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Quantum wells based on magnetic-dipolar-mode oscillations in disk ferromagnetic particles

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Abstract. – We show that magnetic-dipolar-mode oscillations in a normally magnetized ferromagnetic disk have typical atomic properties like discrete-energy levels. Because of the discrete-energy eigenstates of such oscillations resulting from structural confinement, one can describe the oscillating system as a collective motion of quasiparticles —the light magnons. We calculate the energy levels in a magnetic quantum well and the effective masses of the light magnons.

Introduction. – Semiconductor quantum dots are manmade structures in which electrons are confined in all three spatial directions similar to the physical situation in atoms. As they show typical atomic properties like discrete-energy levels and shell structures, they are often referred to as artificial atoms [1]. This provides various implementations of solid-state systems based on semiconductor quantum dots. It is interesting that confinement phenomena for magnetic dipolar modes in a normally magnetized ferrite disk may also show typical atomic properties like discrete-energy levels. Such wave processes reveal very special behaviors of the geometrical effects. As a starting point for this statement, we call the reader's attention to the fundamental difference between the experimental absorption spectra for magnetostatic (MS) modes in the sphere-shaped [2] and the disk-shaped ferrite resonators [3]. The δ-functional character of the multiresonance spectra, that one can see in the case of a ferrite disk resonator, leads to the clear conclusion that the energy of a source of a DC bias magnetic field is absorbing "by portions" or discretely, in other words. On the contrary, the spectrum of a ferrite sphere does not show a series of sharp field-dependent resonances and is characterized by very few and much "spreading" absorption peaks.

The MS-mode characterization in ferrite samples looks as a relatively straightforward and old problem in magnetism. Nevertheless, some aspects of such oscillations should be reconsidered in view of macroscopically quantized methods. In the last years, there has been a renewed interest in the high-frequency dynamic properties of finite-size magnetic structures. In a series of recent publications, confinement phenomena of high-frequency magnetization dynamics in magnetic particles have been the subject of much experimental and theoretical attention (see [4] and references therein). Mainly, these works are devoted to the important studies of the localized spin-wave spectra, but do not focus on the energy eigenstates of a whole ferrite-particle system. Till now there were no (to the best of our knowledge) phenomenological models of a ferrite particle with high-frequency magnetization dynamics, which use the effective-mass approximation and the Schrödinger-like equation to analyze the energy eigenstates of a whole ferrite-particle system similarly to semiconductor quantum wells. To a certain extent, this letter is aimed to make up such a deficiency.

Energy eigenstates of MS oscillations in ferrite disk resonators. – In the case of magnetic dipolar modes in a normally magnetized ferrite disk one can formulate the energy eigenvalue problem for the MS-potential wave function. Some detailed analysis of the experimental spectra is useful at the beginning. The main feature of multiresonance atomic-like spectra, observed in experiments [3], is the fact that high-order peaks correspond to lower quantities of the bias DC magnetic field. Physically, the situation looks as follows. Let $H_0^{(A)}$ and $H_0^{(B)}$ be, respectively, the upper and lower values of a bias magnetic field corresponding to the borders of a region. We can estimate the total depth of the "potential well" as $\Delta U = 4\pi M_0(H_0^{(A)} - H_0^{(B)})$, where M_0 is the saturation magnetization. Let $H_0^{(1)}$ be a bias magnetic field, corresponding to the *main absorption peak* in the experimental spectrum ($H_0^{(B)} < H_0^{(1)} < H_0^{(A)}$). When we put a ferrite sample into this field, we supply it with the energy: $4\pi M_0 H_0^{(1)}$. To some extent, this is a pumping-up energy. *Starting from this level*, we can excite the entire spectrum from the main mode to the high-order modes. When one continuously varies the quantity of the DC field H_0, for a given quantity of the frequency ω, one can see a discrete set of absorption peaks. This means that one has discrete-set levels of the potential energy. The line spectra appear due to the quantum-like transitions between the energy levels of a ferrite disk-shaped particle. As a quantitative characteristic of permitted quantum transitions, there is the probability, which defines the intensities of spectral lines. Certainly, there should be a certain inner mechanism of the quantization of the DC energy absorbed by a small disk-shaped ferrite sample.

