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Digital correlation holograms implemented on a joint transform correlator

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Abstract

A system of two separated computer-generated holograms termed digital correlation hologram (DCH) is proposed and demonstrated. In each hologram a different computer-generated complex function is coded. The reconstructed image is obtained as a result of a spatial correlation between the hologram's two parts. The correlation between the two sub-holograms is implemented on a modified joint transform correlator. When the double-elements hologram is displayed on the correlator input plane, and illuminated by a plane wave, a desired image is constructed on part of the output plane. Experimental results are shown and possible applications of the DCH are discussed.

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Keywords: Fourier optics; Computer-generated holograms; Image forming and processing

1. Introduction

Generally a holographic image is reconstructed as a result of wave propagation from a hologram through some medium (or optical system) characterized by a determined and parameterized kernel function. For instance, the kernel in the case of the Fresnel hologram [1] is a quadratic phase function which represents a free space medium between the hologram and the image, whereas the distance between them should satisfy the Fresnel approximation [2]. On the other hand, in the case of Fourier hologram [3] the light propagates through a linear space-variant system characterized by the Fourier transform phase function. Almost every type of a hologram can be classified according to the kernel associated with the medium (or the system) located between the hologram and the reconstructed image, or between the object and the recorded hologram.

Here we propose a significant generalization of this traditional scheme described above. The proposed hologram is divided into two different sub-holograms. One sub-hologram functions as usual as an input of some linear space invariant optical system. The other sub-hologram is used as the kernel function of the same system. The

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constructed image is obtained on part of the output plane of the system, as a cross-correlation between the two sub-holograms. In other words, between one sub-hologram and the constructed image there is a space-invariant system with an arbitrary kernel function coded inside the other sub-hologram. Both sub-holograms are synthesized by a digital computer and the output image is a result of correlation process between them. Therefore we coin this hologram as digital correlation hologram (DCH). There are two new elements in the present study. This is the first report to present an experimental spatial correlation between two computer-generated holograms which yields a meaningful output image. The idea of correlation between two holograms yielding a meaningful image has already appeared in different versions [4–8]. However, in all these references, the optical correlators have been simulated by digital computers. Here, for the first time, we report on a real optical experiment implementing this idea. The second new element in the present work is the use of computer generated holograms displayed on real positive masks instead of pure phase masks suggested in [4–8]. It is true that using phase holograms makes the optical setup simpler, more efficient and more suitable for security applications. However, pure phase holograms are much more expensive than amplitude holograms such that none of the researchers in [4–8] have optically implemented their proposed systems. With all disadvantages of amplitude holograms, they enable performing the correlation between two holograms in a simple way, using simple, low-cost, SLMs.

There are at least two main methods to perform spatial correlation between any two arbitrary masks. One is known as VanderLugt correlator (VLC) [3] and the other is the joint transform correlator (JTC) [9]. In principle, both configurations can be used as a platform to carry out the DCH. However, because of the less restrictive alignment requirements between the two sub-holograms in the JTC in comparison with the VLC, we choose the JTC (or actually some modified version which will be described in the following) as the correlator for our first demonstration of the DCH. In the JTC, the lateral distance between the two sub-holograms can be changed within a reasonable tolerance, without changing the shape of the output image. Only the image location on the output plane is changed according to the relative distance between the two holographic masks. On the other hand, in the VLC a slight mutual shift between the filter mask and the light distribution, coming from the input mask, considerably modifies the correlation results.

Two complex functions are coded into the DCH. To get maximum diffraction efficiency it is desired, but not required, to use two pure phase functions. One of these functions is chosen once as a random phase function and the other one is computed by an iterative algorithm called JTC-based projection onto-constraint sets (POCS) [10]. The different conditions on the two functions are desired because of the possible applications for the DCH, as it is discussed later. There is, however, a security system suggested in [7] where both functions are synthesized by some iterative algorithm, and both functions can still be coded into the DCH scheme. Nevertheless, in the present study we continue synthesizing iteratively only one phase function. A complete description of the computation process of these phase functions is given in [6], while here we only briefly summarize it.

The paper is organized as follows. DCH encoding and image construction by correlation operation are described in Section 2. In Section 3, experimental optical and computer simulation results are shown. Finally, our conclusions and remarks are given in Section 4.

