Fresnel Incoherent Digital Holograms Directly Recorded by Multiple Viewpoint Projections

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Abstract: We present and experimentally demonstrate a new, efficient, direct, and accurate method of obtaining a modified Fresnel hologram under incoherent illumination by directly processing the projections of the three dimensional scene, and without using approximations.

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1. Introduction

Several kinds of multiple viewpoint projection (MVP) digital holograms have been proposed in the literature [1-3]. In order to generate this kind of holograms, multiple projections of a three dimensional (3-D) scene are acquired from various viewpoints and digitally processed to yield a hologram of the 3-D scene. The projections are acquired with a simple digital camera, under white light illumination, and without most of the disadvantages characterizing conventional holography, namely the need of a powerful, highly coherent light source, typically a laser, and an extreme stability of the optical system. The resulting hologram is equivalent to an optical hologram of the same scene recorded from a single point of view and thus can be reconstructed optically by illuminating the hologram with a coherent plane wave, or alternatively, by using a digital reconstruction technique.

However, it is hard to generate this kind of holograms because of two main reasons: (a) The number of the acquired viewpoint projections of the 3-D scene might be quite large so the camera has to be repositioned many times; (b) The digital mathematical process that has to be performed on the acquired projections is inaccurate and quite complex. In order to solve the first problem mentioned above, we have recently proposed two different methods to reduce the number of viewpoint projections. In the first method, a microlens array is utilized for acquiring the entire viewpoint projections of the 3-D scene in a single camera shot [4], whereas in the second method only a small number of extreme projections are actually acquired, while the middle projections are predicted by the computer using the view synthesis algorithm [5].

The current paper addresses the second problem mentioned above, that is, the complexity and inaccuracy of the digital process performed on the acquired projections. This is carried out by presenting a new and direct method of synthesizing a modified Fresnel hologram using the MVPs. In Refs. [2] and [3], the generation of the MVP Fresnel hologram has not been performed directly, but rather the Fresnel hologram has been calculated by digital convolution of the digital reconstructed images from an MVP Fourier hologram and quadratic phase functions. This process is approximated, inaccurate and requires redundant calculations that can be avoided by using the new direct method proposed herein. By avoiding the redundant calculations, digital errors during the various transformations are prevented. Moreover, the direct Fresnel holography method presented here does not use approximations like the small angle assumption used to generate the Fourier and other types of holograms. Therefore, the direct Fresnel holograms are not limited to small angles and hence their reconstructions are more accurate. Avoiding redundant calculations becomes extremely critical when Fresnel holograms with a large number of pixels have to be calculated. In addition, since the prospective goal of these white light holography methods is the design of a portable digital holographic camera working in real-time, it is important to use as few digital calculations as possible.

2. Methodology

Figure 1 illustrates the optical system used for acquiring the multiple projections of the 3-D scene. We number the projections by *m*, so that $P_m(x_p, y_p)$ is the *m*-th projection, where x_p and y_p are the axes on the projection plane. According to the proposed method, this *m*-th projection is multiplied by a 1-D quadratic phase function, and the result is summed up to get the (*m*,*n*)-th pixel in the final matrix given by the following expression

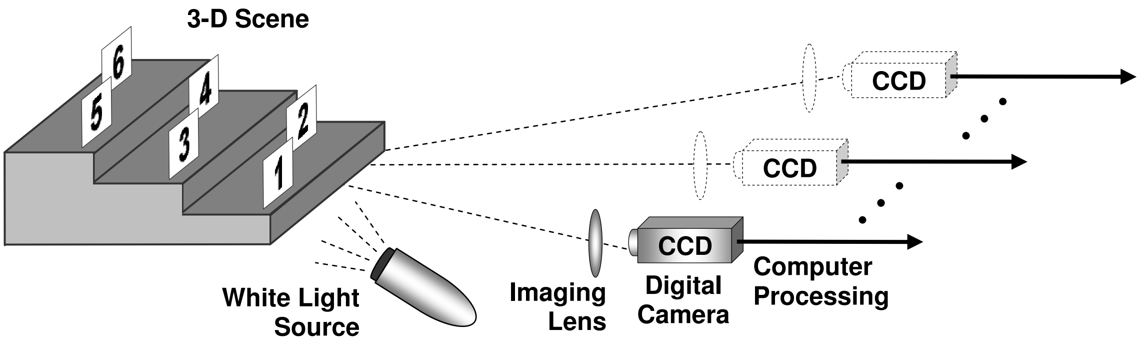

Fig. 1. An optical system for capturing multiple viewpoint projections of a 3-D scene.

