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Longitudinal partial coherence of optical radiation

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Abstract

The spatial coherence between two points, lying on the propagation axis and which are illuminated by a finite extended, quasi-monochromatic, spatially incoherent light source, is studied. We show a Fourier transform relation between the complex degree of coherence and the quadratic radial distribution of the source, and propose two interferometric systems to measure the degree of coherence.

The Van Cittert-Zernike theorem [1] consists of a relation between the two-points spatial coherence function and the intensity distribution of a quasi-monochromatic spatially incoherent light source. The coherence between two spatial points can be measured by interfering the light emitted from these two points [1,2]. In the theorem and in the interferometric measurement it is assumed that the two points are on a transverse plane perpendicular to the propagation axis. In this letter we generalize the theorem for any two points in the 3D space. Furthermore, we derive a Fourier transform relation between the coherence function of two points along the propagation axis and the source distribution. Finally we analyze two interferometric systems that can measure this coherence function. One of them can also be used to perform a longitudinal cosine transform [3] of an aperture illuminated by spatial incoherent light.

Assuming a quasi-monochromatic planar incoherent light source, with an intensity distribution $I_s(s)$, which illuminates among others two arbitrary points $P_{1,2}$, far away from the source plane (see Fig. 1). The complex degree of coherence is defined by the time average of the product of the fields $E_{1,2}(t)$ at the points $P_{1,2}$ [1],

$$\mu(P_1, P_2) = \frac{\langle E_1(t) E_2^*(t) \rangle}{\sqrt{\langle |E_1(t)|^2 \rangle \langle |E_2^*(t)|^2 \rangle}} = \frac{1}{\sqrt{I_1 I_2}} \int_s I_s(s) \frac{\exp[jk(R_1 - R_2)]}{R_1 R_2} ds, \tag{1}$$

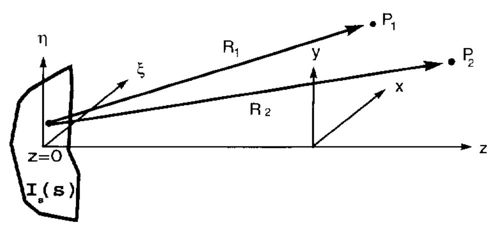

Fig. 1. Schematic illustration of the generalized Van Cittert–Zernike theorem.

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where

$$I_i = \int_s \frac{I_s(s)}{R_i^2}\,\mathrm{d}s, \quad R_i = [z_i^2 + (\xi - x_i)^2 + (\eta - y_i)^2]^{1/2}, \quad i = 1, 2\,,$$

$k = 2\pi/\lambda$ and λ is the wavelength of the light source. Assuming $z_i \gg |x_i - \xi|,\ |y_i - \eta|$, each distance R_i may be approximated by

$$R_i \cong z_i + \frac{(\xi - x_i)^2 + (\eta - y_i)^2}{2z_i}\,, \tag{2}$$

which enables us to write Eq. (1) as

$$\mu(x_1, y_1, z_1; x_2, y_2, z_2) = I_0^{-1} \exp\left[\mathrm{j}k\left(z_1 - z_2 + \frac{x_1^2 + y_1^2}{2z_1} - \frac{x_1^2 + y_2^2}{2z_2} - \frac{x_2^2 + y_2^2}{2z_2}\right)\right]$$
$$\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_s(\xi, \eta) \exp\left\{\mathrm{j}k\left[\xi\left(\frac{x_2}{z_2} - \frac{x_1}{z_1}\right) + \eta\left(\frac{y_2}{z_2} - \frac{y_1}{z_1}\right) + \frac{\xi^2 + \eta^2}{2}\left(\frac{1}{z_1} - \frac{1}{z_2}\right)\right]\right\} \mathrm{d}\xi\,\mathrm{d}\eta\,, \tag{3}$$

where $I_0 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_s(\xi, \eta)\,\mathrm{d}\xi\,\mathrm{d}\eta$. In the case of two points P_1 and P_2 which are placed in the same transverse plane ($z_1 = z_2$), Eq. (3) expresses the well-known Van Cittert–Zernike theorem. On the other hand, if the two points are on the propagation axis, a new Fourier relation is obtained:

$$\mu(\Delta\zeta) = \frac{\exp(4\pi \mathrm{j}\bar{z}^2\Delta\zeta)}{2I_0} \int_0^{\infty} \tilde{I}_s(\sqrt{\rho}) \exp(-2\pi \mathrm{j}\rho\Delta\zeta)\,\mathrm{d}\rho\,, \tag{4}$$

where

$$\rho = r^2 = \xi^2 + \eta^2, \quad \frac{1}{z_2} - \frac{1}{z_1} = \frac{\Delta z}{\bar{z}^2}, \quad \bar{z} = \sqrt{z_1 z_2}, \quad \Delta\zeta = \frac{\Delta z}{2\lambda\bar{z}^2} \quad \text{and} \quad \tilde{I}_s(\sqrt{\rho}) = \int_0^{2\pi} I_s(\sqrt{\rho}, \theta)\,\mathrm{d}\theta\,.$$

