Synthetic spatial coherence function for optical tomography and profilometry: simultaneous realization of longitudinal coherence scan and phase shift

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ABSTRACT

A new scheme for synthesizing three-dimensional longitudinal spatial coherence function is proposed. By manipulating the irradiance of an extended quasi-monochromatic spatially incoherent source with a spatial light modulator, we generated a special optical field that exhibits high coherence selectively for the specified location along the optical axis of propagation and for the specified inclination between the two mirrors in the interferometer. The feasibility of the proposed principle is demonstrated by measuring a step height made by standard gauge blocks. The proposed scheme permits one to perform phase shift without recourse to mechanical movement. The quantitative experimental proof of the principle is presented.

Keywords: interferometry, profilometry, optical tomography, spatial coherence, spatial light modulator, the fringes of equal thickness, coherence theory, optical metrology, and phase shift technique.

1. INTRODUCTION

For the last decade there has been increasing interest in optical tomography[1] and low-coherence profilometry[2]. In these techniques, the characteristic of temporal coherence along the optical axis plays a crucial rule. Because these techniques use a light source with a broad source spectrum, they suffer from serious dispersion problems in many practical applications. Further more, the system needs to have mechanically moving components, such as a precision translator, for the compensation of optical path difference. Recently, Rosen and Takeda[3] proposed an alternative new principle of optical tomography and profilometry that is based on spatial, rather than temporal, coherence. More recently, Wang[2] et al. have given a new interpretation and experimental verifications to this principle. Instead of a point source with a broad spectrum, an extended source with a narrow spectrum was used in this technique. A desired longitudinal coherence function was synthesized by controlling the spatial structure of an extended quasi-monochromatic spatially-incoherence light source. Besides solving dispersion problems, the proposed technique

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enabled a system without mechanical movement for changing the optical path difference. From the experiment performed by Wang et al. [4] a problem was also identified that the system is highly sensitive to a tilt of an object or a reference mirror. It was because of this susceptibility to tilts (and also because of an astigmatic aberration in the optical system) that the synthetic longitudinal spatial coherence functions first experimentally demonstrated by Wang et al. have large fluctuations and significant deviations from the ideal symmetric three-peak functions predicted from the theory. If the first contrast peak disappears because of tilts, one cannot obtain any information about the distance between the object surface and the reference mirror.

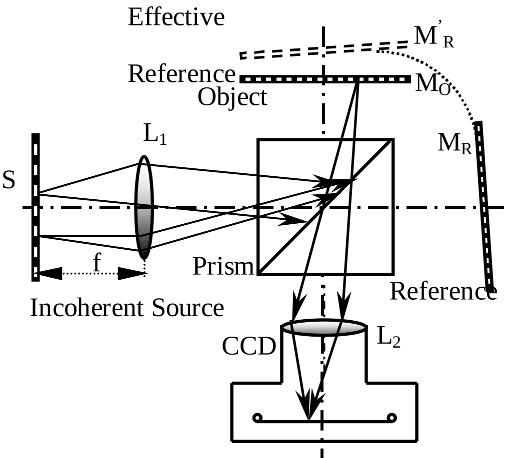


Fig.1. Schematic diagram for measuring degree of coherence.

The purpose of this paper is to study the characteristic of the tilts and exploit it for the simultaneous detection of the distance and the local surface inclination of an object. We will experimentally demonstrate the generation of a special optical field that exhibits high coherence selectively at the specified locations along the axis of beam propagation and in the specified direction of tilt. We also prove the feasibility of the proposed principle by measuring a height step made by standard gauge blocks. In addition, we propose and demonstrate a new technique that enables simultaneous realization of the longitudinal coherence scan and phase shift by means of source intensity modulation using a spatial light modulator.

2. PRINCIPLE

In the previous papers[3,4] the principle of coherence synthesis was described in terms of Fourier optics and fringes of equal inclination (Haidinger fringes). Here we review it briefly in terms of fringes of equal thickness[5] to show that yet another interpretation is possible.