Let us start our analysis by considering an axially magnetized ferrite rod. By an appropriate change of variables, any system of equations describing oscillations in *one-dimensional linear structures with distributed parameters* may be written as (see, *e.g.*, [5])

$$\hat{Q}\vec{u} = \frac{\partial \vec{u}}{\partial t}\,, \tag{1}$$

where $\vec{u}(z,t)$ is a vector function with components $u_1, u_2, \ldots$ describing the system properties and $\hat{Q} = \hat{Q}(z)$ is a differential-matrix operator. Since MS oscillations in a ferrite rod are described by the *scalar wave function* ψ [6], for a lossless structure eq. (1) reduces to

$$a^{(1)}(z)\frac{\partial^2\psi(z,t)}{\partial z^2} + a^{(2)}(z)\psi(z,t) = \frac{\partial\psi(z,t)}{\partial t}\,. \tag{2}$$

This is the Schrödinger-like equation.

Let a one-dimensional linear structure be a waveguide structure with parameters independent of the longitudinal z-coordinate. For harmonic processes, coefficients $a^{(1)}$ and $a^{(2)}$ should be imaginary quantities. Based on the energy balance equation, one obtains the average energy of the MS mode n in a waveguide section restricted by coordinates z_1 and z_2 [7]:

$$\overline{W}_n = -\frac{1}{4a_n^{(1)}} i\omega\mu_0 \int_{z_1}^{z_2}\int_S \psi_n\psi_n^*\,\mathrm{d}s\,\mathrm{d}z + C, \tag{3}$$

where C is an arbitrary quantity independent of time. We can normalize the process by supposing that the constant C is equal to zero. One can see that the energy can be orthogonolized with respect to the known ψ_n eigenfunctions. We represent the MS-potential function in a ferrite rod as $\psi = A\widetilde{\varphi}e^{-i\beta z}$, where A is a dimensional coefficient, $\widetilde{\varphi}$ is a dimensionless membrane function and β is a propagation constant. The membrane functions of MS modes in an axially magnetized ferrite rod give a complete discrete set of functions (on a waveguide cross-section). Based on the Walker equation, one has, for every MS mode in an axially magnetized rod [7],

$$\hat{G}_\perp \widetilde{\varphi}_n = \beta_n^2 \widetilde{\varphi}_n, \tag{4}$$

where

$$\hat{G}_\perp \equiv \mu \nabla_\perp^2; \tag{5}$$

$\nabla_\perp^2$ is the two-dimensional, "in-plane", Laplace operator, μ is a diagonal component of the permeability tensor. For propagating MS modes, the operator $\hat{G}_\perp$ is the positive-definite operator. The fact that the coefficient $a^{(2)}$ is not included in the expression of the average energy gives us the possibility to consider different cases based on certain physical models. One can see that when $a^{(2)} \equiv 0$, eq. (2) resembles the Schrödinger equation for "free particles". This is the case of a constant value of the bias magnetic field $\vec{H}_0$. Certainly, when a ferrite specimen (having the saturation magnetization of a ferrite material), is placed into a bias magnetic field, one has a constant "potential energy" of this ferrite sample in the DC magnetic field.

We define the notion of the *normalized average MS energy of mode* n as the average (on the RF period) energy of the MS waveguide section with unit length and unit characteristic cross-section. This energy for a mode with unit amplitude is expressed as

$$E_n = \frac{1}{4} g\mu_0 \beta_n^2, \tag{6}$$

where g is the unit dimensional coefficient.

Based on the above consideration of the states of MS waves in an axially magnetized ferrite rod, we extend now our analysis to the case of a normally magnetized ferrite disk. In a ferrite disk with a small thickness/diameter ratio separation of variables is possible. The spectrum of "thickness modes" is very "rare" compared to the "dense" spectrum of "flat modes". So, the spectral properties can be entirely described based on the consideration of only a fundamental "thickness mode". For a "flat" mode q in a normally magnetized ferrite disk there are two waves propagating forth and back with respect to the z-axis, the average energy will be twice more than the energy expressed by eq. (6). We have as a result

$$E_q = \frac{1}{2} g\mu_0 \big(\beta_q^{(F)}\big)^2, \tag{7}$$

where $\beta_q^{(F)}$ is the MS-wave propagation constant along the z-axis in a ferrite region ($0 \le z \le h$).

