Design of Transverse Slot Arrays Fed by a Boxed Stripline

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Abstract—An analytical design method applicable to linear transverse slot arrays fed by a boxed stripline is developed. An integral equation involving the unknown electric field distribution in the slot is formulated. The method of moments is utilized to obtain a numerical solution for the field distribution. The insight gleaned from such a computation is used to approximate the field in the slot by a perturbed half-cosinusoid, equiphase distribution. An array design technique which accounts for the external mutual coupling is presented. The design is based on a pair of equations which determine the offsets and lengths of the slots in the array. Experimental data are offered to validate the theoretical approach.

INTRODUCTION

THE DEVELOPMENT of a slot array fed by a boxed stripline traces back to 1952 when such an array, with inclined slots, was tested at Hughes by Strumwasser *et al.* [1]. The transverse electromagnetic (TEM) line was dimensioned so that all waveguide modes were cut off. The inclined series slots were centered over the straight inner conductor. A fundamental difficulty was encountered in that resonant slot lengths were longer than a half-guide wavelength, necessitating loading of the slots if they were not to overlap. The loading was determined empirically but did not reduce the slot lengths sufficiently to permit a standing wave array design. The inclined series slot also generated a cross polarized component in the radiated field, which is equivalent to an efficiency loss and may give undesirably high sidelobes. It is not difficult to understand why this approach was not pursued further.

In 1979 Park [2] completed a Ph.D. dissertation at the University of California, Los Angeles, concerned with centerline *longitudinal* slots fed by a boxed stripline. Coupling was achieved by snaking the stripline under the slots. In this configuration both problems (excessive resonant length and cross polarization) were eliminated. Since all the slots were on the centerline the transverse electric (TE_{10}) mode was not scattered. A larger box width dimension was thereby permitted, and resonant lengths of practical size were obtained. However, a limitation of this array is that the main beam cannot be efficiently scanned close to endfire because of the element pattern.

For this reason, plus the desire to learn how to achieve the other polarization with slots fed by boxed stripline, an investigation has been launched into the case of transverse slots. In this configuration the stripline is offset from the boxed stripline center and passed underneath the slot near one of its ends. The element pattern of such a slot is semicircular in the E-plane and accordingly permits scanning to endfire. In 1980, Park and Elliott [3] investigated this type of array element. Assuming (at resonance) a half-cosinusoid electric field distribution in the slot they designed an array of eight elements. The pattern of the array was well formed and close to theoretical but the input match was badly off. A diagnosis revealed that the difficulty lay in their assumption about the field distribution in the slot. The present paper is concerned with the achievement of a better approximation to the field distribution in the slot, as well as the development of a design method which will give all the fabrication dimensions needed to build an array, with pattern and input impedance specified. Experimental results on an array built to test the design procedure will be presented.

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THE SLOT FIELD DISTRIBUTION

The basic module for a transverse slot fed by a boxed stripline is shown in Fig. 1. The slot is cut in the upper broadwall of a rectangular waveguide. It has a rectangular periphery with length $2l$ and width w, and is offset with respect to the centerline of the broadwall. Its central point is at $(x, b, 0)$ in XYZ coordinates and at $(0, 0)$ in (ξ, ζ) coordinates. The waveguide walls are taken to be infinitesimally thin and composed of perfect conductor. The waveguide is totally filled with a homogeneous, isotropic dielectric of relative dielectric constant ϵ_r.

To avoid the excessive resonant slot lengths encountered by Strumwasser *et al.* [1] the **a** dimension of the waveguide was allowed to be large enough to permit the TE_{10} mode to propagate. To contain this mode and eliminate internal higher order mutual coupling, pin curtains were inserted at each end of the module. In the new geometry the pin curtains complete a cavity for the TE_{10} mode and their location controls the resonant length of the slot. The pin curtains are designed to include a gap that allows the strip (and thus the TEM mode) to pass through. Resonant lengths shorter than a half-waveguide wavelength can be attained with this geometry, and a standing wave array design becomes feasible.

In response to the passage of an incident TEM wave of mode amplitude A_{TEM}, an electric field distribution develops in the slot. We shall assume that the slot is so narrow ($w \ll \lambda$) that this field has only a transverse component in the slot aperture, represented in the functional form $E_z^s(\xi, \zeta)$. Due to the narrowness of the slot this electric field is essentially ζ symmetric. A direct consequence of the ζ symmetry of the field E_z^s is that the backscattered mode amplitude B_{TEM} and the forward-scattered mode amplitude C_{TEM} are negatives of each other. This antisymmetrical scattering is analogous to the scattering from a series element Z_n on a transmission line of characteristic impedance Z_0. This equivalent circuit is subject to the interpretation that, in the boxed stripline, only the TEM mode is of interest because only this mode passes through the pin curtains. All other scattered modes contribute to the value of the series impedance Z_n, but not to the fields which enter and leave the module.