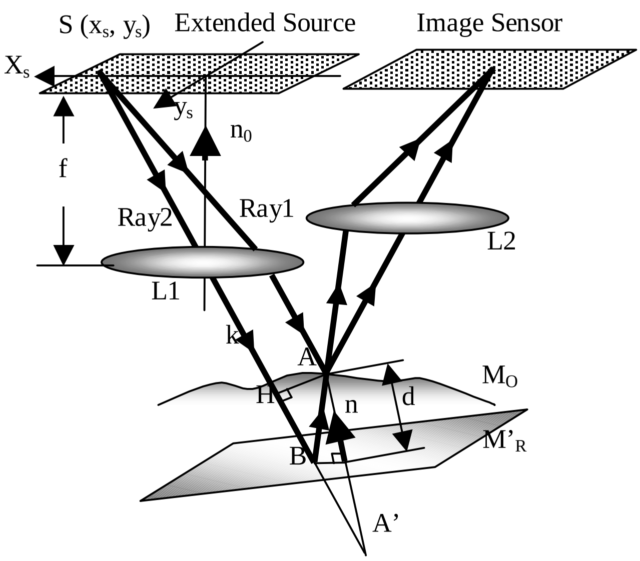


Fig.2. Formation of fringes of equal thickness.

A schematic diagram of the proposed system is illustrated in Fig. 1. A Michelson interferometer is illuminated by an extended quasi-monochromatic spatially incoherent light source (S), which is located in the front focal plane of lens L_1. Light emitted from point source S is collimated by lens L_1 and split into two beams by prism beam splitter P, and the interference fringes generated on the CCD image sensor are the result of combination of images of the two optical field distributions, i.e., the optical field distributions at object mirror M_O and reference mirror M_R. The surfaces of object mirror M_O is exactly focused by lens L_2 and imaged onto a CCD image sensor. From the theoretical analysis below, one can find that our interpretation in terms of the fringes of equal thickness to this proposed system not only provides an insight to longitudinal spatial coherence, but also helps us to find a new technique of phase shift, which is free from mechanical movement.

Formation of fringes of equal thickness for a tilted reference mirror is illustrated in Fig. 2. Light emitted from point source S located at point (x_s, y_s) in the front focal plane of lens L_1 is collimated by lens L_1, which has an optical axis in the direction indicated by unit vector $\mathbf{n_0} = (0,0,1)$. One of the rays in the collimated beam, denoted by Ray1, impinges directly upon point A at position (x, y) on the object mirror surface M_O, and another collimated ray, denoted by Ray2, from the same point source reaches the same point A after reflected at point B by the surface of effective (virtual) reference mirror M'_R. The intensity at point A resulting from interference between these two rays is recorded by an image sensor through lens L_2, which forms the image of the object (mirror) surface M_O onto the image sensor surface of the CCD camera. The phase difference between these two rays is given by

$$\delta(x_S, y_S; x, y) = \mathbf{k} \bullet (2d\mathbf{n}) \quad . \tag{1}$$

Under the paraxial approximation and the assumption of a small tilt, it can be rewritten as

$$\delta(x_S, y_S; x, y) \approx \frac{4\pi d(x,y)}{\lambda} - \frac{2\pi d(x,y)}{\lambda f^2}\left[\left(x_S - f\alpha_x\right)^2 + \left(y_S - f\alpha_y\right)^2\right] \quad . \tag{2}$$

Apart from a constant factor, the intensity as the result of interference between these two rays at point A on object mirror surface is given by

$$\begin{aligned}\hat{I}(x,y;x_S,y_S) &= I_S(x_S,y_S)I_F(x_S,y_S;x,y) \\ &= I_S(x_S,y_S)\left|1+\exp\left[j\delta(x_S,y_S;x,y)\right]\right|^2/2 \\ &= I_S(x_S,y_S)\left[1+\cos\left\{\frac{4\pi d(x,y)}{\lambda} - \frac{2\pi d}{\lambda f^2}[(x_S - f\alpha_x)^2 + (y_S - f\alpha_y)^2]\right\}\right]\end{aligned} \tag{3}$$

where $I_S(x_S, y_S)$ is the intensity of the spatially incoherent light source at point $S(x_S, y_S)$, and $I_F(x_S, y_S; x, y)$ is the intensity of the interference fringe created at point (x, y) on the object surface by the point source of unit intensity located at (x_S, y_S) on the source plane. Because each point source is completely incoherent to any other points on the source, the overall intensity on the image sensor is given by:

$$\begin{aligned}I(x,y) &= \iint \hat{I}(x,y;x_S,y_S)dx_S dy_S \\ &= \iint I_S(x_S,y_S)I_F(x_S,y_S;x,y)dx_S dy_S \\ &= \iint I_S(x_S,y_S)\left[1+\cos\left\{\frac{4\pi d(x,y)}{\lambda} - \frac{2\pi d}{\lambda f^2}[(x_S - f\alpha_x)^2 + (y_S - f\alpha_y)^2]\right\}\right]dx_S dy_S\end{aligned} \tag{4}$$

