Nat Comput
DOI 10.1007/s11047-014-9442-9

Holographic parallel processor for calculating Kronecker product

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Abstract We present a holography based optical architecture that computes Kronecker product of two given binary matrices in a single (configuration) step. We demonstrate the use of the holography capability to enlarge the input of size n into the size n^2 in one step, in contrast to the traditional optical copying techniques that duplicate the input by a fixed constant factor in a single step.

Keywords Optical computing · Holography · Tensor product · Interferometry · Real-time holography · Binary optics

1 Introduction

The tensor product is a widely used computational task, particularly Kronecker product has numerous applications in different fields: physics, statistic, control theory, optimization and more Ballani and Grasedyck (2013); Graham (1981); Van Loan (2000). Therefore, from the algorithmic point of view, it is important to efficiently implement Kronecker product computation.

A straightforward sequential implementation of the task on the conventional architectures has $O(n^2)$ time complexity, where n is the size of the matrices being multiplied. Several parallel algorithms for the task were proposed in Tadonki (2011).

In this work we propose fully optical, holography based, parallel architecture for Kronecker product computation of any two given binary matrices. The novelty of this architecture in its configurational simplicity: the whole computational process performed by the real-time system, which is set up only once at the beginning of the computation and is not reconfigured during the process. The architecture we propose produces n copies of the input (where n is the size of the input) in a single configuration step rather than just a constant number of copies as it is done in the previously designed optical devises Dolev and Fitoussi (2010); Anter and Dolev (2010). Thus our system is one-step, fully optical scheme that produces an output of size larger than the input by factor of n, where n is the size of the input.

To the best of our knowledge this is the first technique that can perform better than constant size copying in one configuration step. In addition, this is the first fully optical implementation of the Kronecker product task. An electrooptical setup for performing Kronecker product computation for the special case of complete Walsh matrices is proposed in Davis (1995). Unlike the system of Davis (1995), our system is purely optical, avoiding any digital electronic computations.

2 Kronecker product of binary matrices

Definition 1 Let $A \in \mathbb{R}_{m\times n}$ and $B \in \mathbb{R}_{s\times t}$. The Kronecker product of A and $B(A \oplus B)$ is a block matrix of the size $ms \times nt$ where block (i,j) contains $a_{ij} \times B$.

We propose an optical architecture, based on holographic approach, that implements the Kronecker product computation of any two given binary matrices. For the

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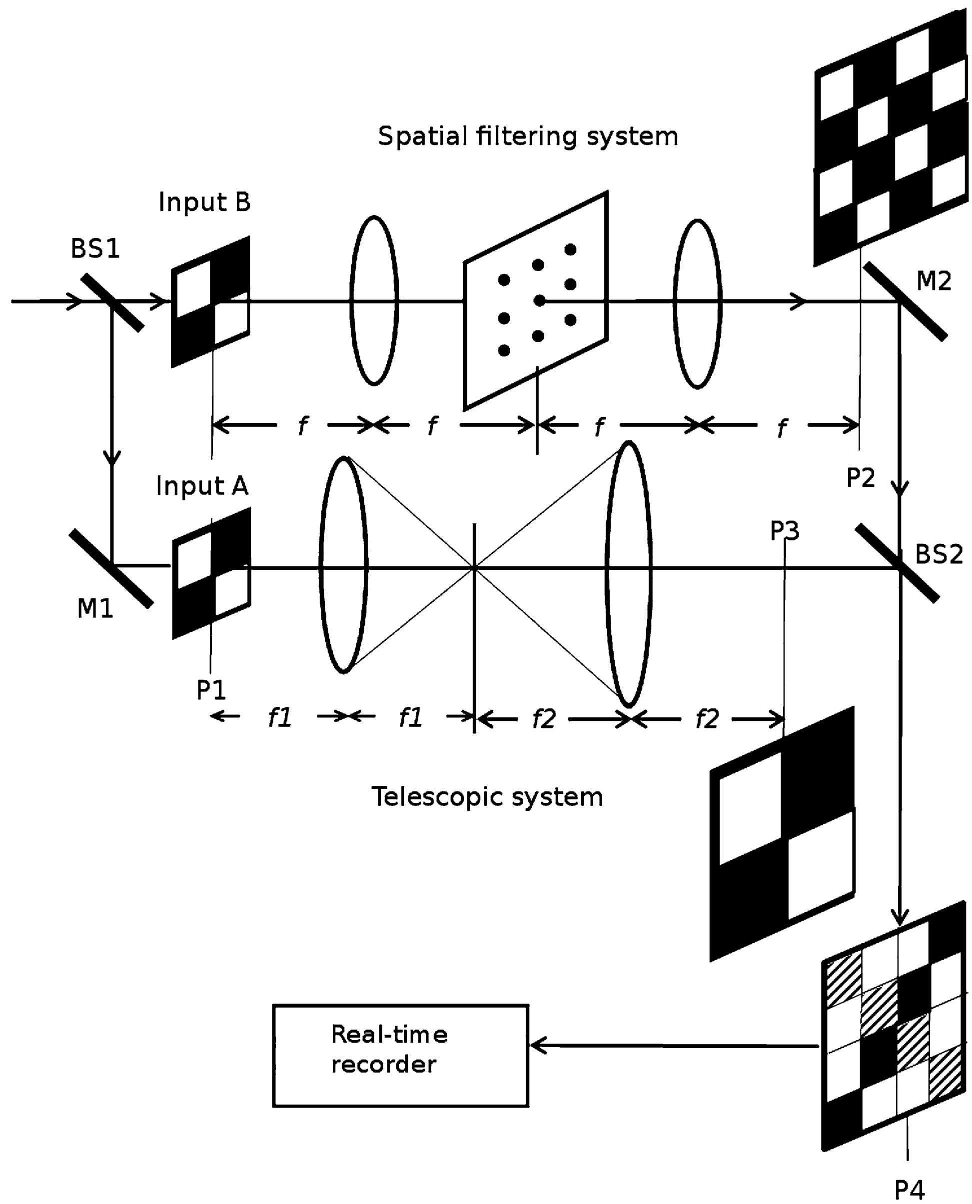
Published online: 04 July 2014