No loss in generality occurs if the ports are taken at the positions $z = \pm\lambda_{TEM}$, with λ_{TEM} the wavelength of the TEM mode. One can show that for such an equivalent circuit the normalized

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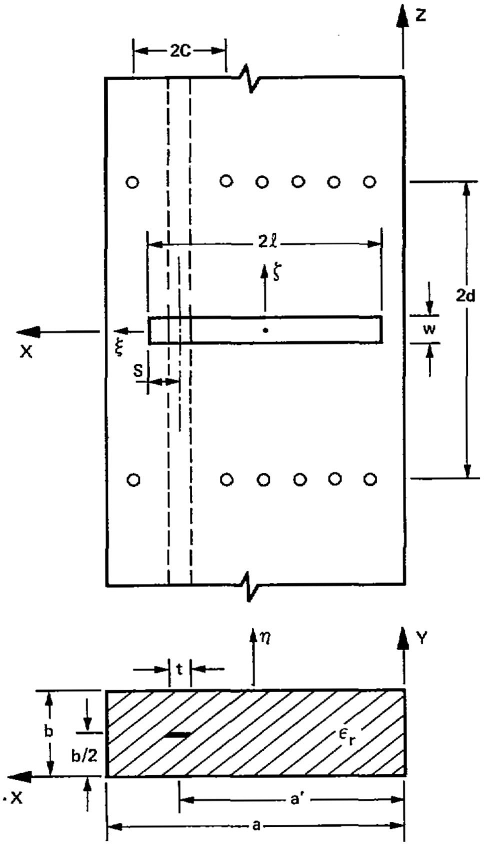

Fig. 1. Transverse slot module.

self impedance at the input port Z_n/Z_0 is related to the backscattered wave B_{TEM} by

$$\frac{Z_n}{Z_0}=\frac{2B_{\mathrm{TEM}}/A_{\mathrm{TEM}}}{1-B_{\mathrm{TEM}}/A_{\mathrm{TEM}}}. \tag{1}$$

The relation between the backscattered wave B_{TEM} and the electric field distribution in the slot has been established by Eyges *et al.* [4] and is given, for the case that B_{TEM} and C_{TEM} are equal and opposite, by

$$B_{\mathrm{TEM}}=-C_{\mathrm{TEM}}=\frac{\int_{\mathrm{slot}} E_z^s(\xi,\zeta)h_{\mathrm{TEM},x}^{(x,b)}e^{-jkz}\,dS}{2\int_0^a\int_0^b \mathbf{e}_{\mathrm{TEM}}^{(x,y)}\mathbf{x}\mathbf{h}_{\mathrm{TEM}}^{(x,y)}\cdot \mathbf{1}_z\,dS} \tag{2}$$

with $\mathbf{e}_{\mathrm{TEM}}(x,y)$, $\mathbf{h}_{\mathrm{TEM}}(x,y)$ the transverse electric and magnetic fields associated with a TEM mode of unit amplitude traveling along the boxed stripline. Park [2] has calculated the TEM field distribution by a doubly modified residue calculus technique and his solution will be adopted in what follows. One can notice that once the electric field $E_z^s(\xi,\zeta)$ is known, then with the aid of (2) and (1) we can compute the backscattered mode amplitude B_{TEM} and the self-impedance Z_n/Z_0.

In the forthcoming derivation we shall assume that the electric field in the slot is separable, i.e., that

$$E_z^s(\xi,\zeta)=\frac{V_s}{w}e_s(\xi)f(\zeta) \tag{3}$$

where V_s is the complex amplitude of the slot voltage, measured across the slot at its center. The magnitude and phase of V_s are dependent on the relative displacement s and the slot length $2l$. $e_s(\xi)$ is the shape of the amplitude distribution with respect to the ξ axis, while $f(\zeta)$ is the field distribution in the ζ direction. In the past, most investigators have made the assumption that $f(\zeta)=$ constant. However, a study of the flux map of the electric field lines in the slot suggests that the flux line density becomes steadily greater as one approaches either edge of the slot, i.e., as $\zeta\to\pm w/2$. Moreover, for a vanishingly thin wall (which is an assumption in the present analysis), the flux density goes to infinity at $\zeta=\pm w/2$. Motivated by the desire to obtain a reasonable physical approximation to the field in the slot Park [5] introduced the double pulse model for $f(\zeta)$, i.e.,

$$f(\zeta)=U\left(\zeta+\frac{w}{2}\right)-U\left(\zeta+\alpha\frac{w}{2}\right)+U\left(\zeta-\alpha\frac{w}{2}\right)-U\left(\zeta-\frac{w}{2}\right) \tag{4}$$

in which $U(\zeta)$ is a unit step function and α is a parameter whose value can be adjusted at the discretion of the designer [6]. This form for $f(\zeta)$ has been adopted throughout the present work.

The slot voltage V_s appearing in (3) can be determined at *resonance* by power balance [7] among the incident power, radiated power, the forward scattered power and the backward scattered power.

The assumption made by previous investigators [3] about the nature of $e_s(\xi)$ has been that

$$e_s(\xi)=\cos k_s\xi \tag{5}$$

in which $k_s=\pi/2l_r$ with l_r the resonant[1] length of the slot. Computation of the slot series impedance, assuming a half-cosinusoid field distribution in the slot, proved to be unsatisfactory. It is necessary to assume a more general form for $E_z^s(\xi,\zeta)$ in order to improve the agreement. Such an attempt has been made in the present work by formulating an integral equation with the field distribution as its unknown.

