Synthesizing Incoherent Digital Holograms with Reduced Number of Projections

Barak Katz*, Natan T. Shaked and Joseph Rosen

Department of Electrical and Computer Engineering
Ben Gurion University of the Negev, P.O Box 653, Beer-Sheva 84105, Israel
**Corresponding author: barakk@cse.bgu.ac.il*

Abstract: We present a method of recording digital Fourier holograms under incoherent illumination, using a significantly reduced number of observed perspective projections and a digital prediction of the middle projections. The method is demonstrated experimentally.

OCIS codes: (090.0090) Holography; (090.0090) Digital holography; (090.1760) Computer holography; (100.6890) Three-dimensional image acquisition; (110.6880) Three-dimensional image processing; (999.9999) View synthesis.

1. Introduction

Synthesizing computer generated holograms (CGH) of realistic three-dimensional (3-D) objects under incoherent white light illumination by capturing projections of the objects from different perspectives and fusing these projections by a specific computing process is presented in Refs. [1-3]. Yet, the required recording process is extremely long and complicated due to the tremendous amount of different perspective projections needed.

Recently, Shaked *et al*. [4] presented a method called integral holography in which the 3-D scene is captured by a microlens array. In the current paper, we propose an alternative method of synthesizing high quality holograms under incoherent white light illumination by significantly reducing the number of the required projections. This method relies on a geometric image interpolation called view-synthesis [5]. We designate the proposed holographic acquisition method as synthetic projection holography (SPH).

2. Synthetic projection holography

In the proposed method, instead of capturing the complete set of observed projections from different perspectives, only a few chosen observed projections are captured by the digital camera. Then, computational process synthesizes a series of synthetic middle projections, located between each two consecutive real observed projections [5]. Next, all of the projections (observed and synthetic) are centered on a chosen reference point by digitally correlating each projection with a known pattern taken from one of the projections. After that, each of the centered projections is multiplied by a horizontally varying linear phase function, the frequency of which is proportional to the serial number of the projection in the entire projection set. The resulting product is summed up to get a single column in the complex amplitude of the 1-D Fourier transform of the 3-D scene. Each projection, therefore, yields a different column in the complex amplitude of the object's 1-D Fourier transform.

At the beginning of the generation of the synthetic projections, two complete correspondence maps are computed. Each element of each complete correspondence map describes the displacement in pixels from one view to another. Three steps are required for generating a complete correspondence map. In the first step, two images of vertical edges are produced by convolving the original views with the following kernel [1 2 1; 0 0 0; −1 −2 −1]. Afterwards, an initial sparse correspondence map is estimated by finding matches between corresponding pixels in the two images of vertical edges. Finally, a complete correspondence map is determined by interpolation of the initial sparse correspondence map. Once the complete correspondence maps are established, a sequence of synthesized views is generated. Each and every synthesized view is estimated according to a pair of observed projections. The location of the synthesized view in the area between the two observed projections is determined by the relative displacement *D/N*, where $N \geq D \geq 0$ and *N* represents the total number of both synthetic and observed projections in the set.

The in-between synthesized views are actually computed in two phases. First, both observed views are warped to the new location of the synthesized view in order to get two warped projections PW_1 and PW_2. The warp functions are computed – based on the two observed projections P_1^O and P_2^O, the correspondence maps $C^1 : P_1^O \rightarrow P_2^O$, $C^2 : P_1^O \leftarrow P_2^O$ and the relative displacement *D/N* of the new synthesized views – as the following

$$PW_1\left(x+\left\lfloor \frac{1}{2}+\frac{D}{N}\times C_1(x,y)\right\rfloor, y\right)=\left(1-\frac{D}{N}\right)\times P_1^O(x,y)$$
$$PW_2\left(x-\left\lfloor \frac{1}{2}+\left(1-\frac{D}{N}\right)\times C_2(x,y)\right\rfloor, y\right)=\frac{D}{N}\times P_2^O(x,y), \quad (1)$$

where *x, y* are the pixels coordinates, the superscript *O* denotes the projections observed by a camera and the symbol $\lfloor\ \rfloor$ stands for rounding down the value to its closest integer. In the second phase, the warped projections PW_1 and PW_2 are summed up ($P_D^S = PW_1 + PW_2$, where the superscript *S* denotes a synthetic projection), giving the final synthesized view P_D^S at the relative displacement *D/N*.

