Array pattern synthesis using neural networks with mutual coupling effect

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Abstract: In the paper a neural network algorithm is presented for the synthesis of array patterns in the presence of mutual coupling. The algorithm is based on wavelet activation functions and is tested on a linear dipole array with a shaped pattern. The excitations and the lengths of the dipoles in the array are determined. The architecture of the neural network is discussed and simulation results are presented. The proposed algorithm makes the synthesis procedure more practical and accurate compared to the synthesis technique without mutual coupling considerations.

1 Introduction

The array pattern synthesis problem is defined as that of finding the array excitations to produce the required antenna radiation pattern. Pattern synthesis is required in different applications of wireless, satellite and radar communication systems. There are numerous synthesis methods, like the Woodward–Lawson method [1], the Orchard–Elliott method [2], the modified Orchard–Elliott method [3] and others. Optimisation algorithms, like genetics algorithm (GA) [4] and neural-network (NN) algorithms [5] have been employed for antenna array synthesis. The NN algorithm is faster than other algorithms, due to its unique parallel structure for real-time applications, and is especially efficient, if the desired pattern belongs to the family of patterns on which the training process was conducted. In [6] it has been shown that, for shaped patterns with sharp transitions like flat top patterns with constant side lobes level (SLL), the wavelet activation function has an edge (in terms of computation time) over other activation functions. Direct implementation of the computed excitations, by these synthesis algorithms, fails to generate the desired radiation pattern due to the mutual coupling effect among the array elements. Hence, a compensation procedure is required to offset the mutual coupling effect and obtain the correct array elements' excitations for the desired pattern. In [7] such a procedure is presented for a shunt slot array fed by a rectangular waveguide, taking into consideration the mutual coupling effect among the elements.

In this paper, a neural-network algorithm with one hidden layer, based on wavelet activation functions and trained with examples generated by the Orchard–Elliott method [2], is presented. The algorithm takes into consideration the mutual coupling effect in a linear dipole array with a shaped pattern. The excitations and the lengths of the dipoles in the array are determined. The mean square error (MSE) criterion is used to quantify the agreement between the desired and computed patterns.

2 Theory

The basic structure of an artificial neural network is an array of processing elements (also called neurons) ordered in layers with a network of interconnection weights, w_j $(j = 1, \ldots, L)$ between the neurons (also called synaptic weights) [8] in different layers. The input data x_j are processed through an activation function $f(\boldsymbol{x})$. The basic model of a single neuron is shown in Fig. 1. The output y of the neuron in this model is given by

$$y = f\left(\sum_{j=1}^{L} w_j \cdot x_j + b\right) \tag{1}$$

where b is the bias parameter of the activation function $f(\boldsymbol{x})$. By adding a new fixed input $x_0 = 1$ and defining the synaptic weight of this input as $w_0 = b$, the bias can be evaluated in the training stage [8].

The inputs to the network are samples of the desired pattern. The number of input nodes (neurons) is dependent on the accuracy needed to represent the desired pattern. The outputs of the network are the amplitude and phase distribution values of the array elements and the lengths of the dipoles in the array. The interconnection weights w_j are generated in the training stage, using patterns with similar characteristics to the desired pattern generated by the Orchard–Elliott method [2].

The NN architecture considered in this paper has one hidden layer and uses wavelet functions as activation functions as described in [6]. Owing to the special characteristics of the wavelet functions, wavelet neural networks (WNN) can approximate better patterns with sharp transitions [6]. The chosen wavelet activation function $f(\gamma_i)$ is the inverse Mexican hat function [9] given by

$$f(\gamma_i) = (\gamma_i^2 - 1) \cdot \exp\left(-\frac{\gamma_i^2}{2}\right) \tag{2}$$

where

$$\gamma_i = \left|\frac{\boldsymbol{x} - \boldsymbol{t}_i}{a_i}\right| = \sqrt{\sum_{j=1}^{L}\left(\frac{x_j - t_{ij}}{a_i}\right)^2} \tag{3}$$