Since the two-dimensional ("in-plane") differential operator $\hat{G}_\perp$ contains $\nabla_\perp^2$ (the two-dimensional, "in-plane", Laplace operator), a double integration by parts (the Green theorem) on S —a square of the "in-plane" cross-section of an open ferrite disk— of the integral $\int (\hat{G}_\perp \widetilde{\varphi})\widetilde{\varphi}^* \, \mathrm{d}S$, gives the following boundary condition for the energy orthonormality for a rod with radius R:

$$\mu \left(\frac{\partial \widetilde{\varphi}}{\partial \rho} \right)_{\rho = R^-} - \left(\frac{\partial \widetilde{\varphi}}{\partial \rho} \right)_{\rho = R^+} = 0. \tag{8}$$

For the operator $\hat{G}_\perp$ the boundary condition (8) corresponds to the so-called *essential* boundary conditions [8]. When such boundary conditions are used, the MS-potential eigenfunctions of the operator $\hat{G}_\perp$ form a *complete basis in an energy functional space* and the functional describing the average quantity of energy has a minimum [8].

To calculate the normalized average MS energies of mode q determined by eq. (7), one should find the MS-wave propagation constants $\beta_q^{(F)}$. Because of the separation of variables, the propagation constants are defined as solutions of a system of two equations. The first equation is the characteristic equation for MS waves in an axially magnetized ferrite rod [6], but with the essential boundary conditions. The second equation corresponds to the characteristic equation for MS waves in a normally magnetized ferrite slab [9].

Now we can formulate the energy eigenvalue problem for MS waves in a ferrite disk resonator as the problem defined by the differential equation

$$\hat{F}_\perp \widetilde{\varphi}_q = E_q \widetilde{\varphi}_q, \tag{9}$$

together with the corresponding (essential) boundary conditions. A two-dimensional ("in-plane") differential operator $\hat{F}_\perp$ is determined as

$$\hat{F}_\perp = \frac{1}{2} g \mu \mu_0 \nabla_\perp^2. \tag{10}$$

The energy orthonormality in a ferrite disk is described as

$$\left(E_q - E_{q'}\right) \int_S \widetilde{\varphi}_q \widetilde{\varphi}_{q'}^* \, \mathrm{d}S = 0. \tag{11}$$

Because of the discrete-energy eigenstates of the MS-wave oscillations resulting from structural confinement in a special case of a normally magnetized ferrite disk, one can describe the oscillating system as a collective motion of quasiparticles —the "light magnons" (lm). When we juxtapose eq. (2) with the Schrödinger equation for "free particles" ($a^{(2)} \equiv 0$), we get the following expression for the effective mass of the light magnon for a monochromatic MS-wave mode:

$$\left(m_{\mathrm{eff}}^{(\mathrm{lm})}\right)_q = \frac{\hbar}{2} \frac{\beta_q^2}{\omega}. \tag{12}$$

The light magnons have a *reflexively-translational motion* between the lower ($z = 0$) and upper ($z = h$) planes of a ferrite disk. The meaning of the term "light magnon" arises from the fact that the effective mass of these quasiparticles is much less than the effective mass of the ("real", "heavy") magnons —the quasiparticles existing due to the exchange interaction. In our description of MS oscillations we neglect the exchange interaction, and the "magnetic stiffness" is characterized by the "weak" dipole-dipole interaction. Expression (12) looks very similar to the effective mass of the heavy magnon for spin waves with a quadratic character of dispersion [10].

Calculations. – Based on the above analysis, we calculated the energy levels in a magnetic quantum well and the effective masses for light magnons. Calculations were made with the use of the disk data given in Yukawa and Abe's paper [3]: $4\pi M_0 = 1790\,\mathrm{G}$, $2R = 3.98\,\mathrm{mm}$, $h = 0.284\,\mathrm{mm}$. The working frequency is $\frac{\omega}{2\pi} = 9.51\,\mathrm{GHz}$. As a starting point, we found a discrete set of propagation constants for MS modes in a ferrite disk. The graphical solutions of a ferrite-rod and ferrite-slab system of equations with respect to the DC magnetic field for the main "thickness mode" and different "flat modes" are shown in fig. 1. Numbers ν are the Bessel function orders and numbers q characterize the radial variations of the MS-patential functions. Figures 2 and 3 show our calculation's results for the energy levels and effective masses for different-type light magnons.

In ferromagnetic resonance one can vary the parameters of the permeability tensor in two ways: a) by variation of H_0 (at constant ω) or b) by variation of ω (at constant H_0) [10].

