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Excitation of Microwave Vortices by Ferrite Disks: the Near-Field and Far-Field Application Problems

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***Abstract*- A small FMR ferrite disk strongly perturbs the cavity field. Graphical representation of the Poynting vector shows that due to the vortex structure the absorption cross-section of a ferrite particle can be much bigger than its geometrical cross-section. Spectral properties of magnetic oscillations in a ferrite disk show unique vortex structures for MS-wave functions.**

***Index Terms*- microwave vortices, ferromagnetic resonance, MS-wave oscillations**

I. INTRODUCTION

In recent years there has been considerable interest in electromagnetic processes with broken symmetry. There are the vortex fields characterizing by different topological effects. It is supposed that these "swirling" entities can, in principle, be used to carry data and point to a new era for communication systems. Application of such generic ideas to microwave systems is of increasing importance in numerous utilizations. For example, any work in the area of a "vortex antenna" is likely to generate many unique microwave systems. e.g. in direction finding and target ranging. Detailed studies of vortex formations in different electromagnetic structures were a subject of numerous investigations. It concerns, for example, vortices of the Poynting vector field in the near zone of antennas [1], vortex formation near an iris in a rectangular waveguide [2], light transmission through a subwavelength slit [3], energy flow in photonic crystal waveguide [4]. Based on the vortex investigations, a new field of research – the singular optics – has been founded. Phase singularities are important both in the near and far fields. As it was shown in [5], the transition of a near-zone phase singularity into a singularity of the radiation pattern may occur. This, in particular, may give a basis for design of fundamentally new EM-wave antennas.

Recent studies in optics [6] show that for a case of a nanoparticle illuminated by the electromagnetic field, the "energy sink" vortices with spiral energy flow line trajectories are seen in the proximity of the nanoparticle's plasmon resonance. It appears that ferrite inclusions with the FMR conditions should provoke creation of vortices in microwaves. In such a question, two main aspects of studies are on the agenda: (a) vortices of the electromagnetic fields due to material (ferrite) gyrotropy and sample geometry and (b) vortices of microwave magnetic oscillations inside a ferrite sample.

In papers [7, 8], ferrite inclusions with the FMR conditions were used to analyze a role of material gyrotropy as a factor leading creation of microwave vortices. No proper analyses of the geometry factor as well as an analysis of the fields inside small ferrite samples were made in these works. The geometry factor and "internal" fields exhibit, however, very unique properties necessary for understanding physics of the vortex creation. The purpose of this paper is to study numerically the fields outside and inside a ferrite disk placed in a rectangular-waveguide cavity. The disk is very thin compared to the waveguide height. Such a nonintegrable system shows very unique vortex properties. There is also a special interest in such ferrite samples in a view of recent studies of interaction of magnetostatic (MS) oscillations with cavity EM fields. Experimental studies show unique properties of interaction of oscillating MS modes in small ferrite disk resonators with microwave-cavity electromagnetic fields [9 – 12]. The character of the experimental multi-resonance absorption spectra leads to a clear conclusion that the energy of a source of a DC magnetic field is absorbing "by portions", or discretely, in other words [13]. The MS modes in a ferrite disk are characterized by dynamical symmetry breaking [14]. As it was assumed in [9 – 12], the cavity-field structure is not strongly perturbed by a ferrite sample. So it was supposed that the acting RF field corresponds to the original cavity field in a point where a ferrite sample is placed. As we will show in this paper, even a small FMR-disk may strongly perturb the cavity field.

It is necessary to note that general scattering and propagation characteristics of microwave waveguides containing cylindrical ferrite samples were under investigations in numerous works during many years (see, e.g. [15]). Nevertheless, an analysis of symmetry breaking in small ferrite samples constitutes a new subject in microwave research. The problem of microwave vortices of the Poynting vector created by ferrite disks looks as very important for many modern applications, e.g., for near-field microwave lenses, for field concentration in patterned microwave metamaterials, for high-Q resonance microwave devices, for new microwave antennas.

