An Efficient Vector Sensor Configuration for Source Localization

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***Abstract*—An electromagnetic vector-sensor enables estimation of the direction of arrival (DOA) and polarization of an incident electromagnetic wave with arbitrary polarization. In this letter, an efficient vector-sensor configuration is proposed. This configuration includes the minimal number of sensors, which enables DOA estimation of an arbitrary polarized signal from any direction except two opposite directions on the *z*-axis. The configuration is obtained by analyzing the Cramer-Rao lower bound (CRLB) for source localization using a single vector-sensor. The resulting vector-sensor configuration consists of two electric and two magnetic sensors. It is shown that this quadrature configuration satisfies the necessary and sufficient conditions for the DOA estimation problem. The CRLB for DOA estimation of signals in the azimuth plane is identical for the quadrature and the complete vector-sensor configurations.**

***Index Terms*—Cramer-Rao lower bound (CRLB), direction-of-arrival (DOA) estimation, electromagnetic vector sensors, Fisher information matrix (FIM), polarization, quadrature vector sensor, source localization.**

I. Introduction

In the last decade, many array processing techniques for source localization and polarization estimation using vector sensors have been developed. Nehorai and Paldi [1] developed the Cramer-Rao Lower bound (CRLB) for this problem and the vector cross-product direction-of-arrival (DOA) estimator. Identifiability and uniqueness issues associated with vector sensors are analyzed in [2]–[6]. Eigenstructure-based source localization techniques, such as ESPRIT and MUSIC using vector sensor arrays have been extensively investigated. Li [7], [8] applied the ESPRIT algorithm to a vector sensor array. ESPRIT-based direction finding algorithms using vector sensor arrays have been further investigated in several papers [9]–[12]. MUSIC-based algorithms for this problem have been applied in [13], [14] . Coherent source localization algorithms are developed in [15].

In [1], it was shown that a complete vector sensor of 6 elements is necessary to estimate the direction of arrival of any source with arbitrary polarization in space. However, in many practical applications, signal DOA in the entire space is not of main concern. For example, in some communication systems, the space of interest is limited to elevations around the azimuth plane (a 2-D problem). In this instance, the required number of sensors can be reduced. Limiting the number of sensors in a vector sensor was first presented in [16] and [17], in which it was shown that measurements from electric *or* magnetic sensors are sufficient for azimuth and elevation estimation. The disadvantage of these configurations is that they are not sufficient for *any* signal polarization.

In this letter, we are interested in finding a minimal configuration for 3-D source localization with arbitrary polarization, and any location in a given space. We confine the space of interest by excluding only two opposite directions on the *z*-axis. A feasibility study is conducted to minimize the number of elements in a vector sensor configuration, capable to estimate the DOA of an arbitrary polarized signal. The CRLB is derived and analyzed for this purpose. It is shown that the resulting vector sensor configuration is unique and consists of two electric and two magnetic dipoles.

This letter is organized as follows. Section II describes the measurement model using a general vector sensor consisting of six sensors as proposed in [1]. In Section III, the CRLB is derived and analyzed to show that the proposed configuration satisfies the necessary and sufficient condition for the source localization problem. Section IV summarizes our conclusions.

II. Problem Formulation and Modeling

Consider a vector sensor consisting of 3 electric and 3 magnetic orthogonal dipoles, as depicted in Fig. 1, (see [1]). The analytical spatial response in matrix notation of the complete vector sensor configuration can be expressed by

$$\mathbf{g}(\theta,\phi,\mathbf{p}) = \underbrace{\begin{bmatrix} -\cos\theta\sin\phi & -\cos\phi \\ \cos\theta\cos\phi & -\sin\phi \\ -\sin\theta & 0 \\ -w\sin\phi & -w\cos\theta\cos\phi \\ -w\cos\phi & w\cos\theta\sin\phi \\ 0 & w\sin\theta \end{bmatrix}}_{\mathbf{A}(\theta,\phi)} \underbrace{\begin{bmatrix} p_\theta \\ p_\phi \end{bmatrix}}_{\mathbf{p}} \quad (1)$$

where w denotes the ratio between the induced voltage in an electric dipole to the corresponding induced voltage in a magnetic dipole. The polarization vector, $\mathbf{p}$, is determined by two real parameters, γ and η: $p_\theta = \sin\gamma e^{j\eta}$ and $p_\phi = \cos\gamma$. In general, an L-size vector sensor may contain part of the 6 sensors shown in Fig. 1.