Fig. 1 Hologram recording



simplicity of representation we discuss the product of two square matrices.

Suppose we are given a binary matrix $A \in \mathbb{R}_{n\times n}$ and a binary matrix $B \in \mathbb{R}_{s\times s}$. The input matrices A and B are recorded on a photosensitive film or configured on a spatial light modulator (SLM). The implementation consists of two conceptual phases: hologram recording, and hologram reconstruction. We implement these two phases as a single real-time process. The phase of the recording is implemented either using real-time planar (thin) or volume (thick) holographic system. The reconstruction of the hologram requires different approaches when using different medium materials. The recording process is identical in both cases. Next we present the implementation of the recording phase, and two different reconstruction techniques, according to the medium used in the recording.

2.1 Recording phase

The architecture for recording the hologram of Kronecker product of two 2×2 matrices A and B is presented in Fig. 1. Input binary matrices A and B are stored by an SLM (or film) and placed on the input plane P_1. A white rectangle on the slide represents a logical 1 in the matrix, and a black rectangle on the slide represents a logical 0.

The process of recording is performed as follows. The laser beam is split into two wavefronts by the beam splitter $BS1$. The wavefront reflected from input matrix B is directed into a spatial filtering system ($4f$ correlator), that is capable of performing a duplication of the input. An array of pinholes in the filter plane of the $4f$ correlator functions as a sampling array in the Fourier plane. The dimension of the matrix cell b defines the gap between

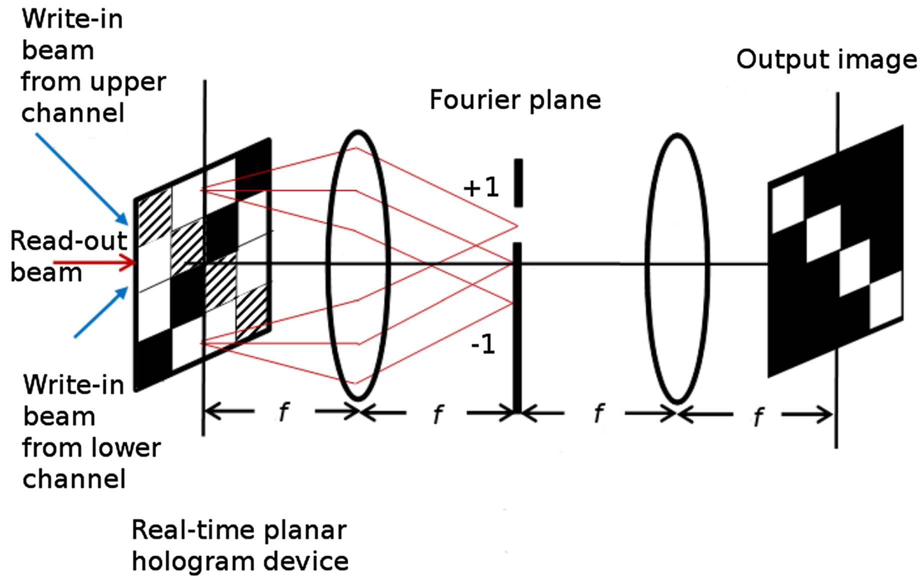


Fig. 2 Hologram reconstruction from photothermoplastic device

every two sequential pinholes to be equal to $\frac{\lambda f}{sb}$, where λ is the laser wavelength and f is the focal distance of the lenses in the channel. This gap limitation guaranties that the various duplications of the input matrix B are displayed on plane P_2 adjusted to each other, without spaces between or overlaps among every two sequential duplicates.

The wavefront reflected from the input matrix A propagates through the two lens magnification system, designed to enlarge the image of the input. The enlarged image of the input matrix A is displayed on the plane P_3. The magnification scale of the system is determined by focal length ratio $M = \frac{f_2}{f_1}$. To obtain the correct result of multiplication, f_1 and f_2 chosen so that $M = s$ (assuming the cells in A and B are of the same size).

