Learning in correlators based on projections onto constraint sets

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The iterative algorithm, projections onto constraint sets, is employed to generate spatial filters for pattern-recognition correlators. Based on all the training sets, all the filters are trained simultaneously.

The problem of teaching a correlator to classify many patterns can be described as an optimization problem.[1] In the case of shift-invariant pattern recognition, we sometimes deal with the optimization of an error function with a huge number of variables, and therefore we must find efficient algorithms that can handle a problem with reasonable complexity and in a relatively short period of time. Recently a method[2] to calculate a synthetic discriminant function[3] (SDF) from a given training set of objects was proposed based on the well-known projections-onto-constraint-sets (POCS) algorithm.[4] The learning procedure has been employed on a simulated joint-transform correlator in order to find a reference function that can then distinguish between two object classes.

In this Letter the learning method and its tasks are modified. The main goal is to introduce a general procedure for synthesizing SDF's from a given training set of any size. At the end of the learning period, these SDF's, configured in correlators, can recognize different objects that belong to the classes on which they have been trained. This learning method can be employed simultaneously, and in parallel, with all the SDF's of the different classes.

The POCS algorithm has been employed in several areas of signal processing (with many other designations), mostly in signal recovery.[4] Basically, it is an iterative process that transfers a function, usually by Fourier transform (FT), from one domain to another. In every domain, it is projected onto a constraint set. The convergence of the process is achieved if and when the function satisfies all the constraints in both domains simultaneously.

Unlike for most POCS algorithms, we transform a function from a SDF plane to a correlation plane by a special case of a correlation operator. Our correlator configuration is designated as a phase-extraction correlator (PEC). In the PEC, only the FT phase distribution of an input object is taken into account during the correlation process. For any input image $f(x)$ (one-dimensional notation is used for simplicity), whose FT is $F(u) = |F(u)|\exp[j\Phi(u)]$, and for a given spatial filter $H(u)$, the output correlation function of the PEC is

$$c(x') = \mathcal{F}^{-1}\{\exp[-j\Phi(u)]H(u)\}, \qquad (1)$$

where $\mathcal{F}$ is the FT operator. The SDF [$h(x)$, which is the FT of the filter function $H(u)$] can be obtained from the correlation function by operation of the inverse PEC, i.e.,

$$h(x) = \mathcal{F}^{-1}\{\exp[j\Phi(u)]\mathcal{F}\{c(x')\}\}. \qquad (2)$$

In contrast to the ordinary linear correlator (i.e., $c = \mathcal{F}^{-1}\{F^*H\}$), the PEC guarantees a nondiverging behavior of the POCS algorithm. Two properties are the reasons for that benefit. First, as follows from Eq. (1), it is linear operator related to $h(x)$ as a system input. Second, the PEC is an energy-conserving operator. This property is easily obtained based on Parseval's theorem, as follows

$$\int |c(x')|^2 \mathrm{d}x' = \int |\exp[-j\Phi(u)]H(u)|^2 \mathrm{d}u = \int |H(u)|^2 \mathrm{d}u = \int |h(x)|^2 \mathrm{d}x. \qquad (3)$$

Our overall goal is to obtain a desired intensity distribution in the correlation plane, and therefore the first constraint set is considered in this plane. In a typical pattern-classification task, we assume K object classes, where in the kth class there are N_k patterns. Our goal is to generate spatial filters $\{H_k(u)\}_{k=1}^{K}$. The kth filter should produce a sharp peak in the correlation plane when an object from the kth class is in the input of the classifier or produce a diffused distribution if an object from the lth class ($l \neq k$) is present. Because the correlator is space invariant, we can handle the problem simultaneously with all the $\sum_k N_k$ objects present at the input plane. All the objects are separated from one another by a distance that enables us to locate the set of the K SDF's between every two objects. That is done in order to avoid overlapping among the various correlation functions. It is convenient to split the input plane into K regions, each one containing the N_k objects of the kth class. As a result, the correlation plane is also split into K regions containing the respective correlation functions of the objects from K classes with the SDF's. The constraint set in this plane requires the appearance of only $\sum_k N_k$ bright correlation peaks corresponding to the centers of the correlations between every object and its proper SDF. Hence the absolute value of the central points of the correlation functions related to the objects of the kth class, with its SDF h_k, and designated region R_1, will be equal to or greater than a predetermined threshold

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level denoted T_1. Since the sharpness of the correlation peaks is not our concern at this point, we do not force any value around the central points of the correlation distributions. These areas without their central points, which are designated as region R_3, will remain unconstrained. The dimensions of every such area are defined as equal to the dimensions of the corresponding object. The magnitude of all other points of the correlation plane, designated region R_2, which are above a second threshold level $T_2 < T_1$, will be constrained to the value of T_2. The ratio between T_1 and T_2 determines the discrimination ratio (DR) and can be adjusted by the user during the synthesis process. The projection operator of the correlation domain P_1 is defined as

$$P_1[c(x')] = \begin{cases} T_1 \exp[j\theta(x')] & \text{if } x' \in R_1 \text{ and } |c(x')| < T_1 \\ T_2 \exp[j\theta(x')] & \text{if } x' \in R_2 \text{ and } |c(x')| > T_2, \\ c(x') & \text{otherwise} \end{cases} \quad (4)$$

where $c(x') \equiv |c(x')|\exp[j\theta(x')]$.