2. Coding and construction of digital correlations holograms

The POCS algorithm is implemented by a digital procedure based on simulating a JTC, in which two phase functions are transformed back and forth between the input and output planes. Appropriate constraints are employed on both planes until the algorithm converges, in the sense that the error between the desired and the obtained image is minimal. The constraints on the JTC input plane are expressed by the need to get two separated, size-limited, phase functions, one is randomly determined once before the first iteration and the other is updated every iteration. The constraint on the JTC output plane reflects the goal to get on part of the plane an intensity pattern close as much as possible to some predefined image. After completing the iterative procedure, the computer has in its memory two complex functions designed for the specific task of constructing a desired image from the cross-correlation between these two functions. Among the two functions, the random one plays the role of a generalized medium between the other function and the reconstructed output image.

The present experimental demonstration is divided into three stages: (1) computation of two complex functions by the JTC-based POCS, as described in [6] and briefly above. (2) Coding the complex functions as a DCH. (3) Construction of the desired image from the DCH in a modified JTC configuration.

We now continue the description from the point that the POCS algorithm has yielded two phase functions $s(x,y) = \exp[i\phi(x,y)]\mathrm{rect}(x/A, y/B)$ and $r(x,y) = \exp[i\varphi(x,y)]\mathrm{rect}(x/A, y/B)$, where (A,B) are the sub-holograms dimensions and $\mathrm{rect}(\cdot)$ represents the rectangle function. We assume that the square magnitude of the correlation between s and r is close enough to some desired image. In case one has an optical medium which can fully modulate the wavefront's phase, it is preferred to display s and r directly on such a transparency, and correlate between them as is described in [6]. However, a good phase transparency is still rarely available and more expensive than any amplitude

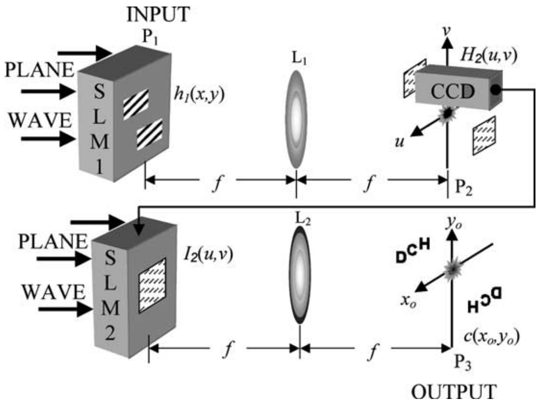

Fig. 1. The JTC used for the experimental demonstration.

transparency. In this work we choose to demonstrate this type of hologram on an amplitude transparency, while postponing the more efficient phase holograms for future work. The two functions are coded into positive, real transparencies and displayed on the input plane P_1 of the JTC (see Fig. 1) as follows:

$$
\begin{aligned}
h_1(x,y) = {} & \{1+\cos[2\pi(\alpha x+\beta y)+\phi(x,y)]\} \\
& \times \mathrm{rect}\left(\frac{x}{A},\frac{y}{B}\right) * \delta(x-a,y-b) \\
& + \{1+\cos[2\pi(\alpha x+\beta y)+\varphi(x,y)]\} \\
& \times \mathrm{rect}\left(\frac{x}{A},\frac{y}{B}\right) * \delta(x+a,y+b),
\end{aligned}
\tag{1}
$$

where $*$ denotes the convolution operation, δ is the Dirac delta function, (α,β) are the carrier spatial frequencies of the holograms, and the two holograms are displayed around the points (a,b) and $(-a,-b)$. $h_1(x,y)$ is a positive real holographic function which, like an usual hologram, contains the phase information $\phi(x,y)$ and $\varphi(x,y)$.

The two holograms are illuminated by a plane wave and jointly Fourier transformed onto plane P_2 by the lens L_1. The complex amplitude on plane P_2 is

$$
\begin{aligned}
H_2(f_x,f_y) = {} & \exp\left[-i2\pi\left(af_x+bf_y\right)\right] \\
& * \left[\sin\left(A\pi f_x\right)\sin\left(B\pi f_y\right)/\pi^2 f_x f_y\right] \\
& + \frac{1}{2}\exp\left\{-i2\pi\left[(f_x-\alpha)a\right.\right. \\
& \left.\left. + \left(f_y-\beta\right)b\right]\right\} S\left(f_x-\alpha, f_y-\beta\right) \\
& + \frac{1}{2}\exp\left\{-i2\pi\left[(f_x+\alpha)a\right.\right. \\
& \left.\left. + \left(f_y+\beta\right)b\right]\right\} S^*\left(f_x+\alpha, f_y+\beta\right) \\
& + \exp\left[i2\pi\left(af_x+bf_y\right)\right] \\
& * \left[\sin\left(A\pi f_x\right)\sin\left(B\pi f_y\right)/\pi^2 f_x f_y\right] \\
& + \frac{1}{2}\exp\left\{i2\pi\left[(f_x-\alpha)a\right.\right. \\
& \left.\left. + \left(f_y-\beta\right)b\right]\right\} R\left(f_x-\alpha, f_y-\beta\right) \\
& + \frac{1}{2}\exp\left\{i2\pi\left[(f_x+\alpha)a\right.\right. \\
& \left.\left. + \left(f_y+\beta\right)b\right]\right\} R^*\left(f_x+\alpha, f_y+\beta\right),
\end{aligned}
\tag{2}
$$

