Reduction in the reconstruction error of computer-generated holograms by photorefractive volume holography

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We suggest a method for coding high-resolution computer-generated volume holograms. It involves splitting the computer-generated hologram into multiple holograms, their individual recording as volume holograms by use of the maximal resolution available from the spatial light modulator, and subsequent simultaneous reconstruction. We demonstrate the recording and the reconstruction of a computer-generated volume hologram with a space–bandwidth product much higher than the limitation imposed by the interfacing spatial light modulator. Finally, we analyze the scheduling procedure of the multiple holographic recording process in photorefractive medium in this specific application.

Binary computer-generated holograms (CGH's) are becoming essential components in optical signal-processing schemes. Apart from their use as a means for storage and reconstruction of images, these holograms can serve as spatial filters, optical elements, and the basic component in interconnection networks. CGH's are created in an electronic computer and are displayed on spatial light modulators (SLM's). This provides real-time variability, which enhances the flexibility of the holograms and makes them suitable for applications such as adaptive optics and reconfigurable interconnects. Unfortunately, most currently available SLM's suffer from a limited information capacity, which substantially deteriorates the space–bandwidth product of the CGH's and results in a undesired difference between the original image and the one actually reconstructed from a CGH.

In this Letter we demonstrate a method for increasing the space–bandwidth product of a binary CGH much above the maximal resolution available from the SLM. It includes splitting the CGH into secondary holograms, each utilizing the SLM's maximal resolution. The individual secondary holograms are sequentially transmitted from the computer to the SLM and imaged onto a volume holographic medium. They are then converted to volume holograms, all recorded with the same reference wave, and stored in the volume of the holographic medium. The subsequent readout results in the reconstruction of the whole original high-resolution image, with a large reduction in the reconstruction error. In this vein, we have used a photorefractive (PR) crystal as the storage medium and also utilize its coupling properties for improving the recording efficiency of the multiple volume (secondary) holograms.

Consider a binary CGH expressed by the spatial distribution (in the plane of the SLM) of $H(u,v)$, which is the Fourier transform of $h(x,y)$. The hologram is designed to reconstruct the image $f(x,y)$ in a subarea A in the $x-y$ plane. The reconstruction error (per pixel in A) is defined as

$$e = \frac{1}{A}\int_A |f(x,y) - \beta\hat{h}(x,y)|^2 \mathrm{d}x\mathrm{d}y\,, \qquad (1)$$

where $\hat{h}(x,y)$ is the part of $h(x,y)$ included in the subarea A and β is a constant that is designed to minimize the reconstruction error.[1] In general, the reconstruction error of a conventional binary CGH decreases when the area of the reconstructed image becomes smaller. We decompose the desired image into simple primitives of a small area and calculate a CGH for each one of them. The whole set of CGH's is sequentially transmitted to an SLM and imaged onto a PR crystal. The CGH's are converted to volume holograms by the method that we introduced in a previous Letter[2] and stored in the crystal as thick gratings. The volume holograms are formed as a result of interference between a nonzero diffraction order (off the SLM), which bears the holographic data, and the zero (nondiffracted) order. The resultant thick gratings differ from each other by their periodicity and orientation and may be addressed (reconstructed) individually as separate data pages[2] or as one, high-resolution, image. Here we utilize the last option and reconstruct the original high space–bandwidth product image by illuminating the crystal with a readout beam directed along the optical axis. In this method, the reconstruction error of the full image is the average of all the errors of the primitives. Therefore as we choose smaller primitives

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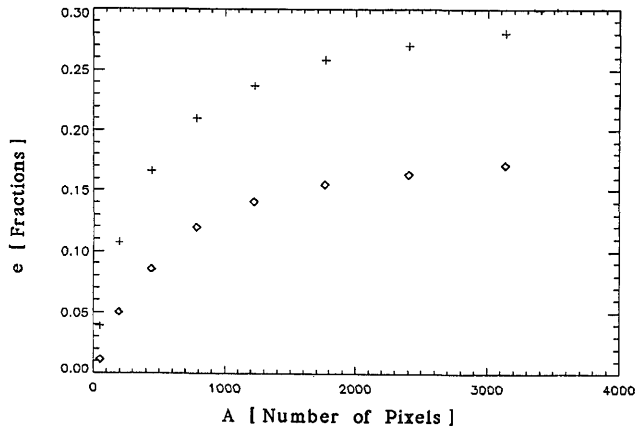

Fig. 1. Reconstruction error e as a function of the reconstructed image area A for images of the letters R (+) and O (◇).

the overall error is reduced. The maximal number of primitives that can be used in this technique is limited only by the scheduling procedure of the holographic recording in the PR crystal.[3]

To illustrate the principle that an image of a small area can significantly reduce the reconstruction error, while larger images include larger errors, we calculate the reconstruction error from two binary CGH's, each of 128×128 pixels, that were designed with the POCS algorithm.[1,2] The CGH's were of two images that extend over different areas: the images of the letters R and O. Figure 1 shows the reconstruction error e [according to Eq. (1)] versus the image area.