Output beams from both systems directed by the mirror $M2$ and the beam splitter $BS2$ to the recording medium, where the interference pattern of their imposition, the hologram (plane P_4), is recorded.

2.2 Reconstruction phase

The reconstruction of the hologram is performed in real-time within the medium. Namely, on the time the hologram is recorded, an additional laser beam (read-out beam) is used by the system to reconstruct the hologram. Next, we discuss two different, well-known, techniques for real-time reconstruction of the hologram.

2.2.1 *Reconstruction from photothermoplastic device (PTPD)*

The recording medium we propose to use in this scheme is photothermoplastic device Hariharan (1996). In Fig. 2 the reconstruction of the hologram from the real-time planar hologram medium is depicted. During recording the two interfering images (output of $4f$ correlator and output of telescopic system) arrive to the recording plane with angle 2θ between the beams (Fig. 1). Whenever there is a high level of light (logic $+1$) in both images an interfering grating is obtained with fringe cycle of $\frac{\lambda}{2sin\theta}$. The PTPD is considered as a thin planar holographic medium, and therefore, the diffracted light from the recorded hologram is composed from a set of diffraction orders according to the Fourier decomposition. The read-out laser plane wave illuminating the PTPD is incoherent to the write-in beams, and therefore, does not affect the hologram transparency distribution. However, the wavefront of the read-out beam is modulated by the hologram, and thus in the Fourier plane of Fig. 2 a Fourier transform of the hologram is obtained with higher than 0 diffraction orders. Because of the grating on part of the area of the hologram, at least 3 considerable diffraction orders are expected. When only the $+1$ order (or -1 order) is allowed to pass, and all the rest of light is blocked, the output image has a high uniform level of light only where there is a grating in the hologram. Therefore the resulting image is indeed $A \oplus B$.

The output image of $A \oplus B$ might be recorded by a digital camera that can feed the input SLM for an additional cycle of duplication, if iterative multiplication is needed. Note that PTPD can be reused for further multiplications.

2.2.2 *Reconstruction by four-wave mixing setup*

We propose an additional reconstruction technique - reconstruction by four waves mixing with a photorefractive crystal (PRC) Yariv (1991). The idea is depicted in Fig. 3. The hologram is recorded exactly as described above for the

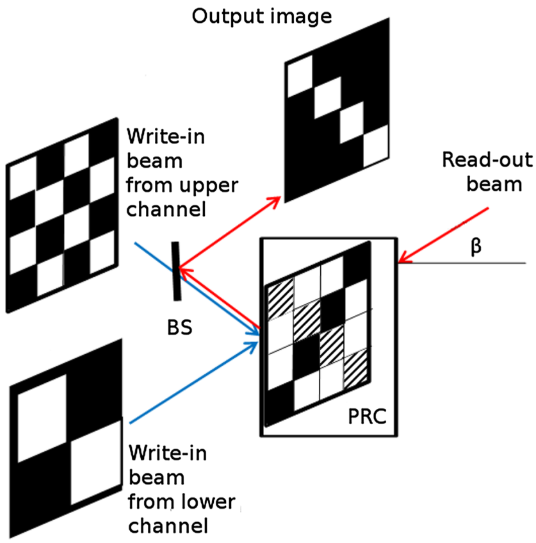


Fig. 3 Hologram recording and reconstruction by four wave mixing

PTPD. Only the read-out mechanism is different, as in that technique a volume thick hologram is recorded, in which the Bragg condition should be satisfied Yariv (1991). If the angle between the two write-in beams is 2α, and their wavelength is λ_1, then the cycle of the grating inside the PRC is $\frac{\lambda_1}{2\sin\alpha}$. According to the Bragg condition, if the wavelength of the read-out beam is λ_2 this beam should be introduced into the PRC in an angle β, which satisfies the condition of $\sin\beta = (\frac{\lambda_2}{\lambda_1})\sin\alpha$. Because the read-out beam is incoherent to the read-in beams, there is no any interaction between the read-out and the write-in beams. Bragg condition can be satisfied only at the areas of the hologram in which the recorded gratings exist, and therefore from these regions the read-out beam is diffracted out to the output image shown in Fig. 3. As before, the process of recording and reconstruction of the hologram can be performed simultaneously, and the diffracted image might be forwarded towards the camera. An advantage of this method is that the PRC can be used as a memory device Yariv (1991) in scenario with multiple Kronecker product multiplications.

3 Concluding remarks and future work

We demonstrate the capabilities of holography in computing, where unlike optical copying, that may imply a constant number of copies in each step, holography can assist in producing $O(n)$ computed copies in a single step. In particular, the computation time is not a function of n. Note, that Kronecker multiplications can be a step in a sequence of computation steps that is concluded with a single value, say, by using a concentrating lens to measure a sum of values, and hence, the complexity of the output is not proportional to the size of the Kronecker multiplications output.

We leave the exploration of additional applications of the holographic capabilities in computing, as well as the general case of Kronecker multiplication of non-binary matrices, for future research.

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