Consider the scenario of a far-field source signal, impinging on the array from direction (θ,ϕ) and polarization vector $\mathbf{p}$. Then, the spatial response of the vector sensor is denoted by $\mathbf{g}(\theta,\phi,\mathbf{p})$, and the data model is given by

$$\begin{aligned} \mathbf{y} &= \mathbf{g}(\theta,\phi,\mathbf{p})s + \mathbf{n} = \mathbf{A}(\theta,\phi)\mathbf{p}s + \mathbf{n} \\ &= \mathbf{A}(\theta,\phi)\boldsymbol{\mu} + \mathbf{n} \end{aligned} \quad (2)$$

where s stands for the complex amplitude of the incident signal and the vector $\boldsymbol{\mu}$ is the modified polarization vector, defined as $\boldsymbol{\mu} \triangleq [\mu_1 \ \ \mu_2]^T = \mathbf{p}s$. The noise vector, $\mathbf{n}$, is assumed to be

Manuscript received November 16, 2003; revised December 18, 2003. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Darren B. Ward.

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Digital Object Identifier 10.1109/LSP.2004.831682

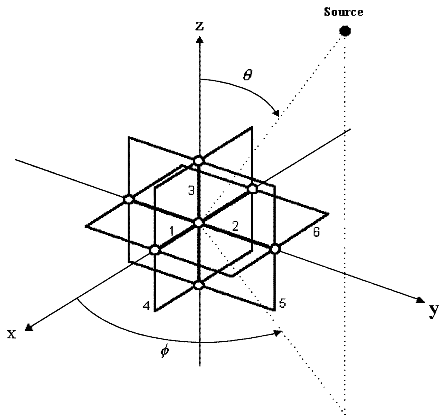

Fig. 1. Complete vector sensor configuration and problem geometry.

zero-mean, complex Gaussian with covariance matrix $\mathbf{R_n} = \sigma_n^2\mathbf{I}$. The measurement and noise vectors, $\mathbf{y}$ and $\mathbf{n}$ are each of size 6, the matrix $\mathbf{A}(\theta,\phi)$ is of size 6×2 whose columns denote the spatial transfer functions for both polarization components of the signal, and $\mathbf{p}$ is a complex vector of size 2 describing the corresponding signal polarization state.

The problem addressed in this letter is determination of the most "efficient" vector sensor configuration. This configuration consists of minimal number of sensors capable to estimate the direction (θ,ϕ) of a single far-field source with an arbitrary polarization and at any direction in space except two singular directions: $\theta = 0°, 180°$. The CRLB is employed and analyzed for this purpose.

III. Design of the "Efficient" Vector Sensor Using CRLB

In this section, the CRLB for source localization using a vector sensor is derived. The "efficient" vector sensor configuration, as described above, is obtained by imposing the following requirements:

R1: Finite CRLB for *any* DOA, θ,ϕ, except two singular directions: $\theta = 0°, \theta = 180°$.

R2: Finite CRLB for *any* polarization vector, $\boldsymbol{\mu}$.

The desired vector sensor configuration is obtained in a two stage process. In the first stage, the necessary sensors are identified for a particular case. For the sake of simplicity, the following particular case is chosen:

- $\boldsymbol{\mu}$, is deterministic known in the form $\boldsymbol{\mu} = [\mu_1 \;\; 0]^T$.
- $\theta = 90°$.

