JOURNAL OF APPLIED PHYSICS VOLUME 95, NUMBER 11 1 JUNE 2004

Mesoscopic quantized properties of magnetic-dipolar-mode oscillations in disk ferromagnetic particles

E. O. Kamenetskii,[a] R. Shavit, and M. Sigalov
Department of Electrical and Computer Engineering, Ben Gurion University of the Negev, Beer Sheva 84105, Israel

(Presented on 8 January 2004)

Magnetic-dipolar-mode or magnetostatic (MS) oscillations in ferrite samples have the wavelength much smaller than the electromagnetic wavelength at the same frequency and, at the same time, much larger than the exchange-interaction spin wavelength. This intermediate position between the ''pure'' electromagnetic and spin-wave (exchange-interaction) processes reveals very special behaviors of the geometrical effects. It was shown recently that magnetic-dipolar-mode oscillations in a normally magnetized ferromagnetic disk are characterized by discrete energy levels resulting from the structural confinement. In this article we give results of the energy spectra in MS-wave ferrite disks taking into account nonhomogeneity of the internal dc magnetic field. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1688271]

I. INTRODUCTION

The magnetostatic (MS)-mode characterization in ferrite samples looks like a relatively straightforward and old problem in magnetism. Nevertheless, some aspects of such oscillations should be reconsidered in a view of the macroscopically quantized methods. In recent years, there has been a renewed interest in high frequency dynamic properties of finite size magnetic structures. In a series of publications, confinement phenomena of high-frequency magnetization dynamics in magnetic particles have been the subject of much experimental and theoretical attention (see Ref. 1 and references therein). Mainly, these works are devoted to the important studies of localized spin-wave spectra, but do not focus on the energy eigenstates of a whole ferrite-particle system. Until now, however, there were no phenomenological models of a ferrite particle with high-frequency magnetization dynamics that use the effective-mass approximation and the Schrödinger-like equation to analyze energy eigenstates of a whole ferrite-particle system, similarly to semiconductor quantum dots.

The recently published theory[2] and calculation results[3] show that the confined effects for MS oscillations in normally magnetized thin-film ferrite disks are characterized by discrete energy levels. When these point disk particles demonstrate such properties with respect to the external electromagnetic fields, they should be referred to as magnetic dots or magnetic artificial atoms, having the mesoscopic quantum bound states. Because of discrete energy eigenstates one can describe the oscillating system as a collective motion of quasiparticles, the light magnons. From the point of view of fundamental studies and applications, a macroscopic quantum analysis for MS oscillations is very important. In particular it underlies the physics of magnetoelectric oscillating spectra observed in ferrite disks with surface electrodes.[4]

It was supposed in Refs. 2 and 3 that the internal dc magnetic field is homogeneous. In this case one can really formulate the spectral problem. At the same time, because of the demagnetizing effects, the internal dc magnetic field in a ferrite disk is essentially nonhomogeneous. This should strongly affect the spectral picture. An analysis of the spectral peak positions for MS oscillations taking into account the dc magnetic field inhomogeneity was made in Ref. 5. Based on an analysis in Ref. 5 one cannot, however, determine the character of the field distribution for MS modes. Therefore the ''spectral portrait'' of MS oscillations in disks with nonhomogeneous internal dc magnetic field becomes unclear. So the physics of interaction of such a particle with the external rf fields becomes absolutely unclear.

In this article we propose analytical models for enough comprehensive characterization of the spectral properties of magnetic-dipolar-mode oscillations in disk ferromagnetic particles taking into account the dc magnetic field inhomogeneity.

II. SPECTRAL PROPERTIES OF MS MODES IN A FERRITE DISK WITH HOMOGENEOUS dc MAGNETIC FIELD

In a ferrite-disk resonator with a small thickness-to-diameter ratio, the monochromatic MS-wave potential function ψ is represented as[2,3]

$$\psi=\sum_{p,q} A_{pq}\tilde{\xi}_{pq}(z)\tilde{\varphi}_q(r,\alpha), \tag{1}$$

where A_{pq} is a MS mode amplitude, $\tilde{\xi}_{pq}(z)$ and $\tilde{\varphi}_q(r,\alpha)$ are dimensionless functions describing, respectively, ''thickness'' (z coordinate) and ''in-plane'' or ''flat'' (radial r and azimuth α coordinates) MS modes. For a certain type of ''thickness mode'' (in other words, for a given quantity p), every ''flat mode'' is characterized by its own function $\tilde{\xi}_q(z)$ and is described by the Bessel functions.