As the analysis progresses, it will become apparent that the technique to be used for finding $E_z^s(\xi,\zeta)$ rests on matching internal and external expressions for the tangential magnetic field $\mathbf{H}$ at a sequence of N equispaced points along the centerline of the slot, i.e., at points $(\xi_i,0)$, with $i=1,2,\cdots,N$. But on the centerline, due to the ζ symmetry of $E_z^s(\xi,\zeta)$, the tangential magnetic field in the slot aperture has only a longitudinal component. Thus the only boundary condition to be satisfied is

$$H_x^e(x,b,0)=H_x^{\mathrm{inc}}(x,b,0)+H_x^i(x,b,0) \tag{6}$$

where the superscripts stand for external, incident, and internal, respectively. If $H_x^{\mathrm{inc}}(x,b,0)$ is taken to be a component of the incident TEM mode, thus becoming the known driving function, and if $H_x^e(x,b,0)$ and $H_x^i(x,b,0)$ are expressed in terms of integrals involving the electric field distribution in the slot, (6) becomes an integral equation in the sought for $E_z^s(\xi,\zeta)$.

The evaluation of the magnetic fields above and under the slot aperture can be accomplished by studying the external and the internal problems independently. For the external problem the electric field distribution $E_z^s(\xi,\zeta)$ in the slot can be viewed as the source of an external field $(\mathbf{E}^e,\mathbf{H}^e)$ in the half-space above the infinite perfectly conducting ground plane in which the

[1] A resonant slot is defined by the condition that the forward scattered wave C_{TEM} and the incident wave A_{TEM} are out of phase.

upper broadwall of the waveguide is assumed to be embedded. Similarly, $E_z^s(\xi, \zeta)$ can be viewed as the source for the scattered internal field $(\mathbf{E}^i, \mathbf{H}^i)$ in the boxed stripline.

Standard computation [6] of the x component of the external radiated magnetic field in the slot aperture on its centerline gives

$$H_x^e(x, b, 0) = -2\mu_0^{-1} \int_{\text{slot}} E_z^s(\xi, \zeta) G_{xx}^e(x/x', z')\, ds' \tag{7}$$

in which $G_{xx}^e(x/x', z')$ is the external Green's function on the centerline of the slot and is given by

$$G_{xx}^e(x/x', z') = -\frac{j\omega\epsilon_0}{4\pi\mu_0^{-1}}\left[1 + \frac{1}{k_0^2}\frac{\partial^2}{\partial x'^2}\right]\frac{e^{-jk_0R}}{R} \tag{8}$$

where

$$R = \sqrt{(x-x')^2 + z'^2}. \tag{9}$$

The internal expression for the tangential magnetic field in the slot consists of the sum of the incident TEM mode and the field scattered by the slot in a profusion of modes. The TEM mode in the boxed stripline has been derived by Park [2]. However, all other modes are, at the outset, unknown. A structure which resembles very much the boxed stripline is the regular rectangular waveguide for which all the modes are well-known [7]. Thus we may consider the boxed stripline as a rectangular waveguide perturbed by the insertion of a strip. Fortunately, all the modes with electric field components E_x, E_z which are zero at the strip location are not perturbed at all and are valid modes in the new structure. In the special case in which the strip is y-centered in the box, those unperturbed modes constitute half of the total modes in the structure and are the TE_{mn}, TM_{mn} waveguide modes with subscript n even. The other half of the waveguide modes are perturbed, but by an investigation conducted with the aid of a technique due to Mittra and Itoh [8], applicable to a relatively narrow strip ($t \ll a$), we have found that the propagation constants of the perturbed modes can be approximated by their counterparts in the rectangular waveguide [7]. Moreover, in the present work the ratio a/b is relatively high (~14) so that even the low order perturbed modes have high propagation constants, compared to the propagation constants of the unperturbed modes. Consequently, they are highly damped and their contribution to the internal scattered field is insignificant in the present work and will be neglected.

The formulation of the internal scattered field utilizes Stevenson's theory [9] for a rectangular waveguide, while extending the set of the waveguide modes to include the boxed stripline TEM mode. In addition we make the following assumptions.

1) The only propagating modes are the TEM and TE_{10} modes; all other modes are considered to be evanescent.
2) The pin curtains constitute two perfectly conducting walls for the TE_{10} mode. The TEM mode passes through without any perturbation, while all other modes decay to an insignificant level before reaching the pins.