where integration is to be done over the area of the extended source, and one may regard $I_F(x_S, y_S; x, y)$ as a point spread function of fringe formation for an impulse point source S on the source plane. After some straightforward algebra, the intensity distribution given by Eq. (4) becomes

$$I(x,y) = B\left\{1+\left|\mu\left[d(x,y),\alpha(x,y)\right]\right|\cos\left[\frac{4\pi}{\lambda}d(x,y)+\phi\right]\right\} \tag{5}$$

where $B = \iint I_S(x_S, y_S)dx_S dy_S$, and $d(x,y)$ and $\alpha(x,y)$ are the distance and the tilt between object mirror M_O and effective mirror M'_R, respectively. The function $\mu[d(x,y),\alpha(x,y)]$ is the longitudinal complex degree of coherence given by

$$\mu[d(x,y),\alpha(x,y)] = \frac{\iint I_S(x_S,y_S)\exp\{-j\frac{2\pi d}{\lambda f^2}[(x_S - f\alpha_x)^2 + (y_S - f\alpha_y)^2]\}dx_S dy_S}{\iint I_S(x_S,y_S)dx_S dy_S} \quad (6)$$

The cosine function in Eq. (5) gives the intensity distribution of fringe pattern and the additional term ϕ is the initial phase of this fringe pattern. Now our problem of how to produce the coherence function of Eq. (6) with a narrow peak can be reduced to the problem of finding a real and nonnegative light source distribution that maximizes modulus of the complex degree of coherence for the specified distance and tilt angle. If one notes that the complex exponential fringe term in Eq. (6) gives a Fresnel zone plate in complex form, and also that Eq.(6) can be interpreted as a projection of the source distribution onto the complex Fresnel zone plate function expressed by the inner product of these two functions, one can find that the above requirement can be satisfied by the circular aperture whose transmittance has the same form of the Fresnel zone plate. We therefore choose the intensity distribution of a light source of the form

$$I_S(x_S,y_S) = \frac{1}{2}\left[1 + \cos\{-2\pi\gamma[(x_S - \xi_x)^2 + (y_S - \xi_y)^2] + \beta\}\right] \quad , \quad (7)$$

where the coefficient of 1/2 is normalization factor, β is the initial phase of the Fresnel zone plate, γ is the scaling parameter that determines the spatial frequency of the Fresnel zone plate, and ξ_x, ξ_y are the coordinates of the center of the Fresnel zone plate. Comparing the Fresnel zone plate source distribution with the corresponding part of complex exponential fringe term in Eq. (6), we find that the modulus of the complex degree of coherence is maximized when we chose the scaling parameter of zone plate source as $\gamma = \pm d/(\lambda f^2)$, and also that γ is proportional to the distance d between the effective reference mirror M'_R and the object mirror M_O. This means that the longitudinal coherence function given by Eq. (6) produces a high coherence peak at the particular position on the axis of beam propagation that satisfies this condition of zone plate matching. Similarly, if we put the shifting parameters of the zone-plate-like source as $\xi_x = f\alpha_x, \xi_y = f\alpha_y$, we find that these source-shift parameters are proportional to the two-dimensional tilt angles between the effective reference mirror M'_R and the object mirror M_O. In other words, Eq. (6) will produce a high coherence peak for the particular angles of tilt that matches the amount of shift introduced to the center of the zone-plate-like source. From the observation above, we can draw our conclusion that, in order to produce a coherence function, $\mu[d(x,y),\alpha(x,y)]$, with a high coherence peak, the scaling parameter γ and shifting parameters ξ_x, ξ_y of zone plate source should satisfy the following matching conditions simultaneously:

Scaling parameter: $\gamma = \pm \frac{d}{\lambda f^2}$

Shifting parameters: $\xi_x = f\alpha_x, \xi_y = f\alpha_y$

In application to tomography and profilometry, one can produce a high coherence peak at the desired location on the optical axis for the specified tilt angle simply by scanning the scaling parameter γ and the shifting parameters ξ_x, ξ_y of the zone plate source.