IEE Proceedings online no. 20045121

doi:10.1049/ip-map:20045121

Paper first received 26th September and in revised form 9th December 2004

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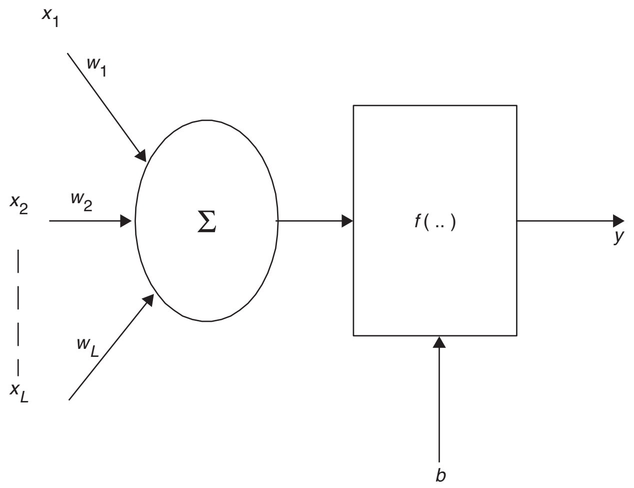

Fig. 1 *Basic model of a neuron*

in which $\boldsymbol{x} = [x_1\ x_2\ \ldots\ x_L]^T$ is the input vector, $\boldsymbol{t}_i = [t_{i1}\ t_{i2}\ \ldots\ t_{iL}]^T$ is the translation parameter vector and a_i is the dilation parameter. The networks weights, translation and dilation parameters are obtained using the back-propagation algorithm as discussed in [5] and [8].

In this study, the array element has been chosen to be a centre-fed cylindrical dipole with length $2l$ and radius a, due to its closed-form self-impedance analytical form. The closed form of such a dipole, in the range of interest $1.3 \leq kl \leq 1.7$ and $0.001588 \leq a/\lambda \leq 0.009525$, [10] is

$$Z = [122.65 - 204.1kl + 110(kl)^2] - j\left[120\left(\ln\frac{2l}{a} - 1\right)\cot kl - 162.5 + 140kl - 40(kl)^2\right] \quad (4)$$

The mutual coupling effect in the array can be evaluated, if we consider two dipoles with lengths and radii $(2l_i, a_i)$ and $(2l_j, a_j)$. The dipoles are z-directed, are placed along the x axis and separated by the distance d_{ij} as shown in Fig. 2. The voltages V_i and V_j are applied across the central gaps of these two dipoles and the input currents I_i and I_j will flow into the dipoles.

Under the assumption of sinusoidal current distribution on each dipole, it can be shown [10] that the real R_{ij} and imaginary X_{ij} components of the mutual impedance, Z_{ij} are

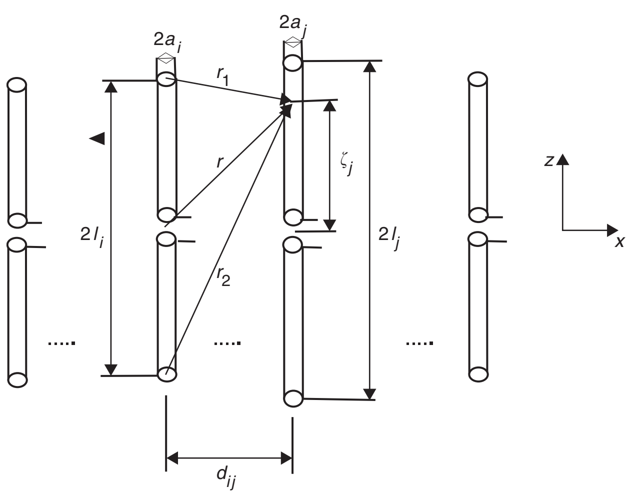

Fig. 2 *Dipole array geometry*

given by

$$R_{ij} = \frac{30}{\sin kl_i \cdot \sin kl_j}\int_{-l_j/\lambda}^{l_j/\lambda}\left(\frac{\sin kr_1}{r_1/\lambda} + \frac{\sin kr_2}{r_2/\lambda} - (2\cos kl_i)\frac{\sin kr}{r/\lambda}\right)\sin k(l_j - |\zeta_j|)d\left(\frac{\zeta_j}{\lambda}\right) \quad (5)$$