Carrying out the details of such an analysis [6], one can show that the internal scattered magnetic field in the slot aperture on its centerline is given by

$$H_x^i(x, b, 0) = \mu_0^{-1} \int_{\text{slot}} E_z^s(\xi, \zeta) G_{xx}^i(x/x', z')\, dS' \tag{10}$$

in which $G_{xx}^i(x/x', z')$ is the xx component of the internal dyadic Green's function on the centerline of the slot, that is,

$$\begin{aligned} G_{xx}^i(x/x', z') = -\mu_0 \Bigg\{ & \frac{h_{\text{TEM},x}^{(x)} h_{\text{TEM},x}^{(x')}}{2S_{\text{TEM}}} e^{-jk|z'|} + \frac{\beta_{10}}{j\omega\mu_0 ab} \sin\frac{\pi x}{a} \sin\frac{\pi x'}{a} \\ & \cdot \frac{\cos\beta_{10}z' + \cos\beta_{10}(2d - |z'|)}{\sin 2\beta_{10}d} + \frac{2}{j\omega\mu_0 ab} \\ & \cdot \sum_{m=1}^{\infty}{}' \sum_{n=0}^{\infty} \frac{\left(\frac{m\pi}{a}\right)^2 - k^2}{\gamma_{mn}} \epsilon_{mn}^2 \sin\frac{m\pi x}{a} \\ & \cdot \sin\frac{m\pi x'}{a} e^{-\gamma_{mn}|z'|} \Bigg\} \end{aligned} \tag{11}$$

where $\Sigma'\Sigma$ denotes that the TE_{10} mode was excluded. In the above,

$$\gamma_{mn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2} \quad j\beta_{10} = \sqrt{k^2 - (\pi/a)^2} \tag{12}$$

and

$$S_{\text{TEM}} = \int_0^a \int_0^b [\mathbf{e}_{\text{TEM}}^{(x,y)} \mathrm{x} \mathbf{h}_{\text{TEM}}^{(x,y)}] \cdot \mathbf{1}_z\, dS \tag{13}$$

Also, $\epsilon_{m0} = \epsilon_{0n} = 1/\sqrt{2}$; $\epsilon_{mn} = 1$ otherwise. Combination of (7) and (10) with the boundary condition (6) gives the integral equation in the unknown electric field in the slot:

$$\begin{aligned} H_x^{\text{inc}}(x, b, 0) = -\mu_0^{-1} \int_{\text{slot}} & E_z^s(\xi, \zeta) \\ & \cdot [2G_{xx}^e(x/x', z') + G_{xx}^i(x/x', z')]\, dS'. \end{aligned} \tag{14}$$

This integral equation in the sought for $E_z^s(\xi, \zeta)$ has been solved numerically by the method of moments [10]. This method reduces the original integral equation to a system of linear algebraic equations in N unknowns. The approach adopted here is a special case of the method of moments and is often called the point matching technique with unit pulses used as basis functions. In our application the slot aperture is divided into N equals rectangular cells and (14) is imposed at the central point of each cell. The net result is that the average value of $E_z^s(\xi, \zeta)$ in each cell is found by matrix inversion.

A test module was constructed with the dimensions a = 3.413 in, a' = 2.55 in and b = 0.25 in (cf. Fig. 1). The strip was 0.1764 in wide and the box was filled with teflon/glass for which $\epsilon_r = 2.5$. The characteristic impedance of the stripline was 50 Ω. The radiating slot had a width of 0.25 in and a variable length. A curtain of plate-through holes was placed 2 in on each side of the slot. The holes were 0.25 in on centers with the holes nearest a sidewall 0.35 in from the sidewall. The operating frequency was 1.75 GHz.

When these dimensions were used in (14) and its related equations, numerical analysis provided the computed shape of the slot field distribution as a function of the offset parameter s. Fig. 2 gives a typical result, for which the offset was $s = -0.1$ in. Comparison to a half-cosinusoid distribution and to the experimental data accomplished by near field measurement is also

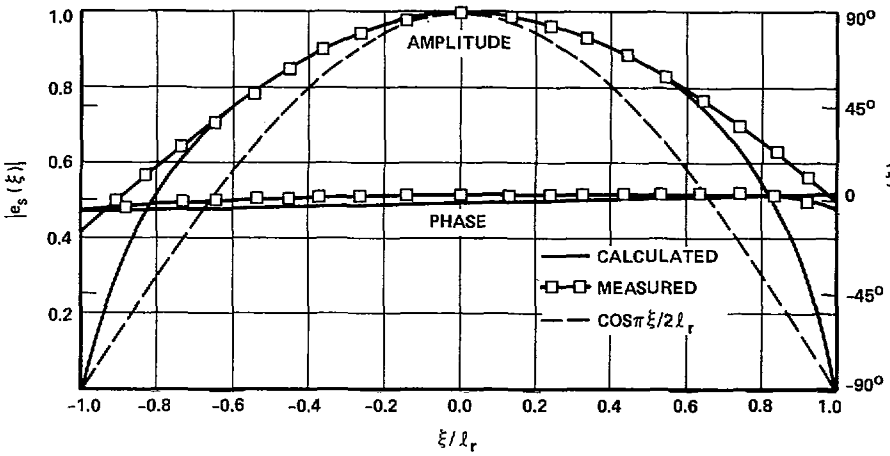

Fig. 2. Field distribution in a resonant slot (s = −0.1 in).

found in Fig. 2. As one can notice, the amplitude shapes of the computed and measured field distributions resemble a half-cosinusoid, but are more bulged (a similar effect has been noted by others who have studied the center-fed dipole [11]); an asymmetry with respect to the slot center can also be noticed (the measured field distribution does not vanish at the slot ends because the probe which measured the near field could not be placed down on the slot surface). Fig. 2 shows also that the computed phase distribution is most nearly uniform at resonance and agreement with the experimental data is good. In general the computed and experimental data agreed fairly well, serving to justify the approximations made in the theory.