In our experiment we have defined the smallest primitive (or the most delicate partition) of a complicated image as a point and recorded a volume CGH for each point that composed the image. Since a binary CGH of a point is merely one rectangular grating (because of the binary nature of the SLM) and bears no additional information, we expect to have the minimal reconstruction error for each one of them. Ideally, every binary grating can be reconstructed into a desired point with no error at all. Reminiscent errors, which may be present as a result of the system's aberrations, are not fundamentally built into the coding method and may be eliminated by iterating corrections of the CGH's.[4]

Our experimental setup is shown in Fig. 2. Each binary CGH is displayed on a SLM of 128×128 pixels, coherently imaged onto a PR crystal ($BaTiO_3$), and converted to a volume hologram.[2] In this process all the CGH's are recorded with the same reference wave: the zeroth order of diffraction from the SLM. Therefore, illumination with a readout beam along the optical axis results in Bragg matching to all the stored holograms and reconstructs all the primitives simultaneously (and hence the whole high-resolution image). The CGH's were sequentially recorded with the incremental[5] method.

Our experimental results are depicted in Fig. 3, where the reconstruction of the amplitude distribution, with a constant phase, of a delicate (made of individual dots) letter R was performed with (a) 1 CGH only (error identical to that in a conventional planar CGH case), (b) 6 CGH's, each designed to reconstruct one primitive line of the letter, and (c) 16 CGH's, one for each point in the R. The improvement in the reconstruction quality is clearly seen.

The scheduling of the recording process of the multiple holograms in the PR crystal requires special attention. Previously suggested schemes for scheduled[3,6] and incremental[5] multiplexing conclude that the diffraction efficiency of one hologram, when N volume holograms are stored, decreases with $1/N^2$. This is because when one hologram is recorded, all previously stored ones are not Bragg matched to the interfering beams and hence experience erasure. Our recording configuration differs from those techniques since all the holograms are recorded with the same reference, which is therefore Bragg matched to all of them. Consequently, during a sequential recording process of one hologram after the other, the reference beam continues to deflect light in the directions of the previously recorded gratings, at any recording time interval. This slows down the erasure process and induces a large asymmetry between the writing and the erasure times. An approximated solution for erasure with one Bragg-matched beam was discussed by Horowitz *et al.*[7] We use this idea to increase the number of stored holograms while maintaining a relatively high diffraction efficiency.

The recording process of multiple holograms stored with the same reference is described by the time-

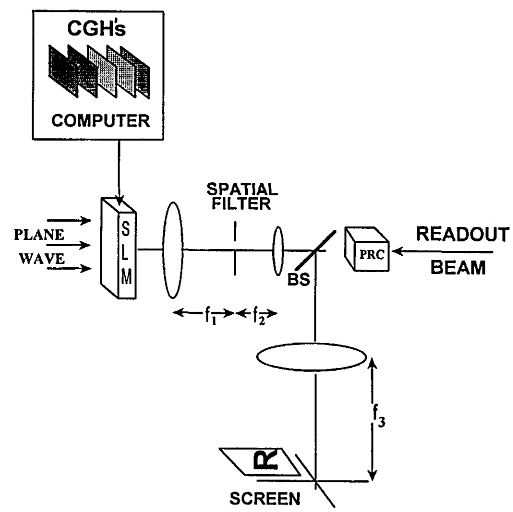

Fig. 2. Experimental setup: BS, beam splitter; PRC, photorefractive crystal.

Fig. 3. Reconstructed images from (a) 1 CGH, (b) 6 CGH's, and (c) 16 CGH's.

