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General configuration for using the longitudinal spatial coherence effect

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Abstract

The Fourier relation between the longitudinal degree of coherence and the radial intensity distribution of a light source can be demonstrated by two configurations of a low spatial-coherence interferometer. We propose a general configuration, which includes the two known types as special cases of the general design. Moreover, by adjusting the distance between the light source and the main lens of the interferometer we can apparently narrow the peak of coherence beyond its known limit. This result might have consequences on the resolution limits of the low spatial-coherence interferometer.

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Keywords: Coherence; Fourier transforms; Interferometry; Low coherence interferometer; Optical coherence tomography

The effect of longitudinal spatial coherence has been extensively investigated by several groups recently. By this effect, the longitudinal spatial degree of coherence of an electromagnetic wave is obtained by one-dimensional Fourier transform of the squared radial intensity distribution of a quasi-monochromatic incoherent light source. This property is derived from the three-dimensional generalization of the Van Cittert–Zernike theorem, as independently noticed several times by various researchers [1–4]. More recently, people have experimentally demonstrated this effect [5–9] and pointed on possible applications like surface profilometry [10] and holographic imaging [11].

There are two basic configurations demonstrating the longitudinal spatial coherence effect; one is without [7–9] and the other with [5,6,10,11] a spherical lens positioned at a focal distance from the source. The

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difference between the two is in the value of the scaling factor of the Fourier transform mentioned above. This difference, although seems only technical, determines different performance merits, like resolution and dynamic range, for the two systems.

In this study, we generalize the system which demonstrates the longitudinal spatial coherence effect such that both known versions become particular cases of the proposed general description. The general case is a system in which the quasi-monochromatic source is positioned in an arbitrary distance from a spherical lens. Obviously, in case this distance is exactly equal to the focal length of the lens, the setup is identical to systems proposed in [5,6,10,11]. We show here that when the source is attached to the lens, the setup is identical to the lensless version [7–9]. In other distances besides these two cases, the degree of coherence is related to the source intensity by the same general mathematical transform, but with a different scaling factor depending on the specific distance between the source and the lens. However the main issue of this study appears in the general expression of the longitudinal degree of coherence. It is implied from this expression that there is a distance between the source and the lens in which the width of the degree of coherence can be narrowed beyond the well-known limit obtained when the source is in the focal plane of the lens. This finding may lead to low spatial-coherence interferometers [10,11] with significantly better depth resolution than the today state of the art.

We start our analysis by observing on the complex degree of coherence between two complex amplitudes behind a spherical lens. These two complex amplitudes are induced by the same incoherent quasi-monochromatic source positioned a distance d in front of the spherical lens L_1 as shown in Fig. 1. In order to simplify the analysis we assume that the lenses in the system have infinite apertures. This assumption, as can be seen in the following, leads at some point to an impractical result regarding the dimension of the degree of coherence. This unrealistic result is later corrected in view of lenses' finite aperture. Let us look first on the complex amplitudes induced from a single quasi-monochromatic point source with a time dependent complex amplitude $u_s(x_s, y_s, t)$. Using Fresnel approximation, the complex amplitude on the lens L_1 plane is

$$g(x_1, y_1, t) = u_s(x_s, y_s, t - \tau_1) \frac{\exp[i(kd - \omega t)]}{i\lambda d} \exp\left\{ i \frac{k}{2d} \left[(x_l - x_s)^2 + (y_l - y_s)^2 \right] \right\}, \quad (1)$$

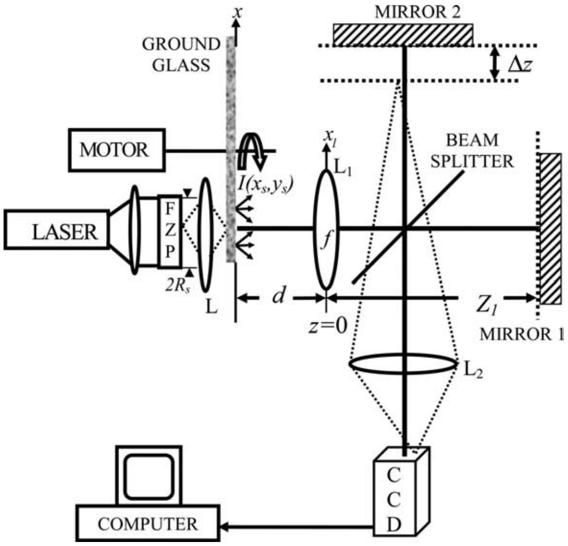

Fig. 1. Schematic of the interferometric system used for measuring the longitudinal spatial coherence effect. FZP = Fresnel zone plate.