Variation of the slot length ±3 percent around resonance showed that the computed shape of the amplitude and phase distribution is fairly constant over this range of slot lengths. Only the level of the phase and amplitude is affected by the length. Theory and experiment agree on this observation.

The computed normalized resonant resistance of the slot compared to the experimental data, and the normalized resonant resistance computed with an assumed half-cosinusoid field distribution are displayed in Fig. 3. It can be observed that the values of R_n^r/Z_0 computed using the method of moments are higher than the measured values. Moreover, the computed resonant length is lower than what was measured. Close scrutiny of the factors which contribute to the calculations of B_{TEM} given by (2) and R_n^r/Z_0 given by (1) reveals that they are extremely sensitive to the shape of the field distribution near the stripline. This sensitivity is caused by the fast decay of the electric field $e_{\mathrm{TEM}}^{(x,b)}$ in the vicinity of the stripline center. Thus a relatively small error in the shape of the computed field distribution can cause a large discrepancy in R_n^r/Z_0. Possible factors which may affect the accuracy of the computed field distribution are the contribution of the perturbed modes (ignored in the present work) and the existence of a longitudinal component $E_x^s(\xi, \zeta)$ in the slot aperture.

However, the results computed via the method of moments as displayed in Fig. 3 show the same trend as the experimental data, thus giving a qualitative confirmation of the theory.

The computed normalized resistance of the slot R_n/R_n^r and the normalized reactance X_n/R_n^r, compared to measured data are displayed in Fig. 4. One can notice that the agreement with the experimental data is quite good in a range ±3 percent around the resonant length. The curves clearly show that the slot behaves like a resonant element with maximum resistance and zero reactance at resonance. Another important conclusion which can be drawn from Fig. 4 is that the impedance data of the slot can be presented by a universal form in a range ±3 percent around resonance. This observation implies that in this range of l/l_r the slot impedance Z_n is separable with respect to the offset s and the relative length l/l_r, i.e.,

$$Z_n = R_r(s)\left[\frac{R_n}{R_n^r}(l/l_r) + j\frac{X_n}{R_n^r}(l/l_r)\right]. \tag{15}$$

The functions $R_r(s)$, $R_n/R_n^r(s)$, and X_n/R_n^r which arise from the measurements can be easily polyfitted for use in the design of the array.

The fact that the experimental data of the resonant resistance R_n^r/Z_0 lies between the values computed by the method of

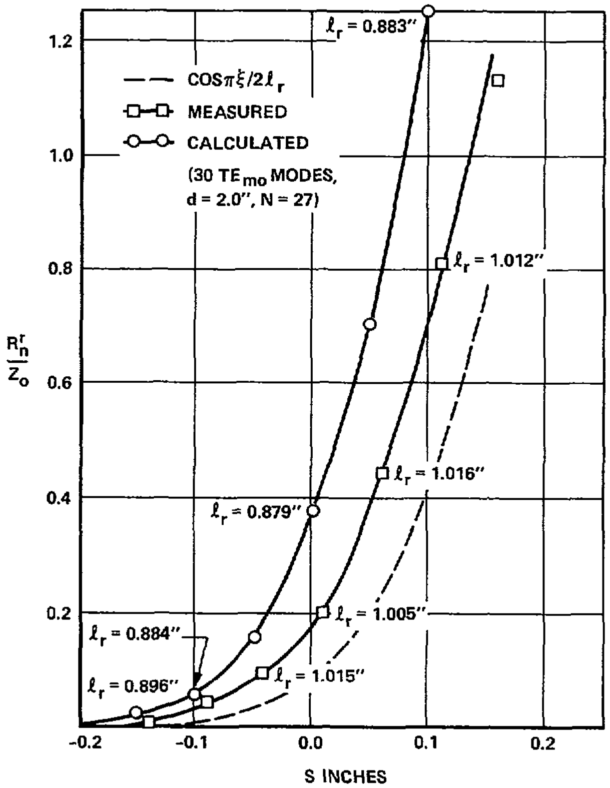

Fig. 3. Normalized resonant resistance of the slot versus the offset s.