This task will be accomplished by analyzing the CRLB on the DOA estimate. In the second stage, we remove the above assumptions, and show that the necessary configuration obtained in the first stage, satisfies the sufficient conditions for a finite CRLB in the general case of an unknown and arbitrary polarization vector, $\boldsymbol{\mu}$.

A. Derivation of the Necessary Configuration

According to the assumption $\mathbf{R1}$, the vector $\boldsymbol{\mu}$ is known, and therefore, the CRLB for estimating the vector parameter $\boldsymbol{\alpha} = [\theta \;\; \phi]^T$ is given by the inverse of the Fisher Information Matrix (FIM)

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{\theta\theta} & \mathbf{J}_{\theta\phi} \\ \mathbf{J}_{\phi\theta} & \mathbf{J}_{\phi\phi} \end{bmatrix} = 2\mathrm{Re}\left\{ \frac{\partial(\mathbf{A}\boldsymbol{\mu})^H}{\partial\boldsymbol{\alpha}} \mathbf{R}_\mathbf{n}^{-1} \frac{\partial(\mathbf{A}\boldsymbol{\mu})}{\partial\boldsymbol{\alpha}} \right\}. \quad (3)$$

Let $\mathbf{J}_l$ denote the FIM for measurements obtained by the lth element of the vector sensor. Under the assumption of spatially white noise, where the noise covariance matrix is given by $\mathbf{R_n} = \sigma_n^2\mathbf{I}$, the FIM from (3) can be rewritten as the sum of FIMs for each sensor type. In the general case with an arbitrary vector sensor configuration, the FIM can be written as: $\mathbf{J} = \sum_{l=1}^{L}\beta_l\mathbf{J}_l$, where β_l is equal to zero or one according to the selected configuration. We are looking for the configuration with minimum number of sensors, capable to estimate the parameters of interest, $\phi,\theta,\forall\phi$, and $\forall,\theta \neq 0°, 180°$.

The additional assumption in the first stage is $\boldsymbol{\mu} = [\mu_1 \;\; 0]^T$. Accordingly, the FIM obtained for the lth sensor type is given by:

$$\mathbf{J}_l = \frac{2|\mu_1|^2}{\sigma_n^2}\mathrm{Re}\left\{ \frac{\partial A_{l1}^H}{\partial\boldsymbol{\alpha}} \frac{\partial A_{l1}}{\partial\boldsymbol{\alpha}} \right\}. \quad (4)$$

Evaluation of (4) for each sensor type yields

$$\begin{aligned}
\mathbf{J}_1 &= \frac{2|\mu_1|^2}{\sigma_n^2} \\
&\quad\times \begin{bmatrix} \sin^2\theta\sin^2\phi & -\sin\theta\sin\phi\cos\theta\cos\phi \\ -\sin\theta\sin\phi\cos\theta\cos\phi & \cos^2\theta\cos^2\phi \end{bmatrix}, \\
\mathbf{J}_2 &= \frac{2|\mu_1|^2}{\sigma_n^2} \\
&\quad\times \begin{bmatrix} \sin^2\theta\cos^2\phi & \sin\theta\sin\phi\cos\theta\cos\phi \\ \sin\theta\sin\phi\cos\theta\cos\phi & \cos^2\theta\sin^2\phi \end{bmatrix}, \\
\mathbf{J}_3 &= \frac{2|\mu_1|^2}{\sigma_n^2}\begin{bmatrix} \cos^2\theta & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{J}_6 = \frac{2|\mu_1|^2}{\sigma_n^2}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
\mathbf{J}_4 &= \frac{2|\mu_1|^2}{\sigma_n^2}\begin{bmatrix} 0 & 0 \\ 0 & w^2\cos^2\phi \end{bmatrix}, \\
\mathbf{J}_5 &= \frac{2|\mu_1|^2}{\sigma_n^2}\begin{bmatrix} 0 & 0 \\ 0 & w^2\sin^2\phi \end{bmatrix},
\end{aligned} \quad (5)$$