where (x_l, y_l) are coordinates of the lens plane, k is the wave number, ω is the angular frequency, λ is the wavelength and τ_l is the time delay between the source and the lens L_1. Using additional Fresnel propagation from the lens toward a plane located z_n ($n = 1,2$) behind the lens, at the nth arm of the interferometer, yields the following complex amplitude,

$$p_n(x_n, y_n, z_n, t) = \frac{\exp[\mathrm{i}(kz_n - \omega t)]}{\mathrm{i}\lambda z_n} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x_l, y_l, t-\tau_n) \exp\left[-\frac{\mathrm{i}k}{2f}(x_l^2 + y_l^2)\right] \times \exp\left\{\mathrm{i}\frac{k}{2z_n}\left[(x_n - x_l)^2 + (y_n - y_l)^2\right]\right\} \mathrm{d}x_l\,\mathrm{d}y_l, \quad n = 1,2. \tag{2}$$

where τ_n is the time delay between the lens L_1 to the point z_n. Substituting Eq. (1) into Eq. (2) yields

$$p_n(x_n, y_n, z_n, t) = -u_s(x_s, y_s, t - \tau_n - \tau_l)\frac{\exp[\mathrm{i}(kz_n + kd - \omega t)]}{\lambda^2 d z_n} \exp\left[\frac{\mathrm{i}k}{2d}(x_s^2 + y_s^2)\right] \times \exp\left[\frac{\mathrm{i}k}{2z_n}(x_n^2 + y_n^2)\right] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[\frac{\mathrm{i}kb_n}{2}(x_l^2 + y_l^2)\right] \times \exp\left\{\frac{-\mathrm{i}k}{z_n d}[(x_s z_n + x_n d)x_l + (y_s z_n + y_n d)y_l]\right\} \mathrm{d}x_l\,\mathrm{d}y_l, \tag{3}$$

where $b_n = (1/z_n + 1/d - 1/f)$. The integral in Eq. (3) can be considered as a two-dimensional Fourier transform. Using Fourier table, the complex amplitude induced by a single source point at the nth arm is,

$$p_n(x_n, y_n, z_n, t) = u_s(x_s, y_s, t - \tau_n - \tau_l)\frac{\exp[\mathrm{i}(kd + kz_n - \omega t)]}{\mathrm{i}b_n \lambda d z_n} \exp\left[\mathrm{i}\frac{\pi}{\lambda d}\left(1 - \frac{1}{b_n d}\right)(x_s^2 + y_s^2)\right] \times \exp\left[\mathrm{i}\frac{\pi}{\lambda z_n}\left(1 - \frac{1}{b_n z_n}\right)(x_n^2 + y_n^2)\right] \exp\left[\frac{-\mathrm{i}2\pi}{\lambda b_n z_n d}(x_n x_s + y_n y_s)\right]. \tag{4}$$

Assuming that the entire optical path differences in the system are much smaller than the coherence length of the quasi-monochromatic light source, all the temporal coherence effects are neglected. Therefore, the complex degree of coherence between two arbitrary points (x_1, y_1, z_1) at the first arm and (x_2, y_2, z_2) at the second arm, behind the lens L_1, for the entire incoherent source points, is defined as [4]

$$\mu(x_1, y_1, z_1; x_2, y_2, z_2) = \frac{\int\int p_1(x_1, y_1, z_1; x_s, y_s, t) p_2^*(x_2, y_2, z_2; x_s, y_s, t)\,\mathrm{d}x_s\,\mathrm{d}y_s}{[\int\int |p_1(x_1, y_1, z_1; x_s, y_s, t)|^2 \mathrm{d}x_s\,\mathrm{d}y_s \int\int |p_2(x_2, y_2, z_2; x_s, y_s, t)|^2 \mathrm{d}x_s\,\mathrm{d}y_s]^{1/2}}. \tag{5}$$