Now let us discuss the role of the initial phase β in Eq. (7). If one compares the form of the light source distribution $I_S(x_S, y_S)$ of Eq.(7) with that of the fringe point spread function $I_F(x_S, y_S; x, y)$ in Eq.(4), one can find perfect similarity between their function forms. This means that these two functions $I_S(x_S, y_S)$ and $I_F(x_S, y_S; x, y)$ can exchange their roles in the evaluation of Eq.(4). Therefore, we can rewrite our longitudinal coherence function in an alternative from:

$$I(x,y) = B'\left\{1 + \left|\mu'[d(x,y),\alpha(x,y)]\right|\cos[\beta + \phi'(x,y)]\right\} \quad (8)$$

where

$$B' = \iint I_F(x_S, y_S; x, y) dx_S dy_S$$
$$= \iint \left[1 + \cos\left\{ \frac{4\pi d(x,y)}{\lambda} - \frac{2\pi d}{\lambda f^2} [(x_S - f\alpha_x)^2 + (y_S - f\alpha_y)^2] \right\} \right] dx_S dy_S$$

is the integration of the virtual light source generated from the fringe point spread function $I_F(x_S, y_S; x, y)$ in Eq.(4), and

$$\mu'[d(x,y), \alpha(x,y)] = \frac{\iint I_F(x_S, y_S, x, y) \exp\{-j2\pi\gamma[(x_S - \xi_x)^2 + (y_S - \xi_y)^2]\} dx_S dy_S}{\iint I_F(x_S, y_S, x, y) dx_S dy_S} \quad (9)$$

is an alternative expression for the complex degree of coherence based on this virtual light source $I_F(x_S, y_S; x, y)$. In Eq.(8), $\phi'(x,y)$ is the phase of this new complex degree of coherence. It should be noted that $\mu(d,\alpha) = \mu'(d,\alpha)$ for the measurement that satisfies the zone-plate matching conditions. Because the cosine function of the interference fringe pattern in Eq.(8) includes the initial phase β of the light-source zone plate, one can perform phase shift simply by changing the initial phase of this light-source zone plate. This can be done with a spatial light modulator (SLM), and no mechanical movement of optical elements is involved for the phase shift.

3. EXPERIMENTS

The schematic illustration of experimental set-up is shown in Fig.3. Linearly polarized light from a 15mW He-Ne laser was expanded and collimated by collimator lens C to illuminate a liquid-crystal-based SLM, which modulates light intensity transmitted by analyzer P placed immediately behind SLM. A computer-generated Fresnel zone plate pattern was displayed on the SLM, and was imaged onto a rotating ground glass GG by a combination of lenses L'p and Lp through pinhole PH, which functions as a spatial filter to smooth out the discrete pixel structure of SLM. The image of the Fresnel zone plate on the rotating ground glass serves as a quasi-monochromatic incoherent light source. The light from the zone plate source was collimated by lens L1 and was introduced into a Michelson interferometer made of prism beam splitter PS, reference mirror M_R, and object mirror M_O, the surface of which is imaged by lens L2 onto the sensor plane of CCD camera.

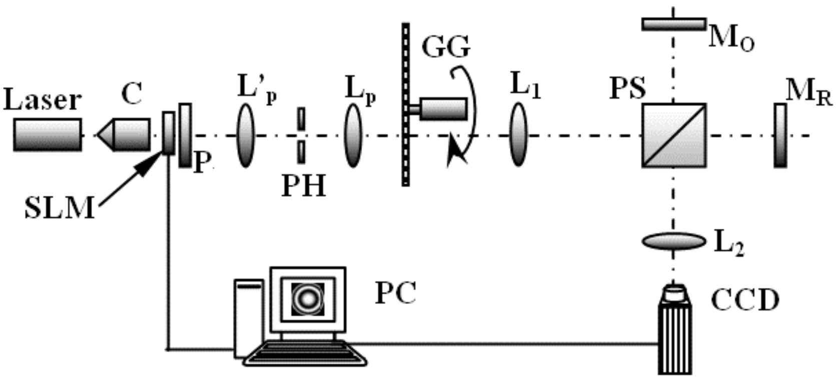


Fig.3. Schematic illustration of experimental setup.