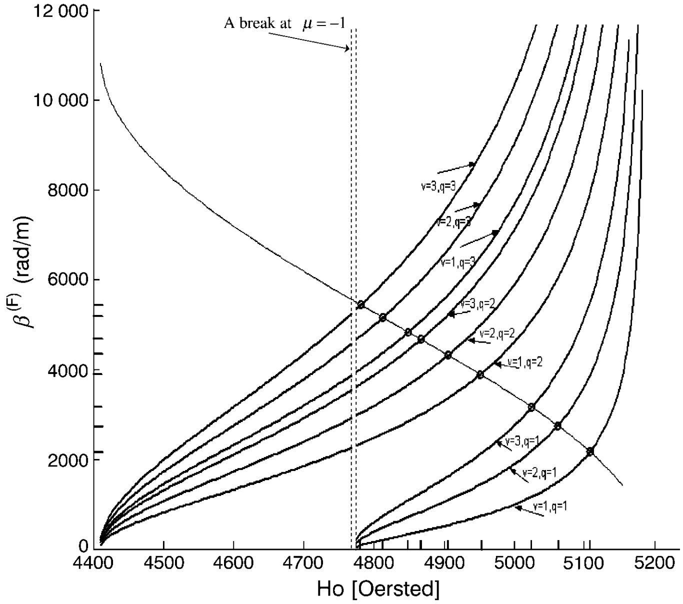

Fig. 1 – The graphical solutions for the main "thickness mode" and different "flat modes".

The above energy eigenvalue problem was formulated for the case of constant ω. *A priori*, there is no foundation to state that for a given quantity of bias magnetic field H_0, when the frequencies ω_m and ω_n are different, the functions $\widetilde{\varphi}_m$ and $\widetilde{\varphi}_n$ are mutually orthogonal.

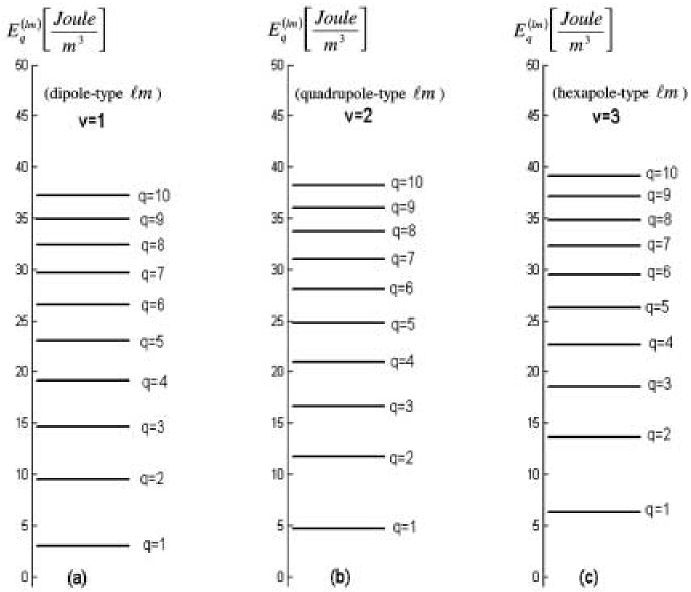

Fig. 2 – The energy levels for different-type light magnons: (a) the dipole-type light magnons ($\nu = 1$); (b) the quadrupole-type light magnons ($\nu = 2$); (c) the hexapole-type light magnons ($\nu = 3$).

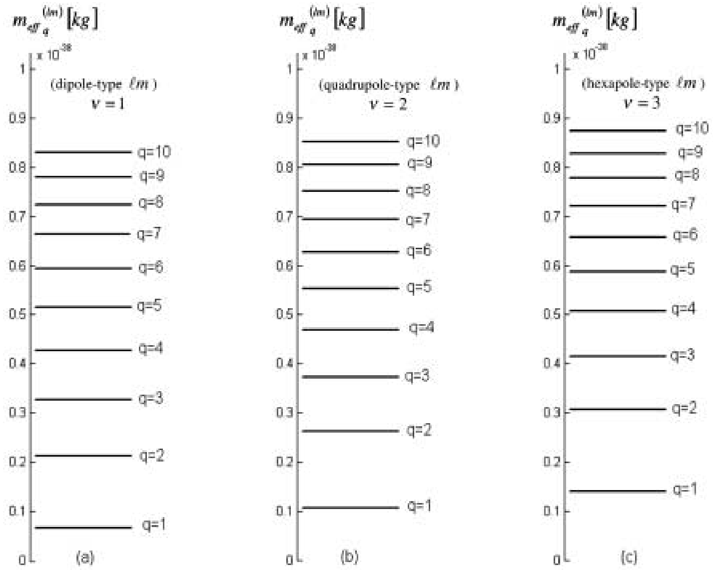

Fig. 3 – The effective masses of light magnons: (a) the dipole-type light magnons ($\nu = 1$); (b) the quadrupole-type light magnons ($\nu = 2$); (c) the hexapole-type light magnons ($\nu = 3$).