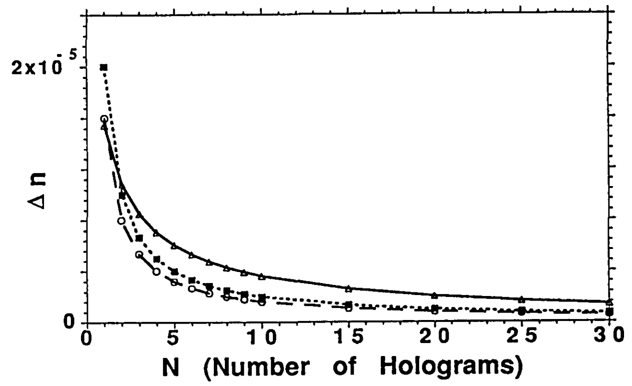

Fig. 4. Photorefractive index perturbation $\langle \Delta n \rangle$ for three cases: from the numerical solution of Eqs. (2)–(4), with an amplitude ratio 1:2 (solid curve); for uncoupled recording beams with an amplitude ratio 1:2 (dashed curve); and for uncoupled beams with an amplitude ratio 1:1 (dotted curve).

dependent wave-mixing process[8]:

$$\partial A_i/\partial z = (ik/n)\Delta n_i A_R\,, \tag{2}$$

$$\partial A_R/\partial z = (ik/n)\sum_{i=1}^{N} \Delta n_i{}^* A_i\,, \tag{3}$$

$$\partial \Delta n_i/\partial t + I_0(t)\Delta n_i = \gamma_i A_i A_R{}^*, \tag{4}$$

where $A_i(z,t)$ is the field amplitude of the image-bearing beam i, $i = 1, 2, \ldots, N$. For this calculation we assume that each band-limited beam A_i is represented by a single plane-wave and $A_R(z,t)$ denotes the reference beam. The overall light intensity in a time interval during the recording process is given by $I_0(t) = |A_R(t)|^2 + |A_i(t)|^2$. Note that $I_0(t)$ is constant for every z plane, and we make the standard low-visibility assumption[8] that $A_R A_i{}^* \ll I_0$. The ith PR perturbation in the refractive index n is $\Delta n_i(z,t)$, γ_i is the PR coupling coefficient, and k is the light wave number in vacuum. We assumed negligible absorption (which does not affect the qualitative process in the transmission geometry anyway), and all units are given in equivalent dark irradiance units.[8] The boundary conditions for recording process are $A_i(0,t)$, $A_R(0,t)$, and $\Delta n_i(0,t)$ (for $i = 1\ldots N$), where the input fields alternate in time in the incremental recording procedure.[5] The initial conditions are $\Delta n_i(z,0) = 0$ for all i, and one may vary the input amplitudes $A_i(0,t)$ to optimize the recording process.

Numerical solution of Eqs. (2)–(4) are presented in Fig. 4 for $N = 1, \ldots 30$ holograms. The input image-bearing beams (A_i's) alternate periodically in time every $\Delta t = 0.05$ (in normalized time units,[8] chosen according to the criterion of Ref. 5) until all Δn_i's reach their steady state and the recording process is terminated. For simplicity, we have used identical coupling coefficients to all the holograms: $\gamma = 5i$ cm^{-1} (where the i indicates the $\pi/2$ phase of γ, which leads to a pure imaginary Δn, responsible for energy coupling between the recording beams) and a crystal length of $L = 0.5$ cm. We optimized the amplitude ratios A_i/A_R at $z = 0$ to obtain the highest Δn_i, and they were approximately 1:2, for all i.

Figure 4 shows the averaged index perturbation $\langle \Delta n \rangle$ (averaging takes place over z, between the high and low values of the steady state, and over the corresponding number of holograms for each case) versus the number of stored holograms N (solid curve). For comparison, we calculate the resultant $\langle \Delta n \rangle$ of the cases in Refs. 5 and 6 (for equal writing and erasure times) with $\langle \Delta n \rangle = \Delta n_{\max}/N$, where $\Delta n_{\max} = \gamma A_i A_R/I_0$ is Δn when a single hologram is recorded. The dashed curve in Fig. 4 shows $\langle \Delta n \rangle$ versus N for the 1:2 amplitude ratio for the uncoupled recording process. We also add the optimal case for uncoupled recording (1:1 amplitude ratio), shown by the dotted curve in Fig. 4. The value of the index perturbation in our case is approximately twice the values in the uncoupled cases, yielding an increase in the diffraction efficiency by a factor of 4.

We have performed a preliminary check on the theoretical results by comparing, in both cases, the maximal number of holograms to be recorded that yielded a given diffraction efficiency. When ordinary-polarized beams were used in the recording process in $BaTiO_3$, the beams did not couple, and we were able to store not more than 20 holograms with a diffraction efficiency of approximately 1% per hologram. When we used extraordinary polarization we successfully stored more than 50 holograms, with the same diffraction efficiency. This qualitatively proves our conclusions.

In conclusion, we have demonstrated a new technique to code high space–bandwidth product computer-generated holograms at a resolution that greatly exceeds the limitations imposed by the interfacing spatial light modulator. Our method is useful for coding masks, filters, interconnects, and optical elements for applications in which high resolution is required.

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