A. Longitudinal coherence distribution for a fixed zone plate source

We produced a zone plate source with the scaling parameter $\gamma = d/(\lambda f^2)$ for d=1mm, λ =633nm and f=300mm, and detected the degree of longitudinal coherence by moving the reference mirror along the optical axis. According to the theory, high coherence peaks will appear at three positions where the distance between the two mirrors is equal to 1mm, -1mm and 0mm. The degree of coherence degree obtained by experiment is shown in Fig.4. As predicted from the theoretical analysis in the previous section, we find three high coherent peaks corresponding to the plus and minus first peaks, and the zero-th peak. The distance between the plus/minus first peak and the zero-th peak was found to be in agreement with the distance of d=1mm used for designing the zone plate source. Shown on the top of Fig.4 are fringe pattern observed at the corresponding locations on the longitudinal axis.

B. Simultaneous realization of phase shift and longitudinal coherence scan by source modulation with SLM

In the next experiment, we fixed the distance between two mirrors to a known distance d=1mm and detected the first coherence peak for this distance by scanning the scaling parameter of zone plate source with the SLM. Simultaneously, we applied our new principle of phase shift by changing the initial phase of the zone plate source as shown in Fig.5(a). As we changed the initial phase of the zone plate source, the intensity of the interference fringes changed accordingly, as shown in Fig.5(b). This shows that the phase shift is introduced to the interference fringes as we intended. Figure 6 shows the change of the degree of coherence as we scanned the scaling parameter of zone plate source with the SLM. The first coherence peak was observed for the scaling parameter corresponding to the given distance of 1mm. The high coherence peak observed in the region of the small scale parameter can be explained from Eq.(9), which gives a high coherence value approaching to unity as γ is reduced toward zero. Remember that the degree of coherence obtained by phase shifting with the zone plate is μ' given by Eq.(9), rather than μ given by Eq.(6). These two coherence functions are equivalent under the condition of practical measurement where the scale parameter of the zone plate is tuned to the given optical path difference, but their behaviors become different in the region where the zone plate is in mismatch to the given optical path difference, as in the region of the small scale parameter shown in Fig.6. We also found that the linear relationship existed between the scaling parameter and the distance between the first and the zero-th order coherence peaks.

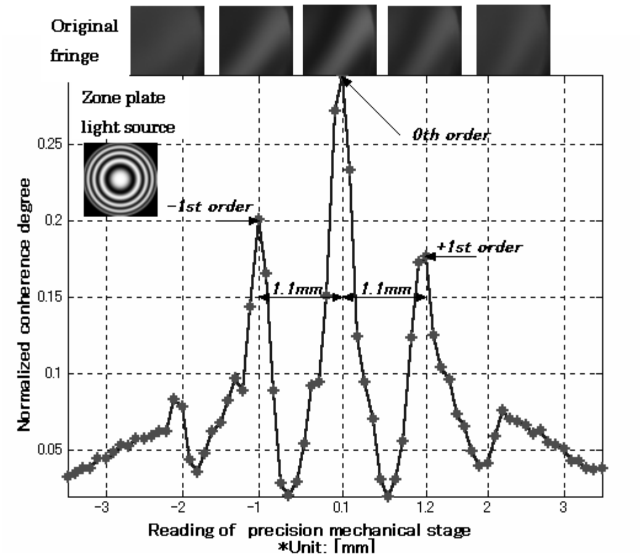


Fig.4. Distribution of degree of coherence along optical axis obtained by experiment.

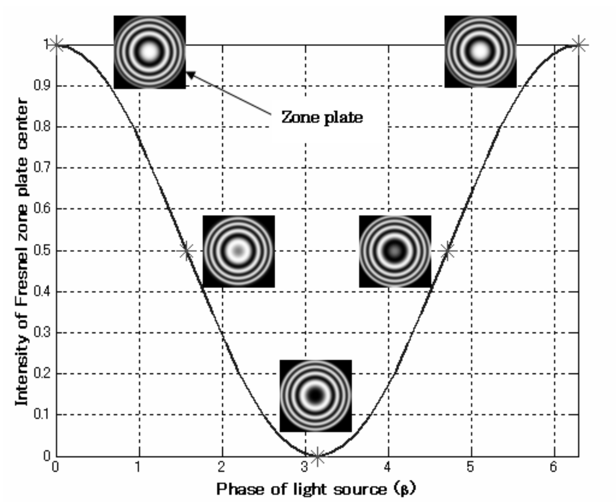


Fig.5(a) Phase shift given to Fresnel zone plate.