II. ELECTROMAGNETIC-WAVE VORTICES ON A THIN FERRITE DISK IN A CAVITY

The main system under investigations is a *X*-band rectangular-waveguide cavity with an enclosed ferrite disk. There is a short-wall cavity with an iris [Fig. 1]. Metal walls of a cavity are made of copper and are characterized by the conductivity of $\sigma = 5.8\times10^7\, Siemens/m$. A ferrite [yttrium iron garnet (YIG), $\Delta H = 0.1 Oe$] is saturated ($4\pi M_s = 1880\, G$) by a DC magnetic field normal to a disk plane. A ferrite disk (diameter 6 mm, thickness 0.1 mm) is very thin compared to the waveguide height and is placed in the middle of the waveguide height. The working frequency (8.7 GHz) and a quantity of a bias magnetic field (5030 *Oe*) correspond to necessary conditions for a ferromagnetic resonance: a diagonal component of the permeability tensor is $\mu/\mu_0 = 23.85$ and an off-diagonal component is equal to $\mu_a/\mu_0 = 22.55$ [16]. The microwave power enters to a cavity through an iris.

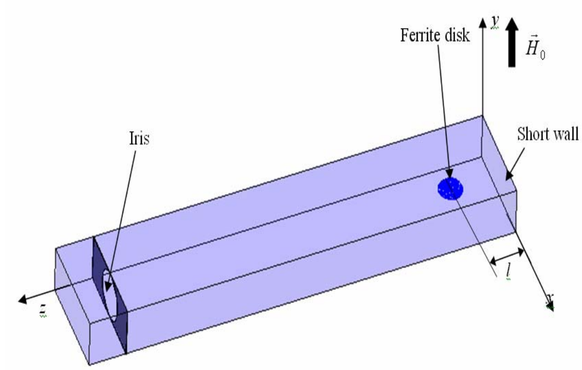

Fig. 1. Rectangular-waveguide TE_{104} cavity with a ferrite disk

The cavity operates at the TE_{104} mode. By virtue of such an elongated structure one can clearly observe the power flow distribution. The cavity resonances were estimated via frequency dependent absorption peaks of the S_{11} parameter of the scattering matrix. Since the nonintegrable nature of the problem precludes exact analytical results for the eigenvalues and eigenfunctions, numerical approaches are required. Using the HFSS (the software based on FEM method produced by ANSOFT Company) and the CST MWS (the software based on FITD method produced by Computer Simulation Technology Company) CAD simulation programs for 3D

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numerical modeling of Maxwell equations, we are able to characterize the complete complex signal including the intensity of the signal and its phase relative to the incoming reference wave. In our numerical experiments, both a modulus and a phase of the fields are determined. It allows reconstructing the Poynting vector at any point within the resonator. The main results of numerical simulations we obtained based on the HFSS program since this program is more relevant for precise analyzing high-resonant microwave objects. The CST program was used just as a test program for some cases. All the below pictures correspond to the HFSS-program numerical results.

As a starting point, we investigated the power flow characteristics for a lossy dielectric disk (disk diameter 6 mm, disk thickness 0.1 mm, $\varepsilon_r = 15$, $\tan\delta = 0.01$) inside a rectangular-waveguide cavity.

A disk is placed at distance $l = \lambda/4$ from a short wall. Fig. 2 demonstrates distribution of the Poynting vector in a cavity with a dielectric disk. One can see how the power flow is distributed with respect to lossy metallic walls and lossy dielectric disk. When one inserts a piece of a lossy magnetized ferrite into the resonator domain, the time-reversal symmetry of a microwave resonator becomes broken. A ferrite disk may act as a topological defect causing induced vortices. Figs. 3 (a), (b) show the streamlines of the Poynting vector when a ferrite disk is inserted in a cavity in a maximum of the RF electric field ($l = \lambda/4$). One can distinguish the clockwise and counter-clockwise rotations of the power flow corresponding to two opposite orientations of a normal bias field.