3. Experimental results

The 3-D scene in our experiment contains three cubes. The distances along the optical axis between the imaging lens of the CCD camera and the first, middle, and last cubes are 30 cm, 37 cm, and 40 cm, respectively. A 1-D Fourier CGH has been generated from a set of 400 observed projections, according to the algorithm described in [1], where the interval between every two successive projections of the scene is 0.1 mm. This fully-observed hologram is compared to four different SPHs according to the method described in section 2. The outcomes of the different steps in the view synthesis algorithm in our experiment are demonstrated in Fig. 1. In this demonstration, the middle (*D*=*N*/2) synthetic projection (g) is synthesized based on two observed projections (a) and (b). The two images of vertical edges [(c) and (d)] and the warped images [(e) and (f)] are presented as well. The SPHs have been generated with 2, 17, 33, and 55 observed projections, where the distance between every two successive projections in each of these four SPHs is equal. The best in-focus reconstructed planes obtained from the fully-observed hologram and the SPHs generated with only 2 observed projections (out of the 400 observed projections in the initial set) are presented in Fig. 2. A quantitative comparison among the different SPHs is presented in Fig. 3 and carried out by measuring the mean-square error (MSE) of the best in-focus reconstructed planes from each SPH and the coinciding planes of the fully-observed hologram. This comparison is restricted to the best in-focus cubes area and not to the entire plane. The MSE calculation is defined by the following equation

$$MSE=\frac{1}{M\cdot K}\sum_{i=1}^{M}\sum_{j=1}^{K}\left[P(i,j)-\beta\cdot\tilde{P}(i,j)\right]^2, \quad (2)$$

where *i*, *j* are the coordinates of each pixel; *M, K* are the dimensions of the considered area; $P(i,j)$ is the scaled reconstructed images from the hologram; $\tilde{P}(i,j)$ is the reconstructed images from the SPH; and β is a factor that scales the reconstructed images to minimize the MSE and given as follows

$$\beta=\left[\sum_{i=1}^{M}\sum_{j=1}^{K}P(i,j)\cdot\tilde{P}(i,j)\right]\Big/\sum_{i=1}^{M}\sum_{j=1}^{K}\tilde{P}(i,j)\cdot\tilde{P}(i,j) \quad (3)$$

According to Fig. 2, the quality of the reconstructed planes of the various holograms is quite good, even for the hologram synthesized with only 2 observed projections (out of the 400 observed projections in the initial set).

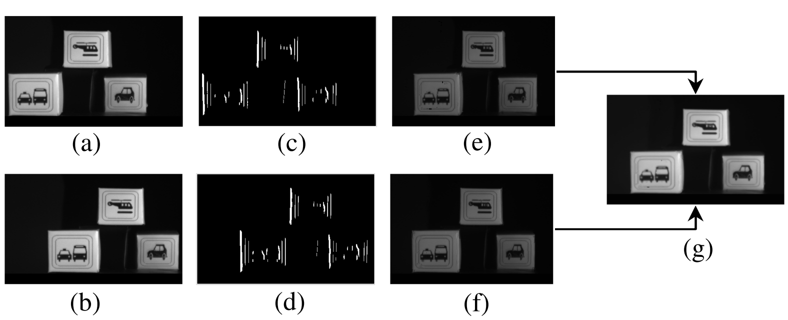

Fig. 1. Results of the different steps in the view synthesis algorithm. Two observed projections (a) and (b); their two images of vertical edges (c) and (d); the warped images (e) and (f), and the final middle synthetic projection (g).