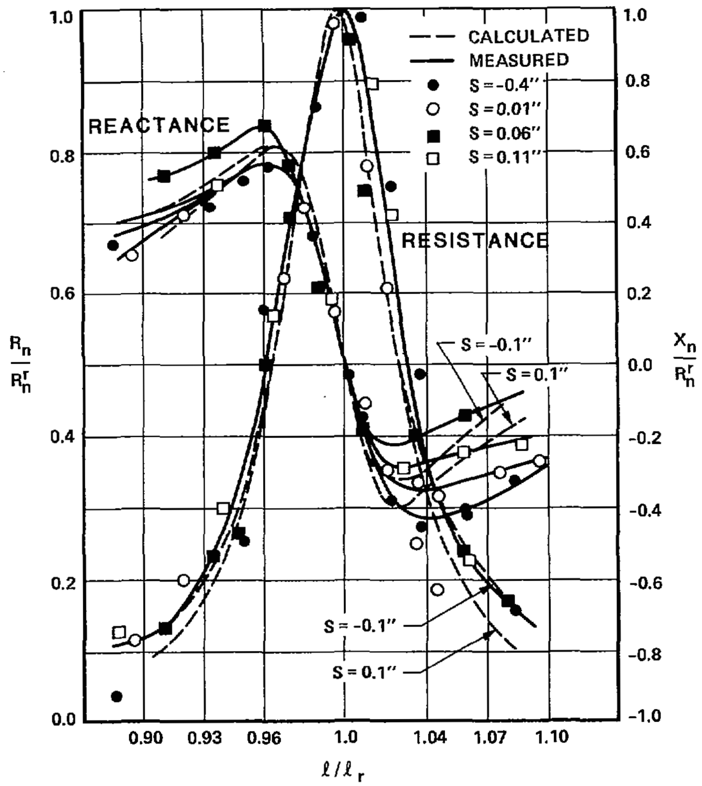

Fig. 4. Normalized resistance and reactance of the slot versus its normalized length.

moments and the values computed with an assumed half-cosinusoid field distribution, suggests that a small perturbation applied to either one will match the computed and measured values of R_n^r/Z_0. Obviously, it is simpler and more cost effective to apply the perturbation to the half-cosinusoid distribution. Thus the perturbed field distribution will be expressed in the form

$$E_z^s(\xi,\zeta)=\frac{V_s}{w}\cos\left[\frac{\pi}{2l}g(\xi)\right]f(\zeta), \quad -l\leqslant\zeta\leqslant l, \quad -\frac{w}{2}\leqslant\xi\leqslant\frac{w}{2} \tag{16}$$

where $g(\xi)$ is a nonlinear function and its boundary conditions are

$$g(\pm l_r)=\pm l_r. \tag{17}$$

A search conducted to detect suitable choices for the function $g(\xi)$ led to a selection which gives a smooth transition from the half-cosinusoid distribution; it involves Rayleigh's function [12] and is given by

$$g(\xi)=\xi+h\,\frac{\xi-l_r}{\xi_a-l_r}\,\frac{\left[e^{\frac{(2l_r^2)-(\xi-l_r)^2}{2\xi_a^2}}-1\right]}{\left[e^{\frac{(2l_r)^2-(\xi_a-l_r)^2}{2\xi_a^2}}-1\right]} \tag{18}$$

where h is the perturbation parameter required to match the computed and measured values of R_n^r/Z_0 and ξ_a is a parameter whose value determines the position of the maximum value of the Rayleigh function [6].

Fig. 5 shows the perturbed compared to the unperturbed half-cosinusoid field distribution. It indicates that a very small perturbation on the half-cosinusoid distribution is required to match the computed (through power balance) and measured values of R_n^r/Z_0 (cf. Fig. 3).

DESIGN THEORY FOR A TRANSVERSE SLOT ARRAY

Consider the module shown in Fig. 1. One-dimensional arrays can be created by arranging a sequence of these modules in tandem (a single branch line). Two-dimensional arrays can be fashioned by placing a number of branch lines side by side.

The succeeding developments of the design procedure of an N element array closely parallel that of Elliott for longitudinal shunt slot arrays fed by a rectangular waveguide [2]. That technique accounts for the external mutual coupling, but neglects the internal higher order mutual coupling.[2] Substitution of (4), (13), and (16) into (2) reveals that the backscattered coefficient $B_{\rm TEM}$ of the nth slot in the array is related to its slot voltage V_n^s by

$$B_{\rm TEM}=-C_{\rm TEM}=V_n^s f_n(l_n,s_n) \tag{19}$$

where

$$f_n(l_n,s_n)=\frac{1-\alpha}{2S_{\rm TEM}}\int_{-l_n}^{l_n}\cos\left[\frac{\pi}{2l_n}g_n(\xi)\right]\cdot h_{{\rm TEM},x}(a'+s_n-l_n+\xi)\,d\xi. \tag{20}$$

In (20) we have replaced the x dependence of the TEM magnetic field by its equivalent in the (ξ,η,ζ) coordinate system.

It will be convenient to consider the total voltage V_n^s to be composed of three parts.

[2] In this application there is no internal mutual coupling because of the presence of the pin curtains.

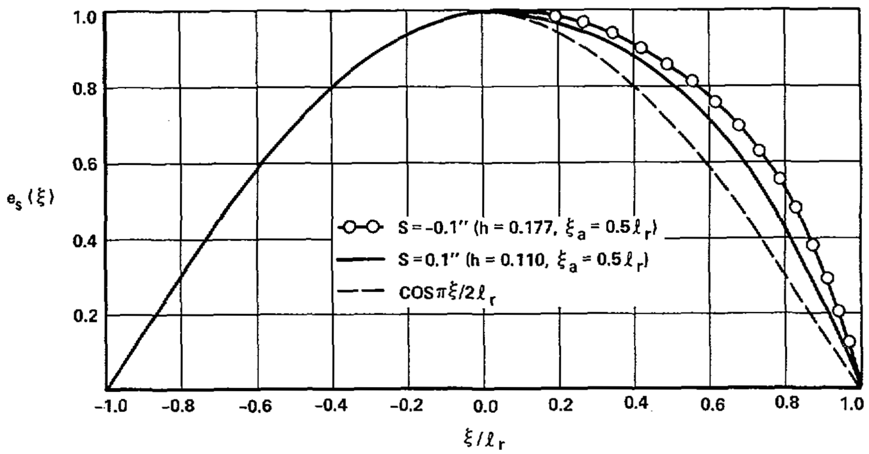

Fig. 5. Half-cosinusoid distribution and two perturbed distributions.