and for $\theta = 90°$, we obtain

$$\begin{aligned}
\mathbf{J}_1 &= \frac{2|\mu_1|^2}{\sigma_n^2}\begin{bmatrix} \sin^2\phi & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{J}_4 = \frac{2|\mu_1|^2}{\sigma_n^2}\begin{bmatrix} 0 & 0 \\ 0 & w^2\cos^2\phi \end{bmatrix}, \\
\mathbf{J}_2 &= \frac{2|\mu_1|^2}{\sigma_n^2}\begin{bmatrix} \cos^2\phi & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{J}_5 = \frac{2|\mu_1|^2}{\sigma_n^2}\begin{bmatrix} 0 & 0 \\ 0 & w^2\sin^2\phi \end{bmatrix}
\end{aligned} \quad (6)$$

with $\mathbf{J}_3 = \mathbf{J}_6 = \mathbf{0}$. This implies that the information from sensors 3 and 6 is zero for the particular case considered. Therefore, the corresponding sensors can be excluded from the relevant configurations. One can observe that the rank of each one of the remaining FIMs is equal to one except for the singular directions of $\phi = 0°, 90°, 180°, 270°$. In order to obtain a finite CRLB, the FIM $\mathbf{J} = \sum_{l=1}^{L}\beta_l\mathbf{J}_l$ is required to be nonsingular. Inspection of (6) shows that for $\phi = 0°, \mathbf{J}_1 = \mathbf{J}_5 = \mathbf{0}$, and therefore sensors $l = 2, 4$ are necessary. Similarly, for $\phi = 90°, \mathbf{J}_2 = \mathbf{J}_4 = \mathbf{0}$, and therefore in this case, sensors $l = 1, 5$ are necessary. Consequently, in the particular case considered here, ($\boldsymbol{\mu}$ is known and equal to $\boldsymbol{\mu} = [\mu_1 \;\; 0]^T$ and $\theta = 90°$), all the four sensors, $l = 1, 2, 4, 5$ are *necessary* to satisfy requirement $\mathbf{R1}$. This combination of the four sensors is referred as the quadrature configuration.

B. Sufficiency of the Quadrature Configuration

In the second stage, we show that this quadrature vector sensor configuration is *sufficient* to satisfy requirements $\mathbf{R1}$ and $\mathbf{R2}$ for the general case in which the assumptions on $\boldsymbol{\mu}$ and θ are removed. For the case of an unknown polarization vector, $\boldsymbol{\mu}$, the vector of parameters to be estimated is given by $\boldsymbol{\xi} = [\boldsymbol{\alpha}^T \ \mathbf{m}^T]^T$ where $\mathbf{m} = [\mu_{1r} \ \mu_{1i} \ \mu_{2r} \ \mu_{2i}]^T$, and the subscripts r and i stand for the real and imaginary parts, respectively. The corresponding FIM is

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{\boldsymbol{\alpha\alpha}} & \mathbf{J}_{\boldsymbol{\alpha}\mathbf{m}} \\ \mathbf{J}_{\mathbf{m}\boldsymbol{\alpha}} & \mathbf{J}_{\mathbf{mm}} \end{bmatrix} = \frac{2}{\sigma_n^2}\mathrm{Re}\left\{\left(\frac{\partial(\mathbf{A}(\theta,\phi)\boldsymbol{\mu})}{\partial\boldsymbol{\xi}}\right)^H \frac{\partial(\mathbf{A}(\theta,\phi)\boldsymbol{\mu})}{\partial\boldsymbol{\xi}}\right\}. \quad (7)$$

Evaluation of (7) using (1) with sensors $l = 1,2,4,5$ yields (after some algebraic operations) (8)–(10) shown at the bottom of the page in which $\mathbf{I}_2$ is an identity matrix of size 2.