$$H(m,n) = \iint P_m(x_p, y_p)[\exp(-j2\pi b x_p^2)\delta(y_p - n\Delta p)]dx_p dy_p, \qquad (1)$$

where b is an adjustable parameter, δ is the Dirac delta function and Δp is the pixel size of the digital camera. The process described by Eq. (1) is repeated for all the projections, so that each projection contributes a different column to the final matrix $H(m,n)$. As can be proved, the obtained 2-D complex matrix $H(m,n)$ represents the 1-D modified Fresnel hologram of the observed scene.

However, contrary to a regular Fresnel hologram, the transverse magnifications M_x and M_y as well as the axial magnification M_z of the proposed hologram are defined as follows

$$M_x = \bar{x}_r / \bar{x}_s = \Delta p / \alpha; \quad M_y = \bar{y}_r / \bar{y}_s = M = f / z_s; \quad M_z = \bar{z}_r / \bar{z}_s = (\bar{z}_s + 2z_s)/(\gamma b f^2 \alpha^2), \qquad (2)$$

where $(\bar{x}_s, \bar{y}_s, \bar{z}_s)$ and $(\bar{x}_r, \bar{y}_r, \bar{z}_r)$ are the sizes of the original object and reconstructed object, respectively, α is the camera (and the imaging lens) movements between two adjacent projections, M is the magnification of the imaging lens, f is the focal length of the imaging lens, z_s is the axial position of the considered object and γ is constant. From Eq. (2), one can see that contrary to a conventional Fresnel hologram, and to the vertical magnification M_y, the horizontal magnification M_x is constant, and is not dependent on the axial positions of the different objects in the 3-D scene. This effect can be eliminated by resampling the reconstructed planes by M/M_x along the horizontal axis, as demonstrated next.

3. Experimental Results

In this section, we experimentally demonstrate the generation of the proposed hologram by the optical system illustrated in Fig. 1. Six planes, each of the size of 2.5 cm × 2.5 cm, containing the digits '1'−'6' are positioned on a dark background and used as the 3-D scene to be recorded. The distances between the plane containing the digit '1' and the planes containing the digits '2', '3', '4', '5' and '6' are 4 cm, 0 cm, 4 cm, 0 cm and 4 cm on the horizontal axis, 0 cm, 3.5 cm, 3.5 cm, 6.5 cm and 6.5 cm on the vertical axis, and 5 cm, 12 cm, 17 cm, 23.5 cm and 28.5 cm on the optical axis, respectively. The distance between the closest plane containing the digit '1' and the imaging lens is 43 cm. The camera used is a CCD camera (PCO, Scientific 230XS) containing 1280×1024 pixels and an 8.6×6.9 mm^2 active area. Figure 2(a) shows three projections, out of 500 projections acquired by the camera across a horizontal range of 10 cm. As shown in this figure, the relative positions of the digits change along the horizontal axis only as a function of the location of the projection in the entire projection set.

Each of the acquired projections is multiplied, according to Eq. (1), by a 1-D quadratic phase function and the result is summed up to get a single column in the 1-D modified Fresnel hologram of the 3-D scene. Figure 2(b) shows the magnitude and phase of the hologram obtained after processing all the acquired projections. Digital reconstruction of the 3-D object recorded into the hologram is obtained by convolving the hologram with a 1-D quadratic phase function. Figure 4(c) shows six chosen reconstructed planes along the optical axis. As shown in this figure, in each plane a different digit is in focus, whereas the other five digits are out of focus. This behavior validates the volumetric information encoded into the hologram. Figure 2(d) shows the best in-focus reconstruction planes resampled along the horizontal axis by M/M_x. After the resampling process, the aspect ratios of the reconstructed objects in each of the best in-focus planes are similar to the aspect ratios of the objects in the original 3-D scene.