Substituting Eq. (4) into Eq. (5) yields

$$\mu(x_1, y_1, z_1; x_2, y_2, z_2) = A\int\int I_s(x_s, y_s) \exp\left\{\frac{\mathrm{i}\pi}{\lambda d^2}\left(\frac{1}{b_2} - \frac{1}{b_1}\right)(x_s^2 + y_s^2) + \frac{\mathrm{i}2\pi}{\lambda d}\left[x_s\left(\frac{x_2}{b_2 z_2} - \frac{x_1}{b_1 z_1}\right) + y_s\left(\frac{y_2}{b_2 z_2} - \frac{y_1}{b_1 z_1}\right)\right]\right\} \mathrm{d}x_s\,\mathrm{d}y_s, \tag{6}$$

where

$$A = \left[\int\int I_s(x_s, y_s)\,\mathrm{d}x_s\,\mathrm{d}y_s\right]^{-1}.$$

From the general expression of Eq. (6) one can easily derive the two well-known cases. First, in the case $d = f$, b_n is equal to $1/z_n$, and the degree of coherence becomes

$$\mu(\Delta x, \Delta y, \Delta z) = A\int\int I_s(x_s, y_s) \exp\left[\frac{\mathrm{i}\pi \Delta z (x_s^2 + y_s^2)}{\lambda f^2} + \frac{\mathrm{i}2\pi(\Delta x x_s + \Delta y y_s)}{\lambda f}\right] \mathrm{d}x_s\,\mathrm{d}y_s, \tag{7}$$

where $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$ and $\Delta z = z_2 - z_1$. The expression in Eq. (7) is naturally the 3-D degree of coherence obtained in the system where the source is positioned in the front focal plane [4–6,10,11].

In the case $d = 0$, the degree of coherence becomes

$$\mu(x_1, y_1, z_1; x_2, y_2, z_2) = A \int\int I_s(x_s, y_s) \exp\left[\frac{i\pi\Delta z(x_s^2 + y_s^2)}{\lambda\bar{z}^2} + \frac{i2\pi}{\lambda}\left(x_s\left(\frac{x_2}{z_2} - \frac{x_1}{z_1}\right) + y_s\left(\frac{y_2}{z_2} - \frac{y_1}{z_1}\right)\right)\right] dx_s\, dy_s, \tag{8}$$

where $\bar{z} = (z_1 + z_2)/2$. Eq. (8) describes a degree of coherence obtained in a lensless system. [4,7–9] This result is well understood, because the attached lens multiplies the source with a complex amplitude of a pure phase distribution, which does not have any influence on the intensity distribution of the source. In other words, a system in which the lens is attached to the incoherent light source is effectively identical to a lensless system.

Back to the general case expressed in Eq. (6), let's concentrate now only in the longitudinal degree of coherence $\mu(0, 0, \Delta z)$. Substituting $(\Delta x, \Delta y) = (0, 0)$ and the approximation $1/b_2 - 1/b_1 \cong (df)^2 \Delta z/(fd + f\bar{z} - \bar{z}d)^2$ into Eq. (6) yields the following longitudinal degree of coherence,

$$\mu(\Delta z) = A \int_0^\infty \tilde{I}_s(\sqrt{\rho_s}) \exp\left[\frac{i\pi f^2 \Delta z \rho_s}{\lambda(df + \bar{z}f - \bar{z}d)^2}\right]_s d\rho_s, \tag{9}$$

where

$$\tilde{I}_s(\sqrt{\rho_s}) = \int_0^{2\pi} I_s(\sqrt{\rho_s}, \theta_s) d\theta_s, \quad \rho_s = x_s^2 + y_s^2 \text{ and } \theta_s = \arctan(y_s/x_s).$$