There is, however, a complete correspondence in the resonance peak positions for two types of spectra: one obtained by variation of H_0 (at constant ω) the other by variation of ω (at constant H_0). This matching is illustrated in fig. 4 for the first three peaks ($q = 1, 2, 3$) at $\nu = 1$. In fact, we are able to place sequentially every peak from the frequency spectrum at the same frequency f' according to the sequence of the DC magnetic-field values. So in the case

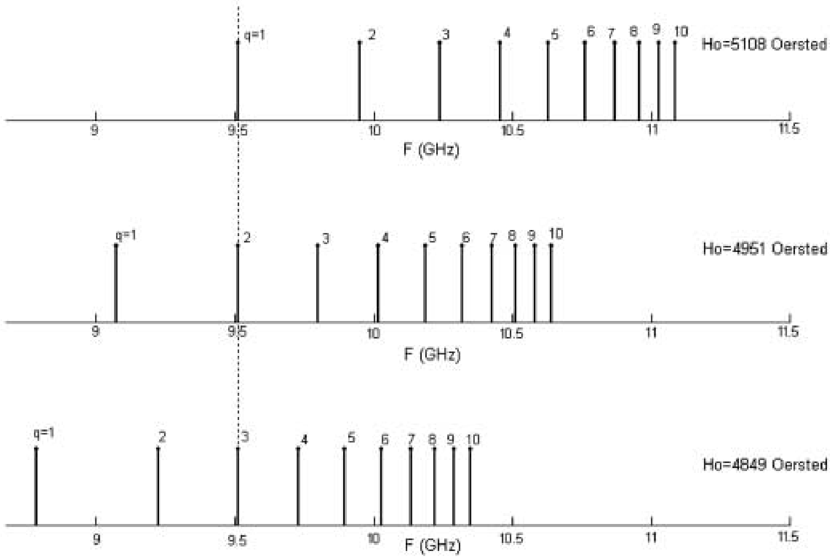

Fig. 4 – Mutual matching for the frequency and magnetic-field spectra.

of variation of ω (at constant H_0), the eigenfunctions constitute the orthonormal functional basis and, as a result, one has the same discrete spectrum of the energies E_q.

Conclusion. – For magnetic-dipolar-mode oscillations in a normally magnetized ferromagnetic disk we showed that a macroscopical quantum model allows using the effective-mass approximation and the Schrödinger-like equation to analyze the energy eigenstates of a whole ferrite-particle system similarly to semiconductor quantum wells.

REFERENCES

[1] Jacak L., Hawrylak P. and Wojs A., *Quantum Dots* (Springer, Berlin) 1997.
[2] White R. L. and Solt I. H. jr., *Phys. Rev.*, **104** (1956) 56.
[3] Dillon J. F. jr., *J. Appl. Phys.*, **31** (1960) 1605; Yukawa T. and Abe K., *J. Appl. Phys.*, **45** (1974) 3146.
[4] Bayer C. *et al.*, *Appl. Phys. Lett.*, **82** (2003) 607; Park J. P. *et al.*, *Phys. Rev. Lett.*, **89** (2002) 277201; Jorzick J. *et al.*, *Phys. Rev. Lett.*, **88** (2002) 047204; Jung S., Ketterson J. B. and Chandrasekhar V., *Phys. Rev. B*, **66** (2002) 132405.
[5] Landa P. S., *Auto-Oscillations in Distributed Systems* (Nauka, Moscow) 1983 (in Russian).
[6] Joseph R. I. and Schlömann E., *J. Appl. Phys.*, **32** (1961) 1001.
[7] Kamenetskii E. O., *Phys. Rev. E*, **63** (2001) 066612.
[8] Mikhlin S. G., *Variational Methods in Mathematical Physics* (Mc Millan, New York) 1964.
[9] Damon R. W. and Van De Vaart H., *J. Appl. Phys.*, **36** (1965) 3453.
[10] Gurevich A. and Melkov G., *Magnetic Oscillations and Waves* (CRC Press, New York) 1996.