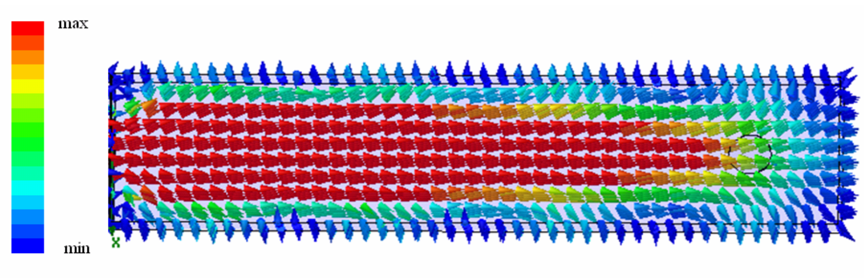

Fig. 2. Rectangular-waveguide TE_{104} cavity with a lossy dielectric disk

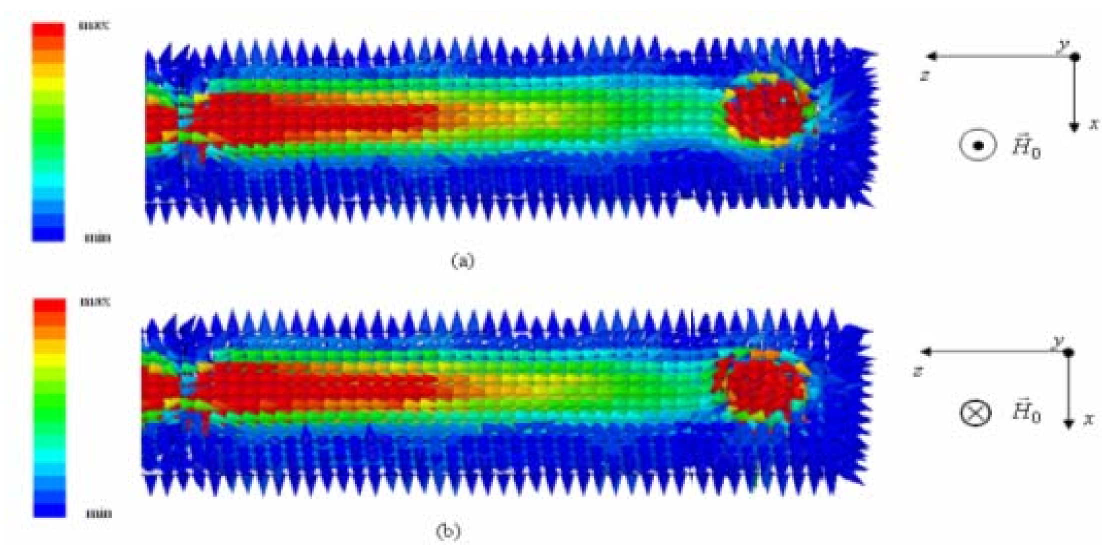

Fig.3. Rectangular-waveguide TE_{104} cavity with a lossy ferrite disk in a maximum of the electric field a. Bias field in the positive y direction, b. Bias field in the negative y direction

The vortex center – the topological singularity – is in a geometrical center of a disk. Fig. 4 shows the Poynting vector distribution immediately inside a ferrite disk.

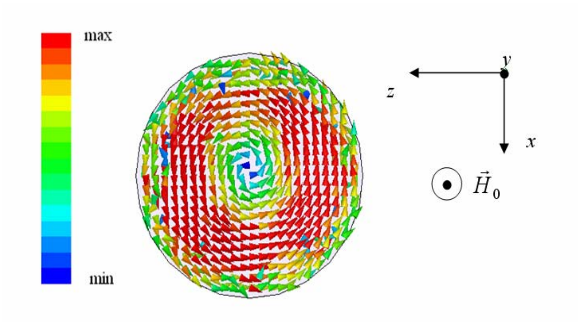

Fig. 4. Poynting vector inside a ferrite disk

Figs. 5 and 6 show, respectively, the electric and magnetic RF fields in a disk. One of the main points of this study is the fact that when a ferrite disk was assumed without losses, no vortices of the Poynting vector were observed.

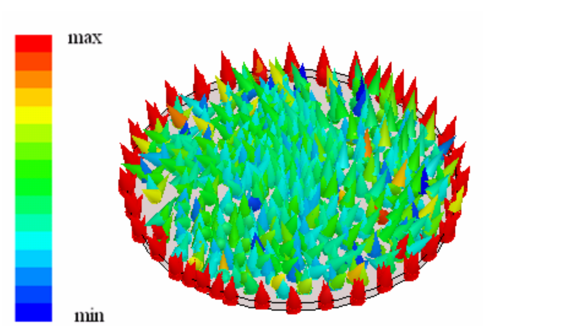

Fig. 5. Electric field vector inside a ferrite disk

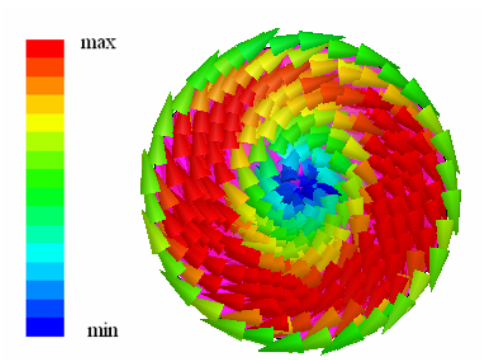

Fig. 6. Magnetic field vector inside a ferrite disk

The vortex picture becomes essentially different when one places a ferrite disk in a maximum of the cavity magnetic field ($l = \lambda/2$). This picture is shown in Figs. 7.

