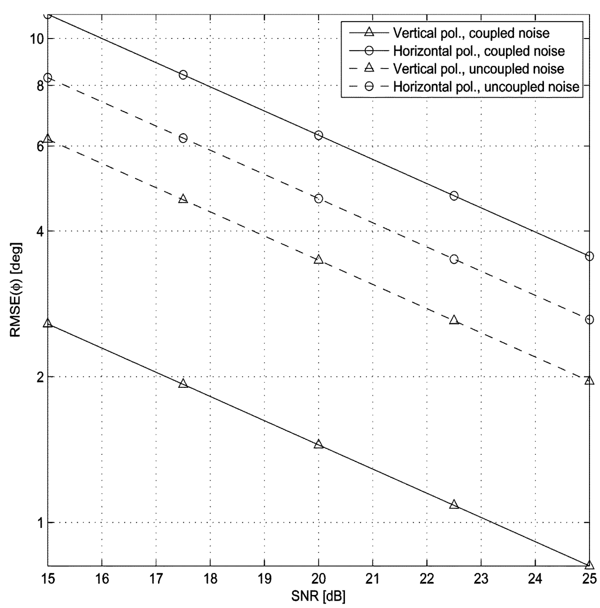

Fig. 5. CRLB curves for DOA estimation with incident signals of vertical and horizontal polarizations, with and without noise coupling, using the TLV antenna array.

of the ML estimators is limited to $(-\pi/2,\ \pi/2)$. This limited search range can be interpreted as additional prior information, which becomes significant at low SNRs, while it is insignificant at high SNRs. On the other hand, the CRLB does not take into account this prior information.

Fig. 5 shows the CRLB curves for DOA estimation of the array for two different cases: a physical array with coupled noise versus a theoretical array with spatial white noise. These curves are obtained separately for vertically polarized and horizontally polarized incident signals. The following conclusions are deduced from the figure.

- The noise coupling improves the DOA estimation performance for vertically polarized signals by 7.6 dB.
- The noise coupling degrades the DOA estimation performance for horizontally polarized signals by 2.6 dB.

One may conclude that the noise coupling may cause significant performance variations of the TLV antenna. Accordingly, the noise coupling is an important factor in the DOA estimation performance and has to be taken into consideration in the antenna design process.

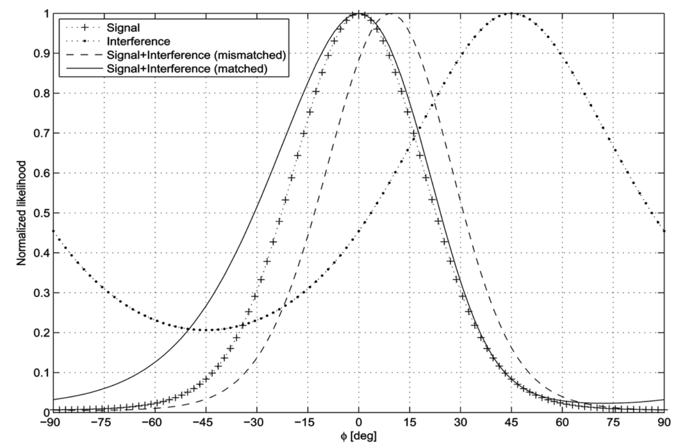

Fig. 6. The likelihood function for source localization: 1) signal without interference noise effect, 2) interference noise only, 3) signal with interference noise (mismatched model), and 4) signal with interference noise (matched model).

In order to demonstrate the effect of the external noise on source localization, the likelihood function [whose logarithm is given in (20)] is evaluated for different cases. In all cases, the receiver center frequency and bandwidth are 900 MHz and 1 MHz, respectively, and the environmental noise temperature distribution is described in (28). In the first case, the scenario of a single source at $\phi = 0^\circ$ and $\theta = 90^\circ$ is considered. In this scenario, the received signal strength is $-120\ \mathrm{dBm}$. In the second case, the signal is removed, and an interferer noise temperature of

$$T(\theta,\phi) = \begin{cases} 5000^\circ\mathrm{K}, & 85^\circ \le \theta < 90^\circ, 35^\circ \le \phi < 55^\circ \\ 0^\circ\mathrm{K}, & \text{otherwise} \end{cases}$$

is added. In the third and fourth cases, the signal and the interference noise sources from the previous cases are combined. In the fourth case, the interferer noise current covariance matrix is taken into account by the ML estimator (matched case), while in the third case it is ignored (mismatched case). Fig. 6 shows the normalized likelihood function for the four cases described above. One can observe that in the first two cases, the likelihood function is maximized at $\phi = 0^\circ$ for the signal and at $\phi = 45^\circ$ for the interference noise, as expected. In the third case (signal and interferer with modeling mismatch), the likelihood function peak moves, and a bias error of about 8° is introduced. In the fourth case, the likelihood function peak returns back to $\phi = 0^\circ$, as desired.

V. Conclusion

In this paper, an approach for evaluating the covariance matrix of the noise currents at the ports of a wire antenna array was presented. This approach takes into consideration the noise coupling between the antenna element ports. The obtained covariance matrix was used to derive the CRLB for source localization with a general antenna array and arbitrary polarized signals. Exploration of the statistical noise coupling behavior has two major contributions. The first contribution is important for the implementation of optimal localization algorithms, which are matched to the noise statistical characteristics. The second contribution is important for obtaining accurate performance evaluation tools in the presence of noise coupling. A numerical example of a two-dipole antenna array was presented to illustrate the noise coupling effect with passive loads. An additional numerical example of a two loop vector antenna array confirmed the fact that the knowledge of the noise current covariance matrix is crucial for achievement of optimal estimation performance.

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