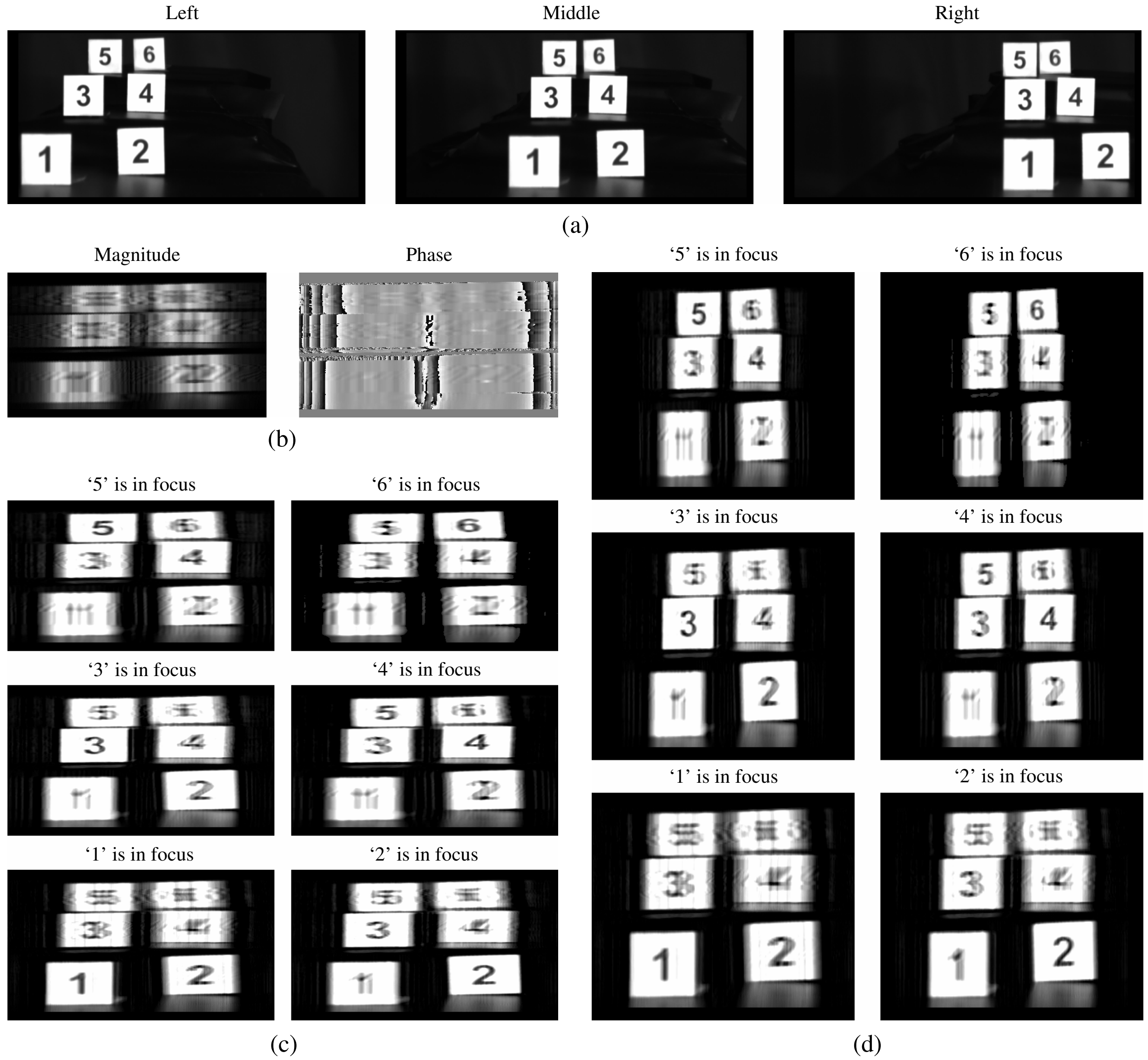

Fig. 2. Experimental results for generating a 1-D modified Fresnel hologram: (a) Three projections taken from the entire projection set; (b) Magnitude and phase of the modified Fresnel hologram; (c) The six best in-focus reconstructed planes along the optical axis; (d) The six best in-focus reconstructed planes along the optical axis after the resampling process of the horizontal axis.

4. Conclusion

We have presented a direct method of obtaining an MVP modified Fresnel hologram of real existing 3-D objects illuminated by white light. This method eliminates redundant digital calculations and thus makes the entire hologram generation process more accurate and faster. Other kinds of directly generated MVP holograms are most likely possible and probably will be explored in the future.

5. References

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