1) $V_{n,1}^s$ is due to a TEM mode of complex amplitude A_{TEM} incident from the left of the nth slot ($z = -\infty$) when all other slots are covered with conducting tape and the boxed stripline containing the nth slot is terminated by a matched load.
2) $V_{n,2}^s$ is due to a TEM mode of complex amplitude D_{TEM} incident from the right of the nth slot ($z = +\infty$) when all other slots are shorted and the boxed stripline is terminated in a matched load.
3) $V_{n,3}^s$ is due to external coupling with the other $N - 1$ slots in the array.

Thus the total voltage V_n^s is given by

$$V_n^s = V_{n,1}^s + V_{n,2}^s + V_{n,3}^s. \tag{21}$$

The situation in which $V_{n,1}^s$ is excited has been considered in the previous section. Accordingly, if we combine (1) and (19) we obtain after rearrangement that

$$V_{n,1}^s = \frac{1}{f_n(l_n, s_n)} \frac{\dfrac{Z_n}{Z_0}}{2 + \dfrac{Z_n}{Z_0}} A_{\text{TEM}}. \tag{22}$$

Similarly,

$$V_{n,2}^s = -\frac{1}{f_n(l_n, s_n)} \frac{\dfrac{Z_n}{Z_0}}{2 + \dfrac{Z_n}{Z_0}} D_{\text{TEM}}. \tag{23}$$

The remaining partial slot voltage $V_{n,3}^s$ can be deduced by an analysis which parallels that of Elliott [13]. The result is that

$$V_{n,3}^s = -\frac{j\omega\epsilon_0(1-\alpha)^2}{4\pi S_{\text{TEM}}} \cdot \frac{1}{f_n^2(l_n, s_n)} \cdot \frac{\dfrac{Z_n}{Z_0}}{2 + \dfrac{Z_n}{Z_0}} \sum_{m=1}^{N}{}' V_m^s p_{mn}. \tag{24}$$

The prime on the summation sign means that the term $m = n$ is excluded. In the preceding,

$$p_{mn} = \int_{-l_m}^{l_m} \cos\left[\frac{\pi}{2l_m} g_m(\xi_m')\right] \left\{ \int_{-l_n}^{l_n} \cos\left[\frac{\pi}{2l_n} g_n(\xi_n')\right] \cdot \left(1 + \frac{1}{k_0^2} \frac{\partial^2}{\partial \xi_n'^2}\right) \frac{e^{-jk_0R}}{R} \, d\xi_n' \right\} d\xi_m' \tag{25}$$

where

$$R = \sqrt{(z_m - z_n)^2 + (\xi_m' - \xi_n')^2}. \tag{26}$$

z_m and z_n are the central axis coordinates of the mth and nth slots in the array.

Equation (24) completes the development of expressions for the three partial slot voltages and opens the way to the formulation of the array design equations.

For a resonantly spaced array (slots λ_{TEM} on centers in a common boxed stripline) the nth slot in the array can be modeled by a series active impedance Z_n^a on an equivalent transmission line of characteristic impedance Z_0. The scattering from Z_n^a is given by [6]

$$B_n = -C_n = \frac{1}{2} \frac{Z_n^a}{Z_0} I_n \tag{27}$$

in which I_n is the mode current at the juncture where the series element Z_n^a is placed. Equations (19) and (27) can be connected by requiring that B_{TEM} and B_n have the same phase at any cross section z and that the backscattered power levels be the same in both cases. Under those conditions the two equations combine to give

$$\boxed{\frac{Z_n^a}{Z_0} = 2\sqrt{S_{\text{TEM}} Z_0} f_n(l_n, s_n) \frac{V_n^s}{I_n}}. \tag{28}$$

Equation (28) is a principal result of the analysis and will be called the first design equation.

On the other hand if we take the difference of the input impedance to the left and to the right of the nth slot (expressed in terms of scattered and incident waves), we obtain

$$\frac{Z_n^a}{Z_0} = \frac{2B_{\text{TEM}}}{A_{\text{TEM}} - B_{\text{TEM}} - D_{\text{TEM}}} \tag{29}$$

where A_{TEM} and D_{TEM} are the incident waves from the left and from the right of the nth slot, respectively. If we use (19) and (22)-(24) to make substitutions in (29), rearrangement gives

$$\boxed{\frac{Z_n^a}{Z_0} = \frac{2f_n^2(l_n, s_n)}{\dfrac{2f_n^2(l_n, s_n)}{Z_n/Z_0} + j\,\dfrac{\omega\epsilon_0(1-\alpha)^2}{4\pi S_{\mathrm{TEM}}} \displaystyle\sum_{m=1}^{N}{}' \frac{V_m^s}{V_n^s} p_{mn}}}. \tag{30}$$

Equation (30) is the second design equation and together with (28), it permits determination of the array dimensions needed to obtain a specified pattern and input impedance.