The CRLB for estimating $\boldsymbol{\alpha}$ is given by $(\mathbf{J}_{\boldsymbol{\alpha\alpha}} - \mathbf{J}_{\boldsymbol{\alpha}\mathbf{m}}\mathbf{J}_{\mathbf{mm}}^{-1}\mathbf{J}_{\mathbf{m}\boldsymbol{\alpha}})^{-1}$, namely, the FIM for estimating $\boldsymbol{\alpha}$ is decreased by $\mathbf{J}_{\boldsymbol{\alpha}\mathbf{m}}\mathbf{J}_{\mathbf{mm}}^{-1}\mathbf{J}_{\mathbf{m}\boldsymbol{\alpha}}$ due to lack of knowledge of the polarization parameters, $\mathbf{m}$. In the sequel, we show that the equivalent FIM

$$\mathbf{CRLB}^{-1} = \mathbf{J}_{\boldsymbol{\alpha\alpha}} - \mathbf{J}_{\boldsymbol{\alpha}\mathbf{m}}\mathbf{J}_{\mathbf{mm}}^{-1}\mathbf{J}_{\mathbf{m}\boldsymbol{\alpha}} \quad (11)$$

is nonsingular $\forall\, \boldsymbol{\mu}, w^2, \phi$ and $\forall\theta \neq 0°, 180°$.

Substitution of (8)–(10) into (11) results in (12) shown at the bottom of the page in which $\nu_1 = 1 + w^2\cos^2\theta$, and $\nu_2 = w^2 + \cos^2\theta$. In order to satisfy requirements $\mathbf{R1}$ and $\mathbf{R2}$, the matrix $\mathbf{CRLB}^{-1}$ in (12) must be nonsingular. One can notice that the bound is ϕ-independent. By definition, this matrix is nonnegative definite, i.e., its eigenvalues are nonnegative. Thus, the matrix $\mathbf{CRLB}^{-1}$ is nonsingular iff its determinant is positive. The determinant of $\mathbf{CRLB}^{-1}$ is given by

$$\det(\mathbf{CRLB}^{-1}) = k(w^2,\theta)Q(\boldsymbol{\mu}, w^2, \theta) \quad (13)$$

where $k(w^2,\theta) \triangleq (2w^2\sin^3\theta/(\sigma_n^2\nu_1\nu_2))^2$ is positive for the parameter range of interest and $Q(\boldsymbol{\mu}, w^2, \theta)$ is defined as

$$Q(\boldsymbol{\mu}, w^2, \theta) \triangleq \left(|\mu_1^2|\,\nu_1 + |\mu_2^2|\,\nu_2\right)\left(|\mu_1^2|\,\nu_2 + |\mu_2^2|\,\nu_1\right) - (1-w^2)^2\sin^4\theta\,\mathrm{Re}^2(\mu_1\mu_2^*). \quad (14)$$

Next, it will be shown that $Q(\boldsymbol{\mu}, w^2, \theta)$ is positive, i.e.,

$$Q(\boldsymbol{\mu}, w^2, \theta) \geq \min_{\theta, w^2} Q(\boldsymbol{\mu}, w^2, \theta) > 0 \quad \forall\boldsymbol{\mu},\, w^2,\, \theta \neq 0°, 180°. \quad (15)$$

Since the minima of ν_1 and ν_2 are obtained at $\theta = 90°$, and the right term of (14) obtains its maximum at this point, then the minimum of $Q(\boldsymbol{\mu}, w^2, \theta)$ with respect to θ occurs at $\theta = 90°$

$$\begin{aligned}\min_{\theta} Q(\boldsymbol{\mu}, w^2, \theta) &= Q(\boldsymbol{\mu}, w^2, \theta = 90°) \\ &= \left(|\mu_1^2| + |\mu_2^2|\,w^2\right)\left(|\mu_1^2|\,w^2 + |\mu_2^2|\right) \\ &\quad - (1-w^2)^2\mathrm{Re}^2(\mu_1\mu_2^*). \end{aligned} \quad (16)$$