$$X_{ij} = \frac{30}{\sin kl_i \cdot \sin kl_j}\int_{-l_j/\lambda}^{l_j/\lambda}\left(\frac{\cos kr_1}{r_1/\lambda} + \frac{\cos kr_2}{r_2/\lambda} - (2\cos kl_i)\frac{\cos kr}{r/\lambda}\right)\sin k(l_j - |\zeta_j|)d\left(\frac{\zeta_j}{\lambda}\right) \quad (6)$$

in which the parameters $r = [d_{ij}^2 + \zeta_j^2]^{1/2}$, $r_1 = [d_{ij}^2 + (\zeta_j - l_i)^2]^{1/2}$ and $r_2 = [d_{ij}^2 + (\zeta_j + l_i)^2]^{1/2}$ as shown in Fig. 2. Accordingly, the input impedance, Z_{in} to the nth element in an M element array is given by

$$Z_{in} = \frac{V_n}{I_n} = Z_{nn} + \sum_{\substack{i=1\\ i\neq n}}^{M}\frac{I_i}{I_n}Z_{ni} \quad (7)$$

The right term of (7) includes the self-impedance and the mutual-impedance effect. Maximum power transfer to the nth dipole in the array is obtained if the imaginary part of its input impedance Z_{in} is equal to zero. This requirement is obtained by varying the length of the dipole.

The neural network is trained with input/output sets of examples generated by the Orchard–Elliott method for a flat-top pattern with constant sidelobes level (SLL). The array excitations from the Orchard–Elliott method and the calculated dipole lengths, which nullify the imaginary part of the active impedance of each of the dipoles in the array, are used to compute the interconnection weights w_j of the NN. The training process purpose is to adjust the network interconnection weights w_j to minimise the error function $E(p)$ defined by

$$E(p) = \frac{1}{2}\sum_{k=1}^{Q}\left[y_k(x_j, w_j) - d_k\right]^2 \quad (8)$$

in which $p = 1, 2, \ldots, P$ is the index of the training set, $k = 1, 2, \ldots, Q$, is the index of the NN outputs, y_k are the neural model outputs (excitation currents and dipole lengths) and d_k are the desired neural model outputs. This is an iterative process using the back-propagation algorithm described in [8]. The weights w_j are updated iteratively. Once the training stage is over and the weights are determined the network is fixed, such that any new input data are processed through the network in one pass from input to output.

The results of the WNN estimation of the array current excitations and the dipoles lengths are fed into the commercial electromagnetic wire analysis software NEC by Nittany Scientific Inc. and the actual array pattern is simulated. This software is based on the method of moments (MoM) technique and is able to simulate the dipole array pattern with the mutual coupling effect.

3 Simulation results

The array used in the simulations is an equispaced 8-element linear array with interelement spacing $d = \lambda/2$. The initial lengths of the dipoles are $\lambda/2$ and the radius to wavelength ratio is chosen as $a/\lambda = 0.00283$. The dipoles radii are kept fixed through the entire process. The training set included a matrix of 75 $(5 \times 5 \times 3)$ flat-top patterns computed by Orchard–Elliott method with sector width intervals of 20°, SLL intervals of 10 dB and ripple intervals of 1 dB. For each pattern, the input power to the array was calculated by

$$P_{in} = \sum_{i=1}^{M} |I_i|^2 \cdot R_i$$

in which I_i is the element excitation and R_i is the real part of the ith active element impedance Z_{in}. This resistance is found using NEC software. All the training patterns were generated by the Orchard–Elliott [2] algorithm. This algorithm generates 2^{N_1} different combinations of current distributions for each desired pattern, where N_1 is the number of roots in the shaped region. The best solution (practical implementation considerations) from all different combinations is the one with minimum ratio of $|I_{\max}/I_{\min}|$. Without loss of generality, the chosen test case studied in this work is the synthesis of a flat-top pattern with –25 dB SLL.