[a]Author to whom correspondence should be addressed; electronic mail: kmntsk@ee.bgu.ac.il

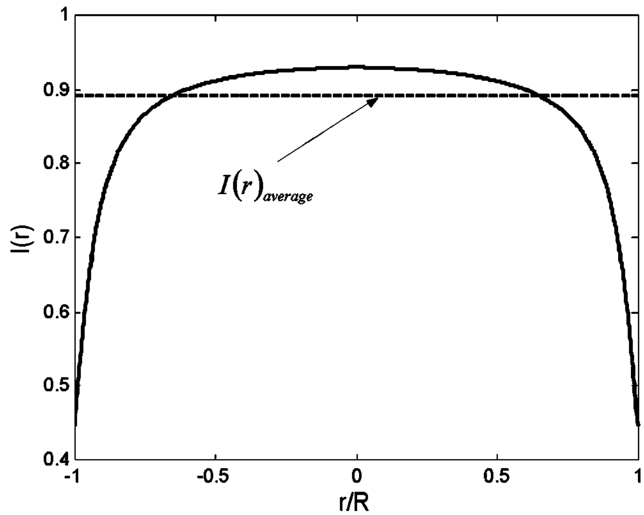

FIG. 1. Radial distribution of a demagnetization factor and averaging on the real-disk-diameter region.

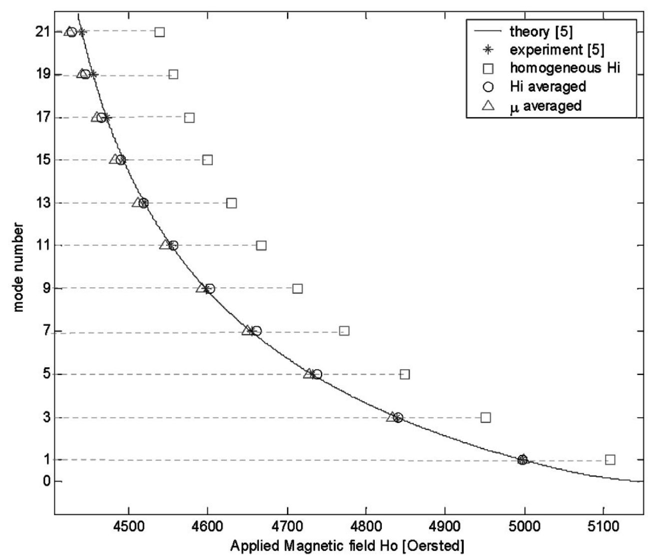

FIG. 2. Mode number positions with respect to the applied magnetic field for different calculation methods.

The spectral problem for MS waves in a ferrite disk resonator can be formulated as the energy eigenvalue problem. The energy orthonormality in a ferrite disk is described as

$$(E_q - E_{q'}) \int_S \tilde{\varphi}_q \tilde{\varphi}^*_{q'} dS = 0. \tag{2}$$

One can consider the oscillating system as a collective motion of quasiparticles—the ''light magnons'' (lm).

The energy levels in a magnetic quantum well and effective masses for the light-magnon modes in a ferrite disk with homogeneous dc magnetic field were calculated in Ref. 3. Calculations were made with use of the disk data given in the Yukawa and Abe paper:[5] saturation magnetization $4\pi M_0 = 1792.7$ G, disk diameter $2R = 3.98$ mm, and film thickness $h = 0.284$ mm. The working frequency is 9.51 GHz.

III. MS MODES IN A FERRITE DISK WITH NONHOMOGENEOUS dc MAGNETIC FIELD

For a normally magnetized thin-film ferromagnetic disk having the small thickness-to-diameter ratio, the demagnetizing field can be considered just as the radius-dependent function. The internal dc magnetic field is determined in Ref. 5 as

$$H_i(r) = H_0 - H_a - 4\pi M_0 I(r), \tag{3}$$

where H_0 and H_a are the applied and the anisotropy fields, respectively, and $I(r)$ is the demagnetizing factor. The function $I(r)$ calculated based on the model described in Ref. 5 is shown in Fig. 1 by a solid line.