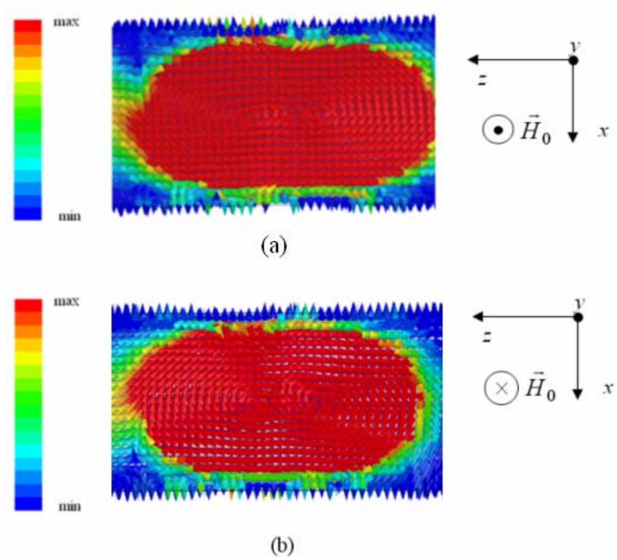

Fig. 7. A lossy ferrite disk in a maximum of the magnetic field:
(a) Bias magnetic field in the *y* direction
(b) Bias magnetic field in the -*y* direction

In this situation one can distinguish two coupled vortices having the same "topological charge" (the same direction of rotation for a given direction of the bias magnetic field). The vortices centers are shifted from a geometrical center of a disk and are situated near the disk border.

III. MAGNETOSTATIC-WAVE VORTICES IN A THIN FERRITE DISK

From a graphical representation of the Poynting vector one sees that the absorption cross-section of a ferrite particle can be much bigger than its geometrical cross-section. When a vortex is created, power flow lines passing through the ferrite particle generate the high energy losses associated with the large microwave cross-section. The reason why a ferrite sample absorbs much more radiation than that given by the geometrical cross-section can be explained by

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the fact that the field enters into the particle not only from the front (with respect to a power source) part of the surface, but also from the back ("shadow") side. In other words, one can suppose that the sample absorbs incident energy through its whole surface.

The obtained pictures of vortices on a thin ferrite disk in a cavity are pure electromagnetic-wave vortices which appear due to the ferrite-material gyrotropy and the geometrical factors. At the same time, inside a ferrite disk may exist eigen modes of MS oscillations. One of the very interesting features of these modes is the fact of dynamical symmetry breaking leading to the handedness properties. This may create a very special behavior of the MS-mode interactions with the EM-wave vortices.

In microwaves, ferrite resonators with multi-resonance MS oscillations have sizes two-four orders less than the free-space EM wavelength at the same frequency [16]. These oscillations occupy a special place between the “pure” electromagnetic and spin-wave (exchange) processes. The energy density of MS oscillations is not the electromagnetic-wave density of energy and not the exchange energy density as well. These “microscopic” oscillating objects – the particles – may interact with the external EM fields by a very specific way. Very unique properties of magnetic-dipolar-oscillation spectra can be found in a case of a normally magnetized ferrite disk. To describe the particle-field interactions, mathematical apparatus similar to the quantum mechanical analysis should be used [13, 14].

For MS-potential wave functions ψ, in a normally magnetized ferrite-disk resonator one has the eigenvalue differential equation [13, 14]: $\mu \nabla_{\perp}^2 \tilde{\varphi}_q = \beta_q^2 \tilde{\varphi}_q$, where $\tilde{\varphi}_q(r,\theta)$ is an "in-plane" function of mode q with radial r and azimuth θ distribution of MS potential, $\nabla_{\perp}^2$ is the two-dimensional, “in-plane”, Laplace operator, β_q is the propagation constant along z-axis, and μ is the diagonal component of the permeability tensor. A double integration by parts (the Green theorem) on S – a square of an “in-plane” cross section of an open ferrite disk – of the integral $\int (\mu \nabla_{\perp}^2 \tilde{\varphi})\tilde{\varphi}^* \, dS$, gives the following boundary condition for the energy orthonormality:

$$\mu\left(\frac{\partial \tilde{\varphi}}{\partial r}\right)_{r=\Re^-} - \left(\frac{\partial \tilde{\varphi}}{\partial r}\right)_{r=\Re^+} = 0, \qquad (1)$$

where $\Re^-$ and $\Re^+$ designate, respectively, the inner (ferrite) and outer (dielectric) regions of a disk resonator with radius $\Re$. The boundary conditions (1) are the so-called essential boundary conditions. In accordance with the Ritz method it is sufficient to use basic functions from the energetic functional space with application of the essential boundary conditions [17]. When such boundary conditions are used, the MS-potential eigen functions form a complete functional basis. The essential boundary conditions differ from the homogeneous electrodynamics boundary conditions at $r = \Re$, which demand continuity for the radial component of the magnetic flux density (together with continuity for potential $\tilde{\varphi}$). The last ones are called as natural boundary conditions [17]. In a cylindrical coordinate system, continuity for a radial component of the magnetic flux density (the natural boundary condition) at $r = \Re$ is described as:

$$\mu(H_\rho)_{\rho=\Re^-} - (H_\rho)_{\rho=\Re^+} = -i\mu_a\,(H_\alpha)_{\rho=\Re^-}, \qquad (2)$$

where μ_a is the off-diagonal component of the permeability tensor, $H_\rho = -\frac{\partial \psi}{\partial \rho}$ and $H_\theta = -\frac{1}{r}\frac{\partial \psi}{\partial \theta}$ are radial and azimuth component of the RF magnetic field, respectively. Supposing that $\tilde{\varphi} \sim e^{-i\nu\theta}$, one can rewrite (2) as:

$$\mu\left(\frac{\partial \tilde{\varphi}}{\partial r}\right)_{r=\Re^-} - \left(\frac{\partial \tilde{\varphi}}{\partial r}\right)_{r=\Re^+} = -\frac{\mu_a}{\Re}\nu(\tilde{\varphi})_{r=\Re^-}. \qquad (3)$$

One can see that “in-plane” functions $\tilde{\varphi}$, being determined by two second-order differential equations (the Bessel equations for functions $\tilde{\varphi}$ inside and outside the ferrite region) and one first-order differential equation (3), are dependent on both a quantity and a sign of ν. So the functions $\tilde{\varphi}$ cannot be single-valued functions for angle θ varying from 0 to 2π. In other words, we have different results for positive and negative directions of an angle coordinate when $0 \le \theta \le 2\pi$. Because of the double-valuedness properties of MS-potential functions on a lateral surface of a ferrite disk resonator, we can talk about the "spinning-type rotation" along a border contour L.

To analyze symmetry properties of MS modes in a ferrite disk we consider the wave propagation in a helical coordinate system. In the Waldron's helical coordinate system (r, ϕ, ζ) [18], solutions of the Laplace and Walker equations for MS-potential wave function ψ are found as

$$\psi(r,\phi,\zeta) = R(r)P(\phi)Z(\zeta), \tag{4}$$

where

$$\begin{aligned} P(\phi) &\sim \exp(\pm iw\phi), \\ Z(\zeta) &\sim \exp(\pm i\beta\zeta). \end{aligned} \tag{5}$$

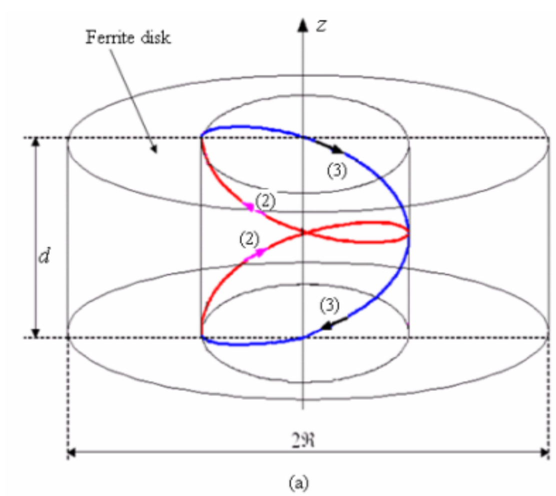
Inside a ferrite region ($r \le \Re$) the solutions are:

$$\begin{aligned}
\psi^{(1)} &= a_1 J_{(w-\bar{p}\beta)}\left[(-\mu)^{1/2}|\beta| r\right] e^{-iw\phi} e^{-i\beta\zeta}, \\
\psi^{(2)} &= a_2 J_{(w-\bar{p}\beta)}\left[(-\mu)^{1/2}|\beta| r\right] e^{+iw\phi} e^{-i\beta\zeta}, \\
\psi^{(3)} &= a_3 J_{(w-\bar{p}\beta)}\left[(-\mu)^{1/2}|\beta| r\right] e^{+iw\phi} e^{+i\beta\zeta}, \\
\psi^{(4)} &= a_4 J_{(w-\bar{p}\beta)}\left[(-\mu)^{1/2}|\beta| r\right] e^{-iw\phi} e^{+i\beta\zeta}.
\end{aligned} \tag{6}$$

For an outside region ($r \ge \Re$) one has:

$$\begin{aligned}
\psi^{(1)} &= b_1 K_{(w-\bar{p}\beta)}(|\beta| r) e^{-iw\phi} e^{-i\beta\zeta}, \\
\psi^{(2)} &= b_2 K_{(w-\bar{p}\beta)}(|\beta| r) e^{+iw\phi} e^{-i\beta\zeta}, \\
\psi^{(3)} &= b_3 K_{(w-\bar{p}\beta)}(|\beta| r) e^{+iw\phi} e^{+i\beta\zeta}, \\
\psi^{(4)} &= b_4 K_{(w-\bar{p}\beta)}(|\beta| r) e^{-iw\phi} e^{+i\beta\zeta}.
\end{aligned} \tag{7}$$

Here J and K are Bessel functions of real and imaginary arguments, respectively. Coefficients $a_{1,2,3,4}$ and $b_{1,2,3,4}$ are amplitude coefficients. Due to interactions between these modes one has two types of the vortex-like resonances. The pictures of such resonant interactions inside a ferrite disk are shown in Figs 8 a, b. Taking into account the boundary conditions above and below a ferrite disk, one obtains an entire picture of the helical MS mode distribution inside and outside a disk. Fig. 9 shows an example of such a picture.



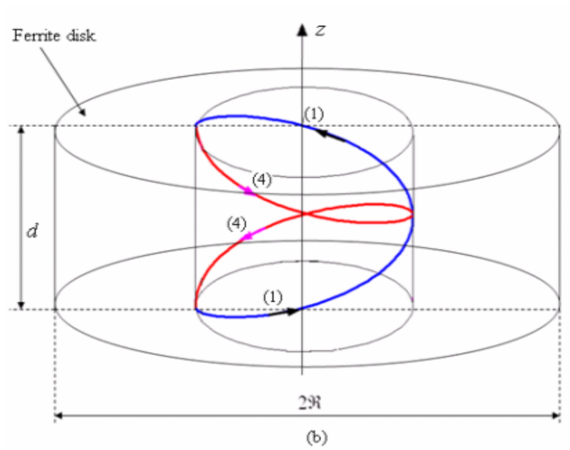


Fig. 8. (a) The "right" resonance caused by the $\psi^{(1)} \leftrightarrow \psi^{(4)}$ interaction. (b) The "left" resonance caused by the $\psi^{(2)} \leftrightarrow \psi^{(3)}$ interaction. Arrows show directions of propagation for helical MS modes.

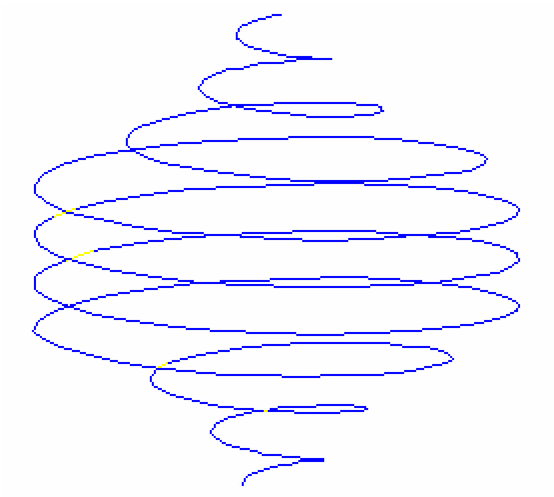


Fig. 9. Illustration of the structure of the helical MS-potential wave function in a ferrite disk

IV. DISCUSSION AND CONCLUSION

The obtained results of the ferrite-disk vortex structures of the EM fields and the MS-wave oscillations open unique possibility for new far-field and near-field microwave applications. First of all, there could be a basis for creation of fundamentally new EM-wave antennas. With a proper microwave design, one can realize the transition of the observed topological singularities of the Poynting vector inside a waveguide resonator into a singularity of the radiation pattern. Another possibility appears with use of unique spectral characteristics of magnetic oscillations for the near-field scanning microwave microscopy (NSMM).