The complex degree of coherence between any two points along the z axis is equal to a linear phase function multiplied by the Fourier transform of the quadratic radial intensity distribution of the source with scaling factor $(2\lambda\bar{z}^2)^{-1}$.

Next we show that, in a complete analogy to the lateral coherence function [1,2], we can measure the longitudinal coherence function by an interferometric system shown in Fig. 2a. A Michelson-like interferometer is illuminated by a quasi-monochromatic planar incoherent light source located at the focal distance f_1 from the input lens L_1. Each mirror in the interferometer is blocked by a centered pinhole, and the length of one channel is adjustable. The observation plane is displayed at the focal distance f_2 from the output lens L_2. This interferometer can be replaced by an equivalent system shown in Fig. 2b, where the illumination points P_1 and P_2 represent the illumination from the holes. The complex amplitude distribution at point (0, 0, z), behind the lens L_1, resulting from the illumination of a single point source at (r, θ), is [3]

$$u(z) = \frac{1}{\mathrm{j}\lambda f_1} \exp\left[\mathrm{j}k\left(z + f_1 - \frac{r^2(z - f_1)}{2f_1^2}\right)\right]. \tag{5}$$

The complex amplitude $O(r_0, \theta_0)$ at the output plane, illuminated by a point source placed at the point (0, 0, z), a distance d from the lens L_2, is given by [4]

$$O(r_0, \theta_0) = u(z)\,\frac{1}{\mathrm{j}\lambda f_2} \exp\left[\frac{\mathrm{j}k}{2f_2}\left(1 - \frac{d}{f_2}\right) r_0^2\right]. \tag{6}$$

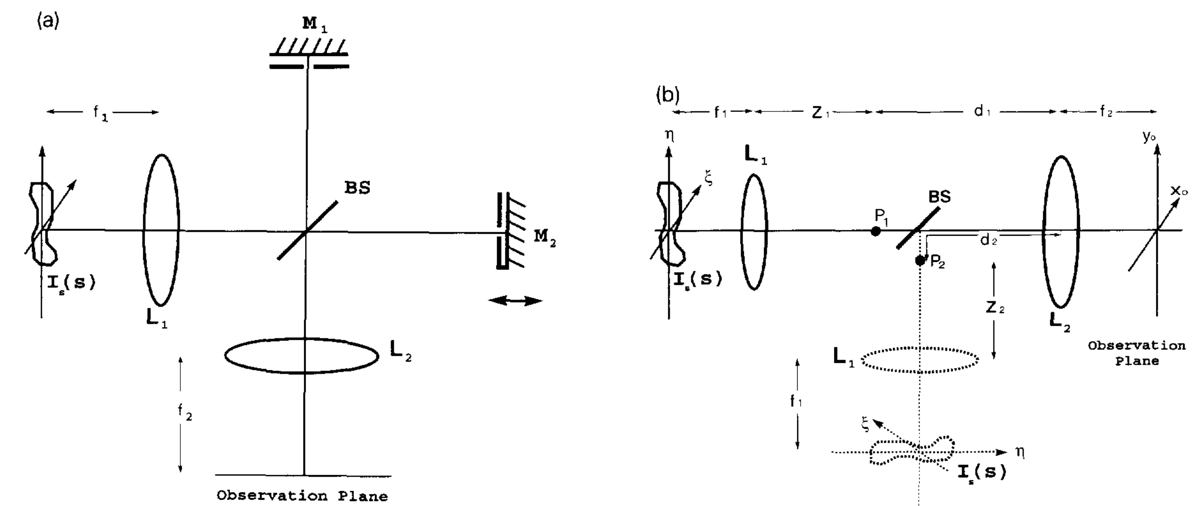

Fig. 2. (a) The optical system for measuring the longitudinal complex degree of coherence. (b) An optical setup which is equivalent to (a).