A sensitivity test of the process was conducted to optimise the number of neurons in the hidden layer. The criterion used was the MSE as given by

$$\mathrm{MSE} = \sqrt{\frac{1}{L}\sum_{q=1}^{L}\left[f_q^{(c)} - f_q^{(d)}\right]^2} \tag{9}$$

in which $f_q^{(d)}$ is the sampled desired pattern, $f_q^{(c)}$ is the sampled computed pattern and L is the total number of samples. As a result of this study, the number of neurons chosen in the hidden layer of the WNN was 20. In addition, a sensitivity study was conducted to determine the maximum difference allowed between the desired radiation pattern and the closest training set parameters, like pattern SLL, sector width and ripple level. The sensitivity study was conducted for flat-top patterns with sector width intervals of 3°, SLL intervals of 2 dB and ripple intervals of 0.25 dB. The study results show that the desired pattern SLL, sector width and ripple level should be less than 15 dB, 15° and 1 dB, respectively, from the closest training set for the MSE to be less than 0.1 dB.

Two examples have been considered to prove the efficiency of the suggested array synthesis algorithm in presence of mutual coupling. In the first example, the desired pattern is a flat-top pattern with sector width of 45°, −25 dB SLL and ripple of ±0.5 dB, and, in the second example, a null at 160° was introduced. Figure 3 shows a comparison with the desired pattern, the Orchard–Elliott pattern without mutual coupling considerations, the NEC computed pattern based on the synthetic Orchard–Elliott excitations with mutual coupling considerations and equal dipole lengths of $\lambda/2$, the NEC computed pattern based on the synthetic Orchard–Elliott excitations with mutual coupling considerations, but with corrected dipole lengths based on a WNN algorithm, and the NEC radiation pattern computed with the corrected current excitations and dipole lengths using the proposed WNN synthesis algorithm. We can observe that the pattern computed using the suggested algorithm follows very well the desired pattern and the synthetic Orchard–Elliott patterns, while the NEC computed patterns with the original Orchard–Elliott current excitations deviates significantly from the desired radiation pattern. Figure 4 shows the current distribution of the pattern computed using the suggested algorithm compared to the current distribution of the synthetic Orchard–Elliott current distribution, with and without mutual coupling effect. The dipoles' lengths computed through the WNN algorithm with mutual coupling effect are shown in Table 1. The variation of the total input power for the pattern computed with mutual coupling considerations compared to the comparable Orchard–Elliott pattern without the coupling effect was less than 0.4%.

Figure 5 shows the radiation patterns of the second example: the desired pattern, the synthetic Orchard–Elliott pattern without mutual coupling considerations, the NEC computed pattern based on the synthetic Orchard–Elliott excitations with mutual coupling considerations and equal dipole lengths of λ/2, the NEC computed pattern based on the synthetic Orchard–Elliott excitations with mutual coupling considerations but with corrected dipole lengths based on WNN algorithm and the NEC radiation pattern computed with the corrected current excitations and dipole lengths using the proposed WNN synthesis algorithm. As in