The minimum of $Q(\boldsymbol{\mu}, w^2, \theta = 90°)$ with respect to w^2 is obtained by differentiating (16) and equating to zero. Back substitution of this result into (16) yields

$$\begin{aligned}Q(\boldsymbol{\mu}, w^2, \theta) &\geq \min_{\theta, w^2} Q(\boldsymbol{\mu}, w^2, \theta) \\ &= \frac{1}{2}(1 + w_o^2)\left(|\mu_1^2| + |\mu_2^2|\right) > 0 \end{aligned} \quad (17)$$

in which w_o^2 is the value of w^2 that minimizes $Q(\boldsymbol{\mu}, w^2, \theta = 90°)$ which completes the proof of the claim in (15).

In conclusion, we have proved that the $\mathbf{CRLB}^{-1}$ is a positive-definite matrix $\forall\, \boldsymbol{\mu},\, w^2,\, \phi,\, \theta \neq 0°, 180°$. This proves that the chosen quadrature vector sensor configuration with sensors $l = 1,2,4,5$ is *necessary* and *sufficient* to satisfy the requirements $\mathbf{R1}$ and $\mathbf{R2}$.

C. Comparison With the Complete Configuration

The complete configuration consists of all the six sensors as shown in Fig. 1. Derivation of the bound for this case can be

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$$\mathbf{J}_{\boldsymbol{\alpha\alpha}} = \frac{2}{\sigma_n^2}\begin{bmatrix} \left(|\mu_1^2| + w^2\,|\mu_2^2|\right)\sin^2\theta & (1-w^2)\mathrm{Re}(\mu_1\mu_2^*)\sin^2\theta \\ (1-w^2)\mathrm{Re}(\mu_1\mu_2^*)\sin^2\theta & (w^2+\cos^2\theta)\,|\mu_1^2| + (1+w^2\cos^2\theta)\,|\mu_2^2| \end{bmatrix} \quad (8)$$

$$\mathbf{J}_{\boldsymbol{\alpha}\mathbf{m}} = \mathbf{J}_{\mathbf{m}\boldsymbol{\alpha}}^T = \frac{2}{\sigma_n^2}\begin{bmatrix} -\sin\theta\cos\theta & [\mu_{1r} \quad \mu_{1i} \quad w^2\mu_{2r} \quad w^2\mu_{2i}] \\ (1+w^2)\cos\theta & [-\mu_{2r} \quad -\mu_{2i} \quad \mu_{1r} \quad \mu_{1i}] \end{bmatrix} \quad (9)$$

$$\mathbf{J}_{\mathbf{mm}} = \frac{2}{\sigma_n^2}\begin{bmatrix} (w^2+\cos^2\theta)\mathbf{I}_2 & \mathbf{0} \\ \mathbf{0} & (1+w^2\cos^2\theta)\mathbf{I}_2 \end{bmatrix} \quad (10)$$

$$\mathbf{CRLB}^{-1} = \frac{2}{\sigma_n^2}\frac{w^2\sin^2\theta}{\nu_1\nu_2}\begin{bmatrix} |\mu_1^2|\,\nu_1 + |\mu_2^2|\,\nu_2 & (1-w^2)\sin^3\theta\mathrm{Re}(\mu_1\mu_2^*) \\ (1-w^2)\sin^3\theta\mathrm{Re}(\mu_1\mu_2^*) & \left(|\mu_1^2|\,\nu_2 + |\mu_2^2|\,\nu_1\right)\sin^2\theta \end{bmatrix} \quad (12)$$

$$\mathbf{CRLB}^{-1} = \frac{2}{\sigma_n^2}\frac{w^2}{\nu_1^c\nu_2^c}\begin{bmatrix} |\mu_1^2|\,\nu_1^c + |\mu_2^2|\,\nu_2^c & (1-w^2)\sin\theta\,\mathrm{Re}(\mu_1\mu_2^*) \\ (1-w^2)\sin\theta\,\mathrm{Re}(\mu_1\mu_2^*) & \left(|\mu_1^2|\,\nu_2^c + |\mu_2^2|\,\nu_1^c\right) \end{bmatrix} \quad (18)$$