Apparently from Eq. (9), the longitudinal degree of coherence is a Fourier transform of the squared radial distribution of the source with scaling factor that depends on the distance between the source and the lens. Moreover, there is a singular point in which the scaling factor $f^2/2\lambda(df + \bar{z}f - \bar{z}d)^2$ can apparently grow to infinity, when the distance d approaches to the value $d = \bar{z}f/(\bar{z} - f)$. This unrealistic result is obtained because of the assumption of an infinite aperture for the lens L_1. Note that the singular point for the longitudinal degree of coherence is located at a distance that also fulfills the imaging condition for the source. This last observation intuitively explains the whole phenomenon; From Eq. (8) one sees that the width of the degree of coherence proportionally grows with $\bar{z}$ – the distance of the measurement point from the source. Therefore, it is expected that when the measurement point getting close to the source's image, the degree of coherence becomes more and more narrow. This last observation indicates on the minimal width of the longitudinal coherence peak that can be achieved in this interferometer. Since we assume that each point on the light source is completely uncorrelated with every other source point, the coherence length at the imaging plane is equal to the thickness of each source point's image. From diffraction considerations, it is well known [12] that the thickness of each point's image is approximately $w \cong \lambda\bar{z}^2/R_L^2$, where R_L is the radius of lens L_1. Therefore, w is the minimal width of the longitudinal coherence peak and the depth resolution limit that can be achieved by our system. Note that from intuitive reasoning, and although we initially assumed that the lens L_1 has infinite aperture, its practical finite aperture dictates the depth resolution limit which can be achieved by this interferometer. Nevertheless, the meaning of this result is that by approaching toward the theoretical singular point, one can narrow a peak of coherence beyond the well-known limit obtained at the source–lens distance $d = f$, from a width $\sim \lambda f^2/R_s^2$ where R_s is the radius of the source, to the approximated value $\lambda\bar{z}^2/R_L^2$. The depth resolution can be improved by the ratio $(fR_L/\bar{z}R_s)^2$, compared to systems where the source is positioned in the front focal plane [4–6,10,11].

We also note that the analogy, mentioned in [5] and [6], between the partial coherence and the scalar diffraction theories does not exist anymore for the case $d \neq f$. To see this, let's calculate the complex amplitude along the optical axis, $u(0, 0, z)$, behind the lens of Fig. 2(a), where an aperture $u_s(x_s, y_s)$ is illuminated

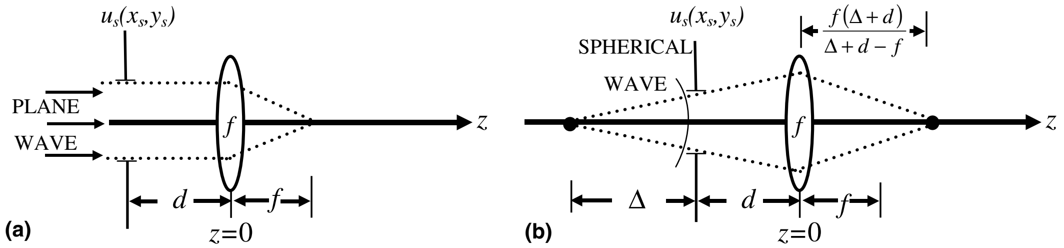

Fig. 2. Schematic of diffraction through a spherical lens from an aperture illuminated by (a) a plane wave and by (b) a spherical wave.

by a coherent plane-wave. This complex amplitude is obtained by an integral on all the contributions of the aperture points, each of which is of the form of Eq. (4), as the following

$$u(0,0,z) = B\int_0^\infty \tilde{u}_s(\sqrt{\rho_s})\exp\left[-\frac{i\pi(z-f)\rho_s}{\lambda(fd+zf-zd)}\right]d\rho_s, \tag{10}$$

where

$$\tilde{u}_s(\sqrt{\rho_s}) = \int_0^{2\pi} u_s(\sqrt{\rho_s},\theta_s)\,d\theta_s,$$