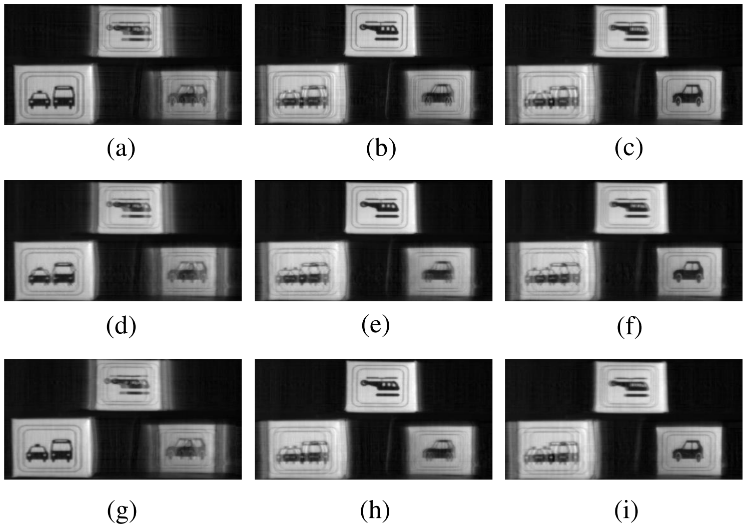

Fig. 2. Best in-focus reconstructed planes obtained from the fully-observed hologram (a, b and c); from the SPH with 2 observed projections (d, e and f) and from the SPH with 55 observed projections (g, h and i).

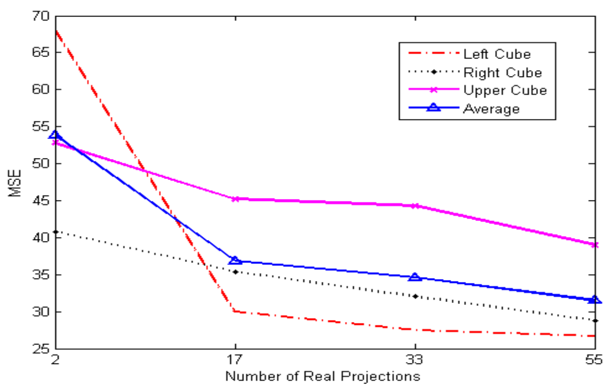

Fig. 3. MSE of the reconstructed images versus the number of observed projections.

Nevertheless, higher quality is achieved by increasing the number of observed projections. The quality improvement is demonstrated by the visual comparison between corresponding images in Fig. 2(d,e,f) and Fig. 2(g,h,i), as well as by the quantitative comparison shown in Fig. 3. For example, the results of the reconstructed planes of the hologram synthesized with 55 observed projections are superior compared to the results of the reconstructed planes of the hologram synthesized with only 2 observed projections.

4. Conclusions

A new method of recording a Fourier hologram under spatially incoherent white-light illumination is presented. In this method, the view synthesis algorithm is integrated into the holographic acquisition in a way that significantly reduces the number of observed projections needed for the generation of the hologram.

5. References

[1] Y. Li, D. Abookasis and J. Rosen, “Computer-generated holograms of three-dimensional realistic objects recorded without wave interference,” Appl. Opt. **40**, 2864-2870 (2001).

[2] D. Abookasis and J. Rosen, “Computer-generated holograms of three-dimensional objects synthesized from their multiple angular viewpoints,” J. Opt. Soc. Am. A **20**, 1537–1545 (2003).

[3] Y. Sando, M. Itoh and T. Yatagai, “Holographic three-dimensional display synthesized from three-dimensional Fourier spectra of real existing objects,” Opt. Lett. **28**, 2518–2520 (2003).

[4] N. T. Shaked, J. Rosen and A. Stern, “Integral holography: white-light single-shot hologram acquisition,” Opt. Express **15**, 5754-5760 (2007). http://www.opticsexpress.org/abstract.cfm?id=133213

[5] D. Scharstein, *View Synthesis Using Stereo Vision-Lecture Notes in Computer Science* (LNCS) **1583** (Springer-Verlag, 1999), Chap. 2.