ARRAY DESIGN

As preliminary preparations to the design procedure of the array we have to compute the desired slot voltage distribution V_m^s/V_n^s from the pattern requirements. In addition, the x component of the TEM magnetic field on the upper broadwall of the box [2], the self-impedance of the slot Z_n given by (15), the resonant length $l_r(s)$, and the perturbation parameter $h(s)$ of the function $g(\xi)$ have to be polyfitted.

Because both design equations are nonlinear, an iterative procedure must be used to obtain a solution. To start the process, all slots can be assumed to be of resonant length and zero offset for the purpose of computing the mutual coupling given by $\Sigma_{m=1}^{N'} V_m^s/V_n^s p_{mn} (n = 1, 2, \cdots, N)$. Then a computer search routine can be used to determine a sequence of couplets (s_n, l_n), each of which will make Z_n^a/Z_0, as it appears in (30), pure real. The proper couplet can then be chosen for each slot so that (28) is satisfied for the specified aperture distribution. In this fashion a preliminary set of slot lengths and offsets is established. Next a test is performed to determine if the sum of all normalized impedances in the array is equal to unity (for an input match). The process is iterated with the new (s_n, l_n) values to improve on p_{mn} calculations and obtain an input match. Experience has shown that convergence normally occurs in a few iterations.

This procedure has been applied to the design of a six-element slot array with the slots $\lambda_g = 0.8\ \lambda_0$ on centers ($\epsilon_r = 2.5$). A Chebyshev amplitude distribution was specified, with a design sidelobe level of −20 dB. An input match was desired at 1.75 GHz. The required slot dimensions computed by the design procedure (outline previously) are given in Table I. The equivalent impedance of the slot was measured by the transmission method. The transmission coefficient T_s of the slot module with the slot not radiating (covered) was read using a network analyzer. Then the transmission coefficient T with the slot uncovered was measured. One can show that those coefficients are related to the normalized impedance of the slot by

$$\frac{Z_n}{Z_0} = 2\left[\frac{T_s}{T} - 1\right]. \tag{31}$$

The measured slot impedance is plotted in Fig. 3. The voltage standing-wave ratio (VSWR) of the array was a minimum (1.07) at 1.738 GHz, which is only 0.6 percent below the design frequency (1.75 GHz). The E-plane pattern of the array was also measured and the best agreement between theoretical and experimental patterns occurred at 1.755 GHz. This comparison is shown in Fig. 6. The measured pattern is well formed and the sidelobe level is within 0.5 dB of the design value at −20 dB. The small shift in the optimum operating frequency can be attributed to the inaccuracy caused by trimming the slots (offsets and lengths) manually. The pattern was measured over an extended frequency range and it was found to maintain its shape, with the SLL not exceeding −18 dB in a band width of ±1.2 percent around the optimum frequency.

TABLE I
COMPUTED ARRAY DESIGN DATA

Element number	Offset (inch)	Slot length (inch)
1, 6	−0.066	2.050
2, 5	−0.031	2.058
3, 4	0.010	2.045

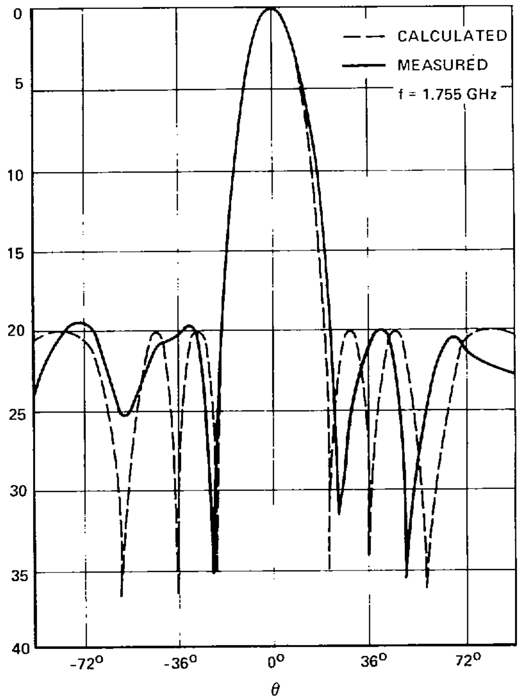

Fig. 6. Calculated and measured patterns for a six-element array, $d = \lambda_g = 0.8\ \lambda_0$, with a 20 dB Dolph-Chebyshev distribution.

CONCLUSION

The purpose of this study was to devise a technique which would permit the design of a transverse slot array fed by a boxed stripline. Solution for the field distribution in the slot, utilizing the method of moments, proved to be successful and gave a qualitative insight to the amplitude shape and phase distribution of the electric field in the slot. The insight gleaned about the field distribution in the slot was utilized to apply a small perturbation on the half-cosinusoid distribution to match the computed and measured values of the resonant resistance R_n^r/Z_0. The theoretical analysis of the field distribution in the slot was coupled with an array design theory which accounts for the external mutual coupling to provide a successful array design technique. The theory has been tested experimentally for a six-element array with 20 dB Chebyshev distribution excitation. In general, the agreement has been found to be quite satisfactory.

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