B is a constant, and since the axial distribution behind a single lens is considered, we substitute into Eq. (4) the relation $(x_n, y_n, z_n) = (0, 0, z)$. Comparing Eqs. (9) and (10), we see that the analogy between partial coherence and scalar diffraction theories is valid only for the case $d = f$, where in all other cases the results are different. In the case of diffraction, as expressed by Eq. (10), and for $d \neq f$, the complex amplitude is not symmetric around the back focal point. This is because the outside variable z appears also at the denominator of the phase expression in the integrand of Eq. (10). Moreover, the phase in the integrand of Eq. (10), near the origin point $z = f$, is approximately $\pi(f - z)\rho_s/\lambda zf$ for any value of d. Therefore, the focal depth cannot be much less than $\lambda f^2/R_s^2$. On the other hand, in the partial coherence system, as expressed by Eq. (9), the degree of coherence is always Hermiticity symmetric around the origin $\Delta z = 0$. This is because the outside variable Δz appears only at the numerator of the phase in the integrand of Eq. (9). Moreover, the distance d can approach the singular point in which the dimension of the degree of coherence goes down, and becomes lens aperture limited rather than source aperture limited.

A similar conclusion is obtained between the partial coherence system and a more general coherent diffraction system shown in Fig. 2(b). This time we consider diffraction from an aperture illuminated by a spherical rather than a plane wave. The complex amplitude along the optical axis beyond the lens is obtained from Eq. (10), where the aperture u_s is multiplied by a quadratic phase function of a spherical wave (under the paraxial approximation) originated a distance Δ before the aperture, as the following:

$$\begin{aligned} u(0,0,z) &= B\int_0^\infty \tilde{u}_s(\sqrt{\rho_s})\exp\left(\frac{i\pi\rho_s}{\lambda\Delta}\right)\exp\left[-\frac{i\pi(z-f)\rho_s}{\lambda(fd+zf-zd)}\right]d\rho_s \\ &= B\int_0^\infty \tilde{u}_s(\sqrt{\rho_s})\exp\left\{-\frac{i\pi[z-f(\Delta+d)/(\Delta+d-f)]\rho_s}{\lambda\Delta(fd+zf-zd)/(\Delta+d-f)}\right\}d\rho_s. \end{aligned} \tag{11}$$

As expected, the origin of the complex axial amplitude is shifted to the point image of the point source, at $z = f(\Delta + d)/(\Delta + d - f)$. Also, from Eq. (11) it is again clear that the analogy between partial coherence and scalar diffraction is valid only for the case $d = f$, where the outside variable z appears only at the numerator of the phase in the integrand of Eq. (11). In all other cases the complex amplitude is not symmetric around the origin, because, again, the outside variable z appears also at the denominator of the phase. Finally, for any value of d, the focal depth is always dependent on the aperture size and there is no any singular point for any d, in which the size of complex amplitude peak becomes lens aperture limited.

A simple experiment has been conducted to demonstrate the validity of the theory described previously. In the experiment a He–Ne laser with $\lambda = 0.63$ μm illuminated a Fresnel zone plate (FZP) projected by the lens L on a rotating ground glass as shown in Fig. 1. The focal length of the lens L_1 was $f = 25$ cm, and the FZP diameter was 3.6 cm with $N = 14.6$ cycles from its center to its perimeter. The distance between the lens and the stable mirror no. 1 was equal to $z_1 = 39$ cm. The intensity distribution of the incoherent FZP source is a binary approximation of the following cosine grating

$$I_s(x_s, y_s) \propto 1 + \cos\{\pi\gamma(x_s^2 + y_s^2) + \beta\}, \quad \sqrt{x_s^2 + y_s^2} \leqslant R_s, \tag{12}$$

where R_s is the overall radius of the source (1.8 cm in this experiment), γ and β are parameters that control the distance between the side peaks of the degree of coherence to the origin and their phase values, respectively. Substituting Eq. (12) into Eq. (9) yields the following complex degree of coherence,