In this case, however, the standard cylindrical-symmetry problem for MS modes cannot be solved. To understand more clearly this fact let us consider the MS-wave solutions for a general case of an axially magnetized ferrite rod with the permeability-tensor components dependent on a radial coordinate: $\vec{\vec{\mu}} = \vec{\vec{\mu}}(r)$. In this case we have the following differential equation for the MS-potential function:

$$\nabla \cdot (\vec{\vec{\mu}}(r) \cdot \nabla \psi) = 0. \tag{4}$$

It is not difficult to show that in a cylindrical coordinate system this equation has the form

$$\mu\left(\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2}\right) + \frac{\partial \mu}{\partial r}\frac{\partial \psi}{\partial r} + i\,\frac{1}{r}\frac{\partial \mu_a}{\partial r}\frac{\partial \psi}{\partial \theta} + \frac{\partial^2 \psi}{\partial z^2} = 0, \tag{5}$$

where μ and μ_a are diagonal and off-diagonal components of the permeability tensor. One can see that in this equation separation of variables is impossible and therefore an analytical solution cannot be found.

The absorption peaks are interpreted in Ref. 5 to be caused by magnetostatic waves propagating radially across the disk with the dc field dependent wave number in the plane of a YIG film. The mode numbers are determined based on the well-known Bohr–Sommerfeld quantization rule. In definition of the spectral peak positions, the Yukawa and Abe model gives good agreement with the experiments. This fact is illustrated in Fig. 2 for MS modes excited by the homogeneous rf magnetic field (here we use the Yukawa and Abe notation for the mode numbers; in the Yukawa and Abe notation there are odd modes). In spite of such good agreement, one cannot, however, rely on the Yukawa and Abe model to physically describe the experimental situation of interaction of small ferrite disks with the external rf fields. When being excited by the homogeneous rf magnetic field, a small ferrite disk should be considered as a magnetic dipole with evident azimuth variations of the ''in-plane'' MS-potential function. At the same time, in the Yukawa and Abe model the ''in-plane'' MS-potential-function distribution is supposed to be azimuthally nondependent. They only find the radial function of the ''in-plane'' wave number without any characterization of the MS-field distribution, which is necessary to describe the excitation problem.

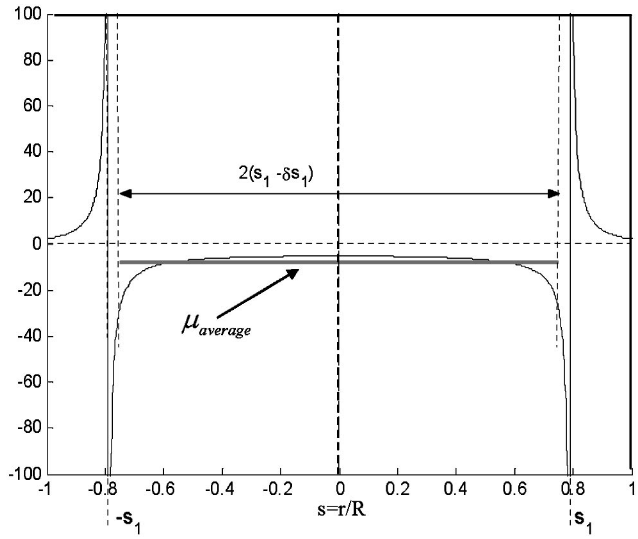

FIG. 3. Averaging procedure for permeability-tensor components on the effective-diameter region.

IV. THE AVERAGE PROCEDURES FOR THE MS-MODE SPECTRAL PROBLEM IN A FERRITE DISK WITH NONHOMOGENEOUS dc MAGNETIC FIELD

The spectral properties of MS modes in a ferrite disk described in Refs. 2 and 3 allow analyzing the real physics of the particle interaction with the external rf fields. In this case, however, the spectral peak positions (found in an assumption of homogeneous internal dc magnetic field and without taking into account the anisotropy field) are rather far from the experimental peak positions. This is illustrated in Fig. 2 by squares. The internal field was calculated based on Eq. (3), but with $H_a=0$ and $I(r)=1$.