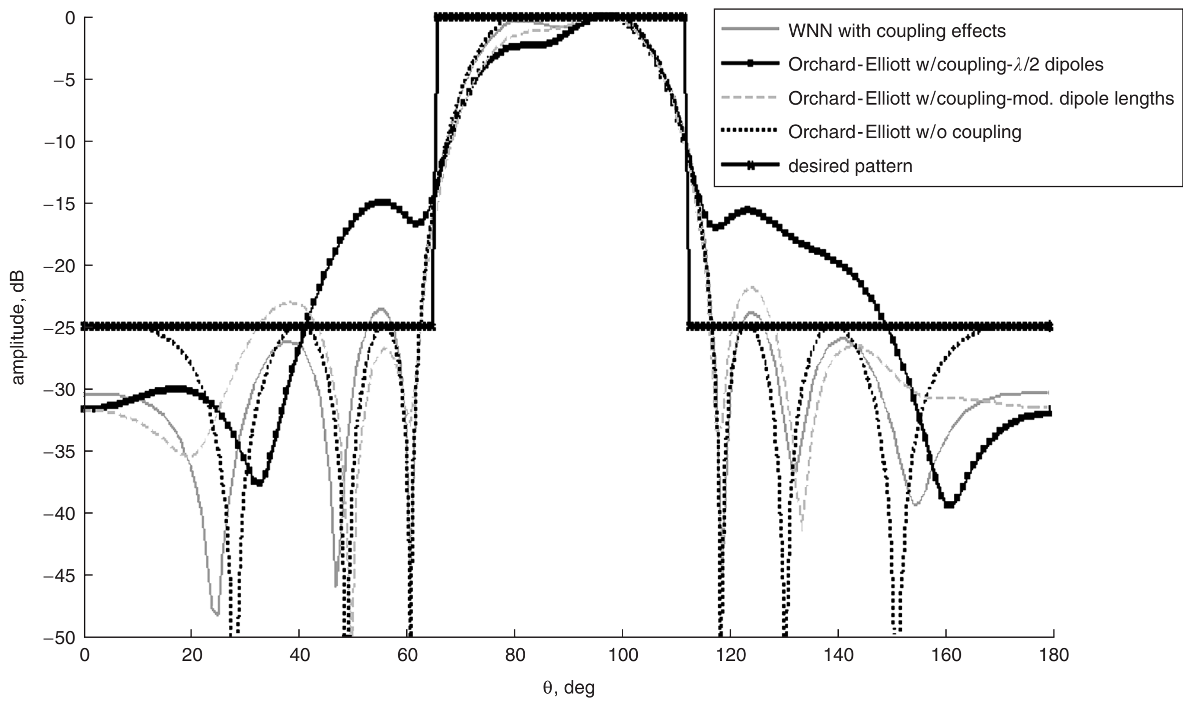

Fig. 3 *Comparison between patterns synthesised by different methods for a flat top pattern with sector width of 45°, −25 dB SLL and ripple of ±0.5 dB*