The overall intensity distribution in the observation plane is obtained by summing the fields radiating from the two points P_1 and P_2 and then integrating over all the intensity contributions of all the source points. The result is

$$I(r_0, \theta_0) = \int_0^{2\pi}\int_0^{\infty} \left|\frac{I_s(\sqrt{\rho}, \theta)}{2\lambda^4 f_1^2 f_2^2}\right| \exp\left\{jk\left[z_1 + f_1 - \frac{\rho}{2f_1^2}(z_1 - f_1) + \frac{r_0^2}{2f_2}\left(1 - \frac{d_1}{f_2}\right)\right]\right\}$$

$$+\exp\left\{jk\left[z_2 + f_1 - \frac{\rho}{2f_1^2}(z_2 - f_1) + \frac{r_0^2}{2f_2}\left(1 - \frac{d_2}{f_2}\right)\right]\right\}\Bigg|^2 \mathrm{d}\rho\,\mathrm{d}\theta$$

$$= \frac{1}{\lambda^4 f_1^2 f_2^2}\int_0^{2\pi}\int_0^{\infty} I_s(\sqrt{\rho}, \theta)\left[1 + \cos\left(k\Delta z - \frac{k\rho\Delta z}{2f_1^2} - \frac{k r_0^2 \Delta z}{2f_2^2}\right)\right]\mathrm{d}\rho\,\mathrm{d}\theta\,, \tag{7}$$

where $\Delta z = z_1 - z_2 = d_1 - d_2$ and $\rho = r^2$. Using the result of the complex degree of coherence (Eq. (4)), while the distance $\bar{z}$ is replaced by f_1, the obtained intensity is

$$I(r_0, \theta_0) = \frac{2I_0}{\lambda^4 f_1^2 f_2^2}\left[1 + \left|\mu\left(\frac{\Delta z}{2\lambda f_1^2}\right)\right| \cos\left(\frac{k r_0^2}{2f_2^2}\Delta z + \measuredangle\mu\right)\right], \tag{8}$$

where $\measuredangle\,\mu$ is the phase angle of μ. This result shows that the complex degree of coherence can be measured by observing the visibility of a concentric interference pattern for different distances Δz. Having $\mu(\Delta\zeta)$, we can calculate the radial structure of the source by the one-dimensional inverse Fourier transform (the inverse transform of the Fourier transform given by Eq. (4)).

As an example we consider a case where the source is a circular disk with a radius a and a uniform intensity. The degree of coherence $|\mu(\Delta\zeta)|$ for this source is

$$|\mu(\Delta\zeta)| = \left|\frac{\sin(\pi a^2 \Delta z / 2\lambda f_1^2)}{\pi a^2 \Delta z / 2\lambda f_1^2}\right|. \tag{9}$$

Since this quantity is equal to the visibility when the overall intensity is identical in both channels, it follows that the interference fringes vanish first when $\Delta z = 2\lambda f_1^2 / a^2$. In comparison, the distance between two holes in the transverse plane of a Young-like interferometer (illuminated by the same source), which causes a vanishing of the

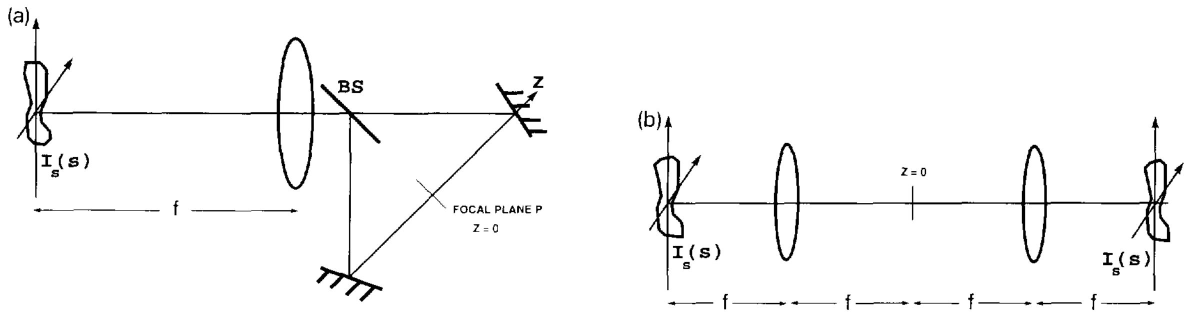

Fig. 3. (a) The optical system for measuring the longitudinal complex degree of coherence and for performing longitudinal cosine transforms. (b) An on-line optical setup which is equivalent to (a).

fringes, is $\Delta x = 0.61\lambda f_1/a$ [1,2]. Based on the assumption that $f_1 \gg a$, we realize that, for the same source, the coherence distance of the longitudinal coherence function is $3.3f_1/a$ times longer than that of the lateral coherence function. This last result can be meaningful in choosing the interferometer for the light sources analysis, when a high accuracy is needed. We realize that, for systems with the same dimensions, the longitudinal coherence interferometer (Fig. 2) has larger dynamic range than that of the Young-like interferometer, although the first one is limited to an analysis of radial symmetric sources.