The NSMM have proven useful for unique experiments extracting material properties of many condensed matter systems for both fundamental and applied physics. Recent results create the opportunity for a new class of NSMM experiments on nanometer length scales [19]. It becomes clear that new perfect lenses that can focus beyond the diffraction limit could revolutionize near-field microscopy. Our studies show that a new type of near-field microwave lenses can be realized based on the oscillation spectral properties of MS modes in a normally magnetized ferrite disk. Since the characteristic sizes of the MS-wave oscillations are two-four orders of the magnitude less than the EM-wave oscillations at the same microwave frequency, the proposed NSMMs should give extremely high resolution compared to the known NSMMs. Because of the nature of the MS oscillations, there will not be problems with the effective shielding off the far-field components of the high-order modes [19]. Because of special symmetry properties of the oscillating spectra in a ferrite disk, the proposed microwave microscope will allow measuring chirality in chemical and biological objects on nanometer length scales.

REFERENCES

[1] F. Landstorfer, H. Meinke, and G. Niedermair, Nachr. tech. Z. **25**, 537 (1972).
[2] R. W. Ziolkowski and J. B. Grant, IEEE Trans. Microw. Theory Tech. **MTT-34**, 1164 (1986).
[3] H. F. Schouten, T. D. Visser, and D. Lenstra, J. Opt. B **6**, S404 (2004).
[4] T. Søndergaard and K.H. Dridi, Phys. Rev. B **61**, 15688 (2000).
[5] H. F. Schouten, T. D. Visser, G. Gbur, and D. Lenstra, Phys. Rev. Lett. **93**, 173901 (2004).
[6] M. V. Bashevoy, V.A. Fedotov, and N.I. Zheludev, Opt. Express **13**, 8372 (2005).
[7] P. So *et al*, Phys. Rev. Lett. **74**, 2662 (1995); U. Soffregen *et al*, Phys. Rev. Lett. **74**, 2666 (1995); H. Schanze *et al*, Phys. Rev. E **71**, 016223 (2005).
[8] M. Vraničar *et al*, J. Phys. A: Math. Gen. **35**, 4929 (2002).
[9] J.F. Dillon Jr., J. Appl. Phys. **31**, 1605 (1960).
[10] T. Yukawa and K. Abe, J. Appl. Phys. **45**, 3146 (1974).
[11] A.K. Saha, E.O. Kamenetskii, and I. Awai, Phys. Rev. E **64**, 056611 (2001); J. Phys. D: Appl. Phys. **35**, 2484 (2002).
[12] E.O. Kamenetskii, A.K. Saha, and I. Awai, Phys. Lett. A **332**, 303 (2004).
[13] E.O. Kamenetskii, Phys. Rev. E **63**, 066612 (2001); E.O. Kamenetskii, R. Shavit, and M. Sigalov, Europhys. Lett. **64**, 730 (2003); E.O. Kamenetskii, M. Sigalov, and R. Shavit, J. Phys.: Condens. Matter **17**, 2211 (2005).
[14] E.O. Kamenetskii, Europhys. Lett. **65**, 269 (2004); Phys. Rev. E **73**, 016602 (2006); J.Magn. Magn. Mater. **302**, 137 (2006).
[15] N. Okamoto, I. Nishioka, and Y. Nakanishi, IEEE Trans. Microw. Theory. Techn. **MTT-19**, 521 (1971); T. Yoshida, M. Umeno, and S. Miki, IEEE Trans. Microw. Theory. Techn. **MTT-20**, 739 (1972).
[16] A. Gurevich and G. Melkov, *Magnetic Oscillations and Waves* (CRC Press, New York, 1996).
[17] S. G. Mikhlin, *Variational Metods in Mathematical Physics* (Mc Millan, New York, 1964).

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[18] R.A. Waldron, Quarterly J. Mech. Appl. Math. **11**, 438 (1958).
[19] A. Imtiaz *et al*, J. Appl. Phys. **97**, 044302 (2005).