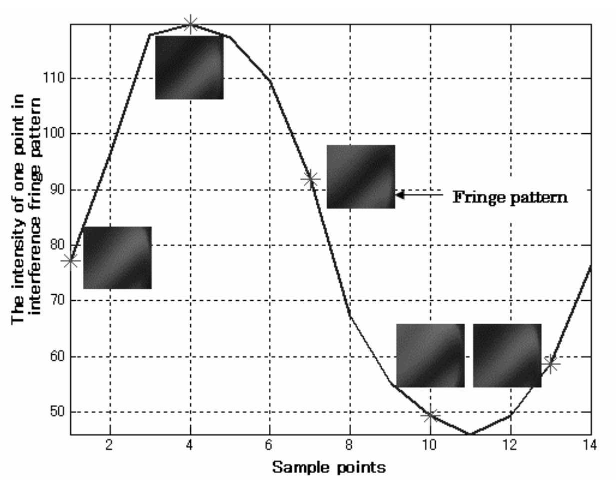


Fig.5(b) Phase shift observed in fringe pattern.

C. Detection of surface inclination by shifting the center of zone plate source

From the analysis of the previous section, the inclination between two mirrors can be measured by scanning the shifting parameters of the zone plate source. In this experiment, we fixed the distance between the two mirrors to 3mm, and adjusted the scaling parameter of the zone plate source so as to bring the first coherence peak to this mirror distance. Then we gave a lateral displacement to the center of the zone plate source and observed the change in the degree of coherence. The result of the measurement of the degree of coherence is shown in Fig.7, where the sequence number of the zone plate that gives the highest coherence peak identifies the amount of the inclination between the two mirrors. Noting that we cannot determine the sign of the tilt angle from a single fringe pattern obtained by conventional interferometry, we performed an experiment to demonstrate a potential application of the proposed zone plate shift technique. By fixing the distance between the mirrors to 1.5mm, we switched between the two particular states of mirror tilts, where the mirrors have the same inclination angles but opposite signs in reference to the optical axis. These two states are not distinguishable in conventional interferometry. Figure 8 shows the change of the degree of coherence as we moved the center of the zone plate source. Two peaks are observed for the tilts with opposite signs. As seen in the figure, we can clearly distinguish the signs of the tilts from the position of the peak of the degree of coherence.

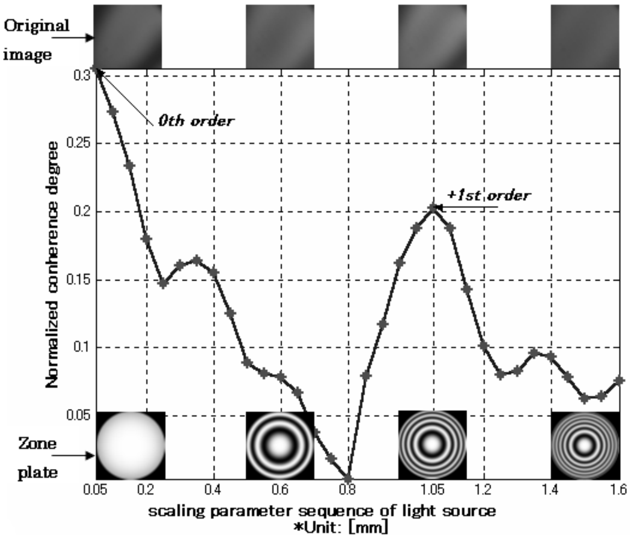


Fig.6. Variation of the degree of coherence with the change of scale parameter.

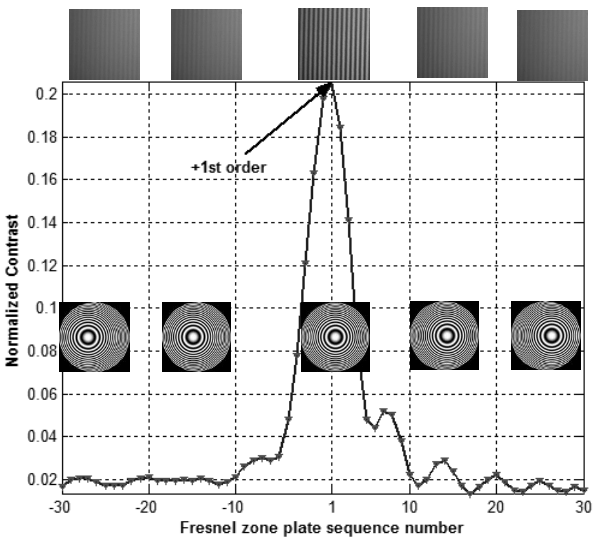


Fig.7. Variation of the degree of coherence with the lateral shift of the zone plate source.