The second domain of the POCS algorithm is the SDF plane in which the SDF's are reshaped in every iteration. Our aim in this process is to find SDF's that classify many objects that are simultaneously displayed on the input scan, regardless of their specific positions. This goal can be achieved if the dimensions of every SDF are similar to that of an object (assuming that all the objects in the training set have approximately the same dimensions). Therefore the second constraint forces every SDF to have a predefined finite extended area that is similar to the object's area. Since all the SDF's are calculated in parallel, they should all appear in the SDF plane together. In order to avoid overlapping among the different correlation functions, the SDF's are separated such that between every two SDF's there is enough space to locate any object. After the projection, the nonconnected region that the SDF's should occupy (according to the above-mentioned guiding) is denoted S. The second projection is given by

$$P_2[h(x)] = \begin{cases} h(x) & \text{if } x \in S \\ 0 & \text{otherwise} \end{cases}, \quad (5)$$

where $h(x)$ is a complex function composed from all the K SDF's, i.e., $h(x) = \sum_k h_k(x - d_k)$, and d_k is the distance of h_k from the origin. It is well understood that, in the final classifier, each $h_k(x)$ is transformed separately to a spatial filter and configured alone in one of the correlation channels. The complete scheme describing the POCS algorithm is shown in Fig. 1.

In order to evaluate the convergence of the proposed algorithm we present a nondiverging process, following Fienup's study[5] regarding the phase retrieval. The two error measures defined here are the mean-square errors between functions in the ith iteration and their projected versions in every domain, i.e.,

$$e_{1,i} \triangleq \int |c_i(x') - P_1[c_i(x')]|^2 \mathrm{d}x,$$

$$e_{2,i} \triangleq \int |h_i(x) - P_2[h_i(x)]|^2 \mathrm{d}x. \quad (6)$$

Based on the above-mentioned linearity and energy-conserving properties, the error value $e_{1,i}$ can be written as

$$e_{1,i} = \int |P_2[h_i(x)] - h_{i+1}(x)|^2 \mathrm{d}x. \quad (7)$$

By definition, the projected function of h_{i+1} is the nearest value to h_{i+1}, and therefore

$$e_{1,i} \geq \int |h_{i+1}(x) - P_2[h_{i+1}(x)]|^2 \mathrm{d}x = e_{2,i+1}. \quad (8)$$

Similarly, when we consider the error at the SDF plane, it is

$$e_{2,i+1} = \int |h_{i+1}(x) - P_2[h_{i+1}(x)]|^2 \mathrm{d}x = \int |P_1[c_i(x')] - c_{i+1}(x')|^2 \mathrm{d}x'. \quad (9)$$

Here again, the projected function of c_{i+1} is the nearest value to c_{i+1}, and therefore

$$e_{2,i+1} \geq \int |c_{i+1}(x') - P_1[c_{i+1}(x')]|^2 \mathrm{d}x = e_{1,i+1}. \quad (10)$$

The overall conclusion is that the values of the errors series cannot increase and that, in the worst case, they stay the same at each successive iteration, i.e., $e_{1,i+1} \leq e_{2,i+1} \leq e_{1,i}$. The series $\{e_{1,i}\}_{i=1}^{\infty}$ is most important for our purpose, since it indicates how close the output distribution is to the desired result. We also see a generalization of the nondiverging analysis to all POCS algorithms with a linear and energy-conserving transform operator. The FT as well as the PEC is a transform operator with those properties.

In our experiment we chose four versions of the digit 2 for the first class and four versions of the digit 3 for the second class, as shown in Fig. 2. We calculated the phase function $\exp[j\Phi(u)]$ once by Fourier transforming the pattern in Fig. 2 and extracting

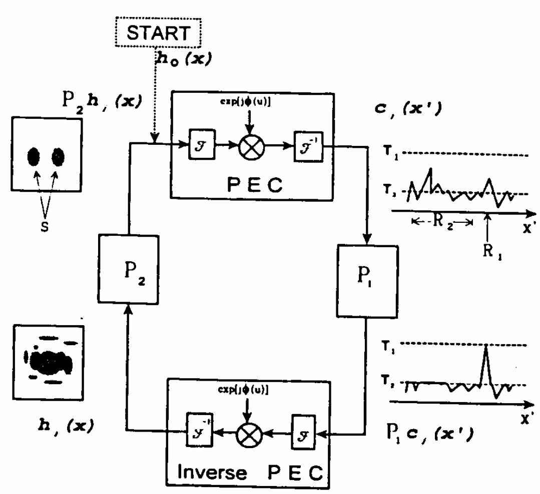

Fig. 1. Block diagram of the POCS process.