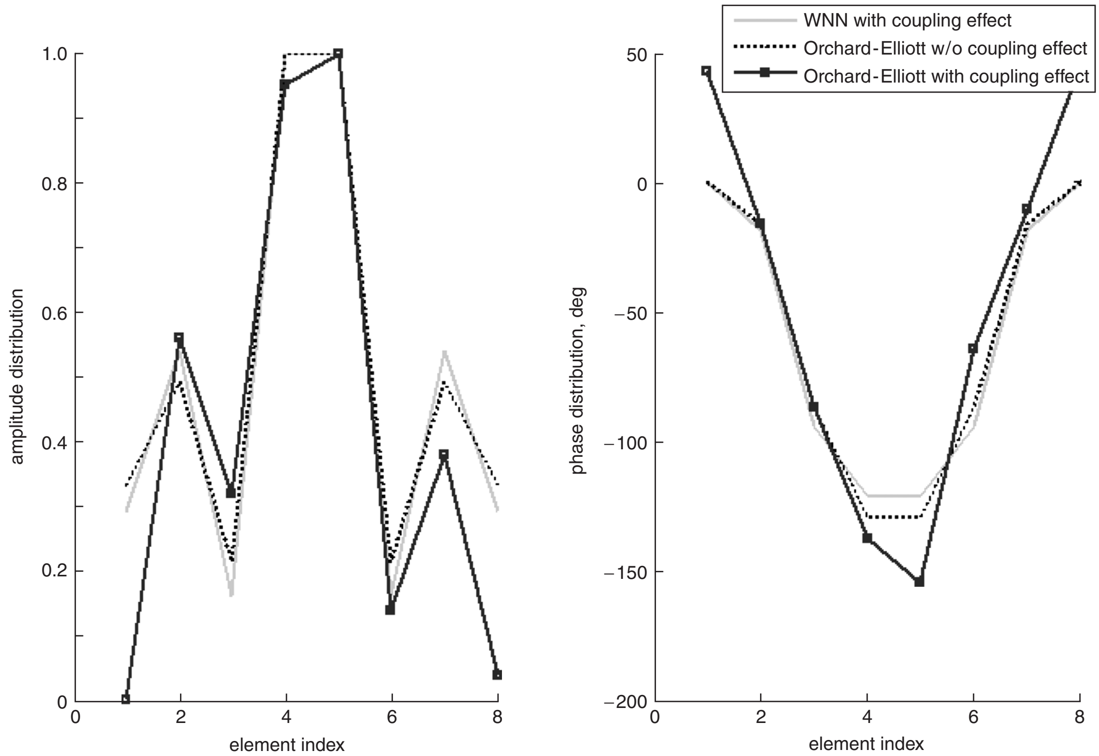

Fig. 4 *Normalised current amplitude and phase distribution for a flat top pattern with sector width of 45°, −25 dB SLL and ripple of ±0.5 dB*

Table 1: Dipole lengths found for a flat top pattern with sector width of 45°, SLL of −25 dB and ripple of ±0.5 dB in the flat region

Element number	1	2	3	4	5	6	7	8
Dipole length, λ	0.487	0.496	0.49	0.493	0.493	0.49	0.496	0.487

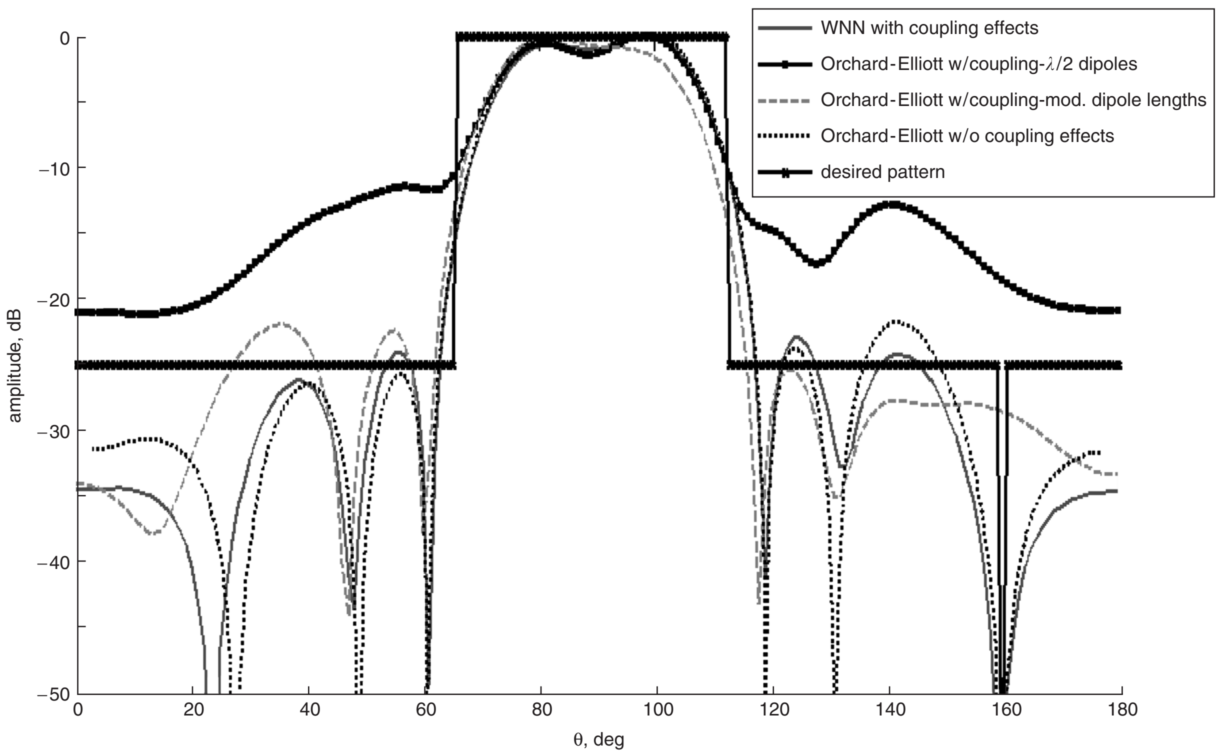

Fig. 5 *Comparison between patterns synthesised by different methods for a flat top pattern with sector width of 45°, −25 dB SLL, ripple of ±0.5 dB and null at 160°*