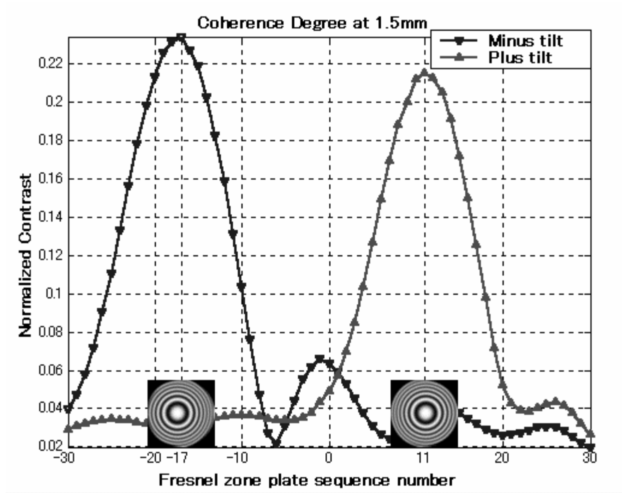
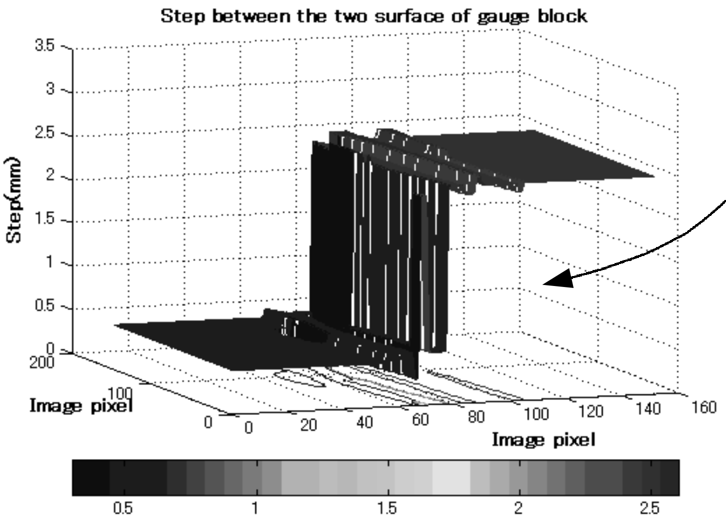


Fig.8. Variation of the degree of coherence with the lateral shift of the zone plate source. The sign of the

D. Three-dimensional shape measurement by means of longitudinal spatial coherence scanning

The feasibility of the proposed principle is verified by measuring the shape of a discontinuous three-dimensional object with a 2 mm step height made by a set of standard gauge blocks from Mitutoyo Company. According to the theory, high degree coherence is realized on each surface of the step only under the condition that both scaling parameter and shifting parameters of the zone plate source match to the distance and the inclination between each of the surfaces and the reference mirror simultaneously. The schematic illustration of the height step and the result of measurement are shown in Fig.9. The obtained step height was in good agreement with the height given by the standard gauge blocks, which demonstrates the potential of the proposed principle for the practical application to optical profilometry and tomography.



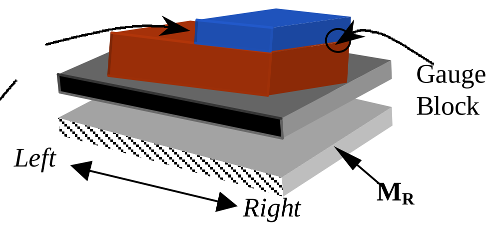


Fig.9. Schematic illustration of 2mm step height made by standard gauge blocks (right), and the result of measurement (left).

4. CONCLUSION

From the interpretation of the fringes of equal thickness, a new scheme for synthesizing three-dimensional longitudinal spatial coherence function is proposed. By manipulating the irradiance of an extended quasi-monochromatic spatially incoherent source with a spatial light modulator, we generated a special optical field that exhibits high coherence selectively for the specified location along the optical axis of propagation and for the specified inclination between the two mirrors in the interferometer. The feasibility of the proposed principle was demonstrated by measuring a step height made by standard gauge blocks. The proposed scheme permits one to perform phase shift without recourse to mechanical movement. The quantitative experimental proof of the principle was presented.

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