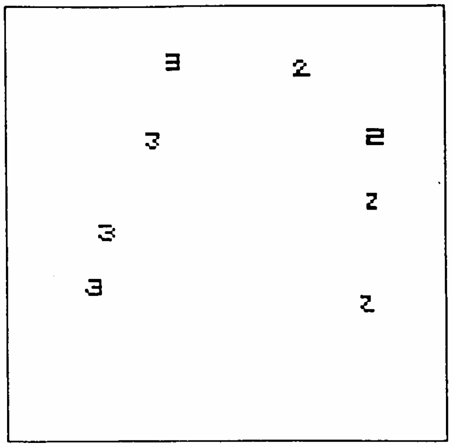

Fig. 2. Input training set.

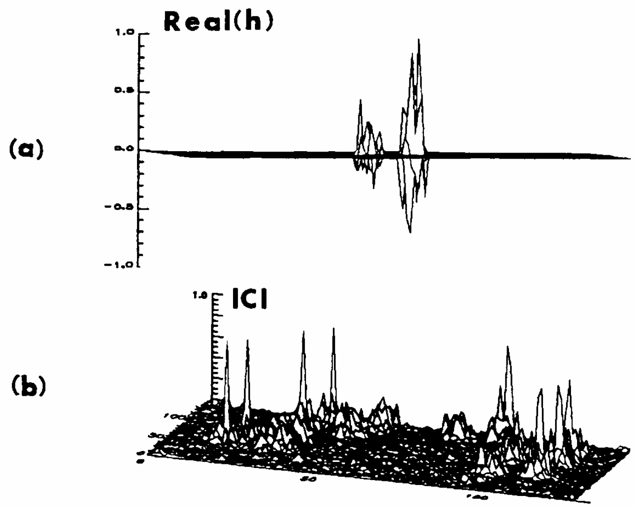

Fig. 3. (a) SDF plane, including two separated SDF's (the real part). (b) The correlation plane after 100 iterations (the absolute value).

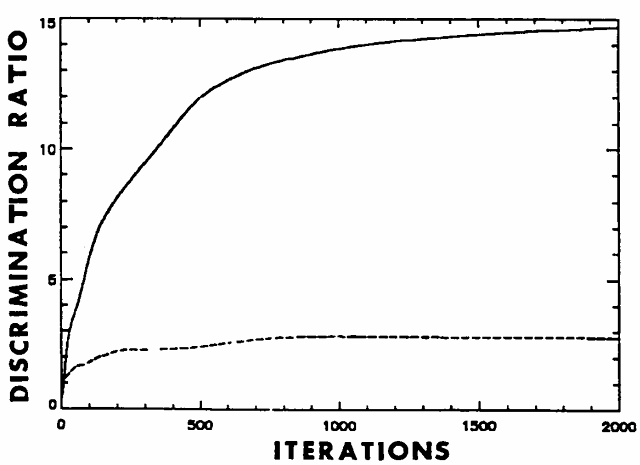

Fig. 4. DR versus the number of iterations for the PEC learning system (solid curve) and for the linear correlator (dashed curve).

the phase distribution. Matrices of 128 × 128 pixels are used, and the digits are limited by a rectangle of 5 × 5 pixels, whereas every SDF is limited by a rectangle of 7 × 7 pixels. The POCS process starts with randomly distributed $h(x)$. In Fig. 3(a) we see the real part of the SDF after 100 iterations. As mentioned, the two SDF's are displayed and calculated together. The respective absolute value of the correlation function is shown in Fig. 3(b). Note that there are eight strong peaks; however, the four leftward peaks shift to the left-hand side, whereas the four rightward peaks shift to the right-hand side. That phenomenon is due to the fact that the four leftward peaks are obtained as the result of the correlation with the left-hand SDF, whereas the four rightward peaks are obtained from the right-hand SDF. The desired DR chosen for this experiment was $\gamma_d = 16$. The actual DR is given by

$$\gamma_a \triangleq \left(\underset{x' \in R_1}{\mathrm{MIN}}\{c(x')\}\right)^2 \Big/ \left(\underset{x' \in R_2}{\mathrm{MAX}}\{c(x')\}\right)^2. \qquad (11)$$

The behavior of γ_a during 2000 iterations is shown in Fig. 4 (solid curve). The DR after 2000 iterations is 14.5 and is greater than 2 just after 50 iterations.

The fact that the learning system is based on the PEC does not impose such restrictions on the final classification system. If the goal is to perform the classification process with the same DR as obtained at the end of the learning phase, we may continue using the PEC as classifier. The PEC can be implemented by a digital computer or as an optical correlator.[6] On the other hand, in the most realistic cases the principal information of an image is borne in the phase distribution of its FT.[7] Therefore a SDF calculated on the phase distribution alone can be sufficient to distinguish among the various classes, even in a linear correlator. For comparison, the DR in the linear correlator after every ten iterations of the learning process is depicted in Fig. 4 (dashed curve). The SDF, synthesized by the PEC learning system, succeeds in classifying the different objects in a linear correlator ($\gamma_a \approx 2.5$ after 2000 iterations) even with lower DR than the PEC, as expected.

In conclusion, we demonstrated an example of SDF learning by the POCS algorithm based on a training set. The definitions of the constraint sets are not rigid, and therefore the method can be easily adopted to solve many other correlation problems and can satisfy other various constraints on filters or SDF's.

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