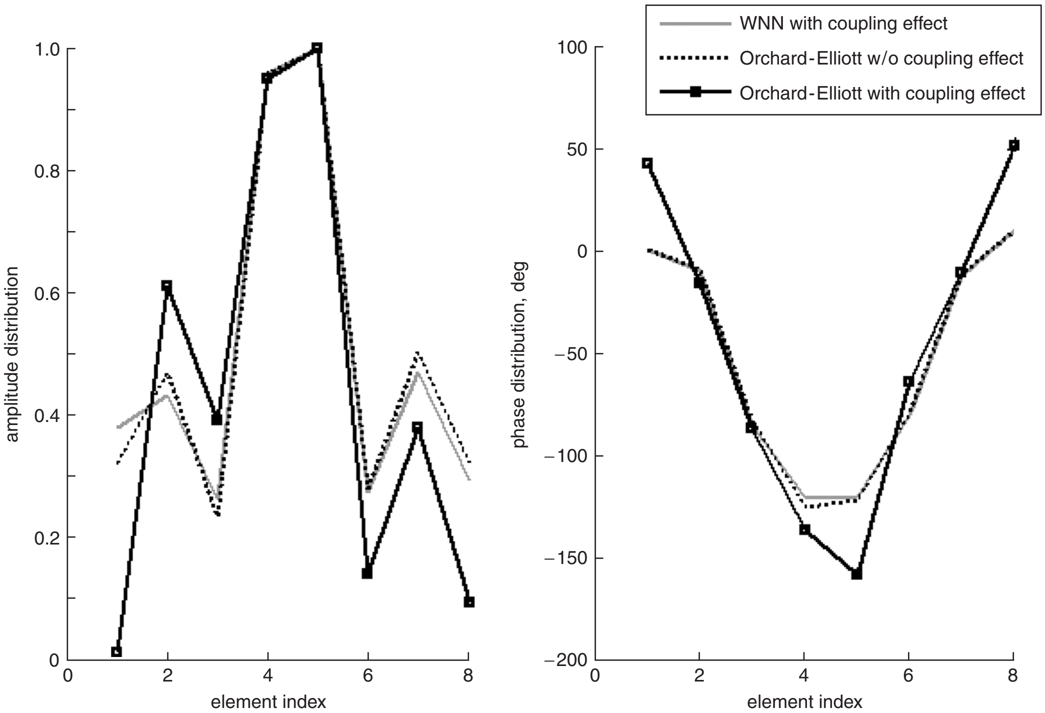

Fig. 6 *Normalised current amplitude and phase distribution for a flat top pattern with sector width of 45°, −25 dB SLL, ripple of ±0.5 dB and null at 160°*

Table 2: Dipole lengths found for a flat top pattern with sector width of 45°, SLL of –25 dB, ripple of ±0.5 dB in the flat region and a null at 160°

Element number	1	2	3	4	5	6	7	8
Dipole length, λ	0.491	0.483	0.49	0.494	0.493	0.49	0.486	0.485

the previous example, we can observe that the pattern computed using the suggested algorithm follows nicely the desired pattern and the synthetic Orchard–Elliott patterns, while the NEC computed patterns with the original Orchard–Elliott current excitations deviates significantly from the desired radiation pattern. Figure 6 shows the current distribution of the pattern computed using the suggested algorithm compared to the current distribution of the synthetic Orchard–Elliott current distribution with and without mutual coupling effect. The dipoles' lengths computed through the WNN algorithm with mutual coupling effect are shown in Table 2.

4 Conclusions

An NN algorithm for the synthesis of array patterns in the presence of mutual coupling has been presented in this paper. The algorithm is based on a wavelet activation function and was tested on a linear dipole array with a shaped pattern. It computes the excitations and the lengths of the dipoles in the array. The proposed algorithm makes the synthesis procedure more practical compared to the synthesis technique without mutual coupling considerations. The algorithm can be used in a vast range of applications, in which the network can be trained offline with suitable data, to carry out real-time processing to find array element excitations.

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