Another consequence of this observation is in holography with nonlaser sources [5]. The coherence between a reference and an object in an in-line holography setup [6] is obtained from Eq. (9). Therefore, the coherence requirements of in-line holograms are less than that of off-axis holograms [6]. For the same degree of coherence, the distance between two interfering points in the in-line arrangement is $3.3f_1/a$ times longer than the distance between two points in the off-axis arrangement, assuming the source is a circular disk with a radius a.

An additional observation can be made concerning the nature of the source. We showed that the angular distribution of the source I_s had no influence on the longitudinal degree of coherence. Furthermore, the assumptions concerning the coherence nature of the source along a line of constant radius become unnecessary. In other words, we can take a narrow ring (δ-function in ρ) as a source, and, as far as longitudinal coherence is concerned, regardless of the state of coherence along the ring it acts as a coherent source. The ring source can be a collection of source points of any degree of coherence (between every two points), and still every two points along the z axis in the far field are completely coherent to each other. If for complete coherence between two laterally separated points we need a point source, then for complete coherence between two longitudinal separated points it is sufficient to use a ring-like (with an infinitesimal width) incoherent source.

A completely different method to measure the degree of coherence is described in Fig. 3. The incoherent source is present in the input plane of a ring interferometer. The optical path between the source to the lens is equal to the focal length f, and the forward focal plane of the lens is at plane P. The equivalent system of the interferometer is shown in Fig. 3b. Unlike the system of Fig. 2, in this setup we consider the intensity distribution along the propagation axis behind the lens. The beam from every point in the source plane is split into two beams (which are mutually coherent) which propagate through different channels. The longitudinal intensity distribution is obtained by summing the fields of any two source points and integrating the intensity contributed from all source points. The result is

$$I(z) = \int_0^{2\pi}\int_0^{\infty} \left| \frac{\sqrt{I_s(r,\theta)}}{j\lambda f} \exp\left[jk\left(z + 2f - \frac{r^2 z}{2f^2} \right) \right] + \frac{\sqrt{I_s(r,\theta)}}{j\lambda f} \exp\left[-jk\left(z + 2f - \frac{r^2 z}{2f^2} \right) \right] \right|^2 r\,dr\,d\theta$$

$$= \frac{I_0}{\lambda^2 f^2} \left[1 + \frac{1}{I_0} \int_0^{\infty} \tilde{I}_s(\rho) \cos\left(2kz + 4kf + \frac{kz\rho}{f^2} \right) d\rho \right]. \qquad (10)$$

Using the result of Eq. (4) (replacing $\bar{z}$ by f and Δz by $2z$), we can write the output longitudinal intensity as

$$I(z)=\frac{I_0}{\lambda^2 f^2}\left[1+\left|\mu\left(\frac{z}{\lambda f^2}\right)\right|\cos(4kf+\sphericalangle\mu)\right]. \quad (11)$$

The intensity distribution along the z axis is related to the complex degree of coherence between a point at the origin ($z=0$) and any other point lying on the propagation axis.

Considering Eq. (10), we may say that the system shown in Fig. 3a yields an intensity profile along the z axis that is equal to the cosine transform of the quadratic radial intensity distribution of the incoherent source (plus a constant term) with a scale factor of $(\lambda f^2)^{-1}$. The practical meaning of this result is that the synthesis of longitudinal coherent beams proposed in Refs. [3,7–9] can be applied also to apertures illuminated incoherently. Instead of looking at the complex amplitude of the aperture and of the amplitude of the longitudinal output profile, we consider the intensity distribution of both of them.

In conclusion, we introduced the longitudinal coherence function and proposed two methods to measure it. This coherence function can be used as a tool for calculating the intensity distribution of radial symmetric, spatially incoherent sources. In comparison to the conventional method, the proposed device has a higher dynamic range with the same degree of complexity. The degree of longitudinal coherence can be measured by multiple exposures (see Fig. 2), each with a different optical path between the two channels, or alternatively, by a single exposure as an axial distribution (see Fig. 3). The second system is also a device for implementing incoherent longitudinal holography, i.e. creating an arbitrary axial intensity distribution using an incoherent source.

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