$$\mu(0,0,\Delta z) \propto \operatorname{sinc}\left(\frac{f^2 R_s^2 \Delta z}{2\lambda[fd + f\bar{z} - \bar{z}d]^2}\right) * \left[2\delta(\Delta z) + \exp(i\beta)\delta\left(\Delta z + \frac{\gamma\lambda}{f^2}(fd + \bar{z}f - \bar{z}d)^2\right) + \exp(-i\beta)\delta\left(\Delta z - \frac{\gamma\lambda}{f^2}(fd + \bar{z}f - \bar{z}d)^2\right)\right], \tag{13}$$

where the asterisk denotes convolution, δ is Dirac delta function and $\operatorname{sinc}(x) = \sin(\pi x)/\pi x$. According to Eq. (13), a source in the shape of FZP produces a longitudinal degree of coherence in a shape of three coherence peaks, with a gap between every two successive coherence peaks of,

$$\Delta z_g = \frac{2N\lambda}{(R_s f)^2}(fd + \bar{z}f - \bar{z}d)^2, \tag{14}$$

where the relation $\gamma = 2N/R_s^2$ is employed. According to Eq. (13), the width of any order of the spatial degree of coherence, between the center and the first zero of the sinc, is

$$\Delta z_w = \frac{2\lambda[fd + \bar{z}f - \bar{z}d]^2}{f^2 R_s^2}. \tag{15}$$

However, because of technical difficulties in measuring the orders' width, in the present experiment we measured only the gap between the coherence orders. Of course these coherence peaks are not directly observed like diffraction pattern, but obtained by measuring the interference visibility versus the path difference between the interferometer mirrors. For every value of the distance d, we scan along some range with the moving mirror in order to find the value of the mirrors' gap in which the visibility is locally maximal. The gap between the central (zero) and the first order was measured for eight values of the distance d. The results are shown in Fig. 3, where the triangles are the measured half gap versus the distance d, and the solid line is the calculated graph according to Eq. (14). In Fig. 3 we measured and calculated only the half gap $\Delta z_g/2$ because the path difference between the mirrors in a Michelson interferometer is double the actual shift of the mirror. Note that in this graph there are three measuring points beyond the distance $d = f = 25$ cm, and indeed the gap between the central and the first order is monotonically decreased according to Eq. (14). Since the degree of coherence is symmetric and the two first orders are located at an equal distance from the point $\Delta z = 0$, from its two sides, the overall dimension of the degree of coherence is decreased with the increase in d. The good agreement between the measured and the calculated results verifies that the main result of our analysis, expressed by Eq. (9), is correct.

The consequence of this analysis is that by properly adjusting the distance between the source and the main lens, one apparently can narrow any peak of coherence beyond the known limit of approximately

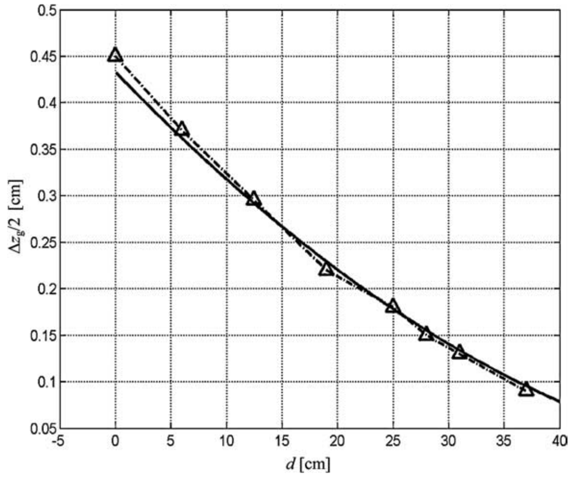

Fig. 3. The half gap between the central (zero) and the first coherence orders versus the distance d between the source and the lens L_1. The solid line is calculated according to Eq. (14). The triangles denote the experimental results.

$\lambda f^2/R_s^2$ without considerable efforts. He does not need to decrease the wavelength of the light source, and by that to invest more energy in the system. He also does not have to increase the system's numerical aperture and by that to introduce intolerable aberrations into the system. In order to improve the system's resolution he just needs to increase the source–lens gap beyond the focal distance up to the point where the system becomes lens aperture, rather than source aperture, limited. The unique feature of improving resolution performances without meaningful efforts might make this interferometer more attractive for applications of tomography and profilometry.

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