To take into account the internal field nonhomogeneity preserving, at the same time, the quantized "spectral portrait" of MS oscillations in a ferrite disk one has to develop certain models of averaging by integrating over the disk diameter. For the initial efforts to create such models one should direct to possible radial averaging the function $\vec{\vec{\mu}}(r)$. If such averaging for diagonal and off-diagonal components of the permeability tensor becomes relevant, one can use the separation of variables and obtain, as a result, the spectral characteristics for the oscillating modes. Figure 3 illustrates typical distribution of $\mu(r)$ calculated based on Eq. (3) with use of the Yukawa and Abe data of the disk parameters and for a certain applied field H_0. Distribution of $\mu_a(r)$ has a very similar character and practically coincides with the dependence $\mu(r)$, shown in Fig. 3. From Fig. 3 one can see that the "internal region"—the region where $\mu<0$ and so the oscillating modes exist—is plateau-like almost in all the range $r<r_1$, where r_1 is a the break radius of μ. The region where μ becomes sharply dependent on radial coordinate r is very closely abutting to r_1. This gives a real possibility of introducing a certain procedure for μ averaging in a region $r<r_1$. We made μ averaging for a ferrite disk with an effective diameter $2(s_1-\delta s_1)$, where $s_1=r_1/R$ is a relative break radius and δs_1 is the deviation of a relative break radius. In our calculation we took $\delta s_1 \leqslant 0.1 s_1$. For μ averaged in the above region and in supposition that a ferrite disk has an effective diameter $D_{\mathrm{eff}}=2(s_1-\delta s_1)$ we solved the spectral problem based on the method described in Refs. 2 and 3. The calculation results depicted in Fig. 2 by triangles show good agreement with the Yukawa and Abe experimental data. Figure 3 gives a concrete picture of the $\mu(r)$ function corresponding to the first mode in Fig. 2. The first mode has the smallest diameter D_{eff}. The higher the mode number, the closer the diameter D_{eff} is to a real disk diameter $D=2R$.

The μ averaged calculation results lie down almost completely on a curve calculated based on the Bohr–Sommerfeld quantization rule for MS modes used in Ref. 5 (see Fig. 2). This gives us the possibility to make further justification of our μ-averaged method. The Bohr–Sommerfeld integral for MS modes is:[5]

$$n=\frac{2}{\pi}\frac{D}{h}\int_0^{s_1}\frac{1}{\sqrt{-\mu}}\tan^{-1}\frac{1}{\sqrt{-\mu}}\,ds. \tag{6}$$

One can see that the integrand aims to zero for $|\mu|\to\infty$. So the role of the abutting region δs_1 becomes negligibly small in definition of the mode number.

The above procedure of radial averaging the function $\vec{\vec{\mu}}(r)$ gives good agreement with the experimental results. Nevertheless, the fact that for different modes one has different effective diameters makes it impossible to formulate classical spectral problems, where the domain of definition of a differential operator should be the same for different modes. To solve correctly the spectral problem one has to use another averaging procedure—averaging the function $I(r)$ in the region of the real disk diameter. Figure 1 shows the quantity $I(r)_{\mathrm{average}}$ based on which the peak positions were calculated. The calculation results shown in Fig. 2 by circles give good agreement with experimental peak positions.

V. CONCLUSION

Based on the above models of averaging, the energy spectra in MS-wave ferrite disks taking into account nonhomogeneity of the internal dc magnetic field can be effectively calculated. This makes the problem of interaction of such ferrite particles with the external rf fields solvable.

[1] C. Bayer *et al.*, Appl. Phys. Lett. **82**, 607 (2003); J. P. Park *et al.*, Phys. Rev. Lett. **89**, 277201 (2002); J. Jorzick *et al.*, *ibid.* **88**, 047204 (2002); S. Jung, J. B. Ketterson, and V. Chandrasekhar, Phys. Rev. B **66**, 132405 (2002).
[2] E. O. Kamenetskii, Phys. Rev. E **63**, 066612 (2001).
[3] E. O. Kamenetskii, R. Shavit, and M. Sigalov, Europhys. Lett. **64**, 730 (2003).
[4] A. K. Saha, E. O. Kamenetskii, and I. Awai, Phys. Rev. E **64**, 056611 (2001); J. Phys. D **35**, 2484 (2002).
[5] T. Yukawa and K. Abe, J. Appl. Phys. **45**, 3146 (1974).