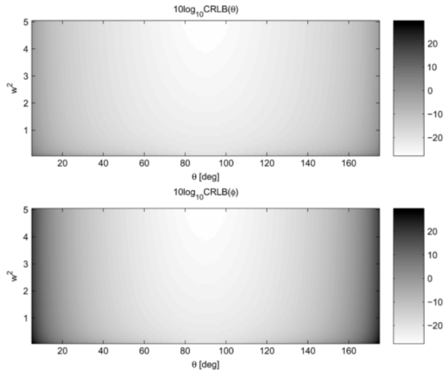

Fig. 2. CRLB on DOA estimation for the quadrature configuration as a function of w^2 and elevation angle θ with $\mathrm{SNR} = 20$ dB.

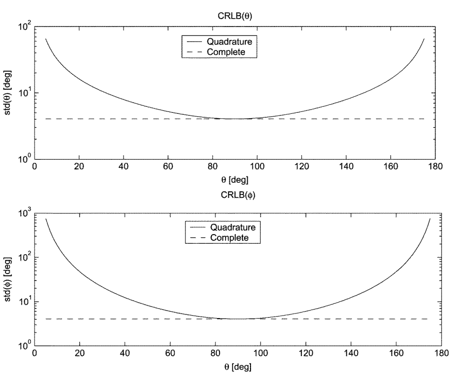

Fig. 3. CRLB on DOA estimation versus the elevation angle for the quadrature and complete configurations with $\mathrm{SNR} = 20$ dB.

performed in a similar fashion to the quadrature configuration outlined in the previous subsection. The result for $\mathbf{CRLB}^{-1}$ in this case is similar to the quadrature configuration as shown in (18) at the bottom of the previous page in which $\nu_1^c = 1$ and $\nu_2^c = w^2$. It can be noticed that in this case, the bound is finite for any θ. One can observe that for $\theta = 90°$ which is the case in many practical applications (such as cellular communication systems), the bounds in both configurations are identical. This implies that the quadrature configuration achieves the optimal performance at $\theta = 90°$.

D. Example

Fig. 2 shows the CRLB on θ and ϕ for $\mathrm{SNR} = 20$ dB with the quadrature vector sensor configuration. The SNR is calculated as $||\boldsymbol{\mu}||^2/\sigma_n^2$. One can notice that the bound is finite for any w^2 and θ except the directions $\theta = 0°, 180°$.

Fig. 3 presents a comparison between the CRLB on DOA estimation using the proposed quadrature and the complete configurations where $w^2 = 1$ and $\mathrm{SNR} = 20$ dB. It can be observed that at $\theta = 90°$ the CRLB is identical in both cases. This implies that the missing sensors in the quadrature configuration do not contribute to the accuracy of DOA estimation in the azimuth plane. The performance of the quadrature configuration is degraded as θ deviates from 90°.

IV. Conclusion

In this letter, an efficient vector sensor configuration for source localization was proposed. This configuration consists of two electric and two magnetic sensors and is referred as quadrature vector sensor. The configuration was obtained by analyzing the CRLB for source localization. This quadrature configuration consists of the minimum number of sensors capable to estimate the DOA of an arbitrary polarized signal from any direction except two opposite directions on the *z*-axis. It was shown that the CRLB for DOA estimation of signals in the azimuth plane, is identical for both quadrature and complete vector sensor configurations. The performance of the proposed configuration was tested and compared to the complete configuration via an example.

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