where $(f_x,f_y) = (u/\lambda f, v/\lambda f)$, (u,v) are the spatial coordinates of plane P_2, λ is the wavelength of the plane wave, f is the focal length of lens L_1,

* denotes the complex conjugate and the functions S and R are the Fourier transforms of s and r, respectively.

In our modified JTC only part of the joint spatial spectrum around the point $(f_x, f_y) = (\alpha, \beta)$ is observed by a CCD having a limited observation frame of the size $\Pi_x \times \Pi_y$. This size of the frame is exactly equal to the bandwidth of the functions s and r, or in other words, to the size of the functions S and R. Thus, from the entire terms of Eq. (2), only the second and the fifth terms are recorded by the CCD. These terms are separated from the other terms if the inequality $\sqrt{\alpha^2 + \beta^2} \geqslant (A^2 + B^2)^{-1/2} + \sqrt{\Pi_x^2 + \Pi_y^2}/2$ is satisfied. The CCD also changes the origin of the coordinates such that its new coordinates satisfy the relation: $(\tilde{f}_x, \tilde{f}_y) = (f_x - \alpha, f_y - \beta)$. The intensity distribution recorded by the CCD is

$$\begin{aligned} I_2(\tilde{f}_x, \tilde{f}_y) &= \left| H_2\left(\tilde{f}_x, \tilde{f}_y\right) \mathrm{rect}\left(\frac{\tilde{f}_x}{\Pi_x}, \frac{\tilde{f}_y}{\Pi_y}\right) \right|^2 \\ &= \left| \exp\left[-\mathrm{i}2\pi\left(a\tilde{f}_x + b\tilde{f}_y\right)\right] S\left(\tilde{f}_x, \tilde{f}_y\right) \right. \\ &\quad \left. + \exp\left[\mathrm{i}2\pi\left(a\tilde{f}_x + b\tilde{f}_y\right)\right] R\left(\tilde{f}_x, \tilde{f}_y\right) \right|^2 \\ &= \left| S\left(\tilde{f}_x, \tilde{f}_y\right) \right|^2 + \left| R\left(\tilde{f}_x, \tilde{f}_y\right) \right|^2 \\ &\quad + \exp\left[-\mathrm{i}4\pi\left(a\tilde{f}_x + b\tilde{f}_y\right)\right] S\left(\tilde{f}_x, \tilde{f}_y\right) \\ &\quad \times R^*\left(\tilde{f}_x, \tilde{f}_y\right) + \exp\left[\mathrm{i}4\pi\left(a\tilde{f}_x + b\tilde{f}_y\right)\right] \\ &\quad \times S^*\left(\tilde{f}_x, \tilde{f}_y\right) R\left(\tilde{f}_x, \tilde{f}_y\right). \end{aligned} \tag{3}$$

The intensity pattern of Eq. (3) is displayed on a spatial light modulator (SLM) indicated as SLM2 in Fig. 1. This SLM is illuminated by a plane wave such that the Fourier transform of the SLM transparency is obtained on the back focal plane P_3 of lens L_2. Assuming the focal length of L_2 is identical to that of L_1, the Fourier transform of $I_2(\tilde{f}_x, \tilde{f}_y)$ is,

$$\begin{aligned} c(x_0, y_0) = s(x_0, y_0) \otimes s(x_0, y_0) + r(x_0, y_0) \otimes r(x_0, y_0) \\ + [s(x_0, y_0) \otimes r(x_0, y_0)] \\ * \ \delta(x_0 - 2a, y_0 - 2b) \\ + [r(x_0, y_0) \otimes s(x_0, y_0)] \\ * \ \delta(x_0 + 2a, y_0 + 2b), \end{aligned} \tag{4}$$

where $\otimes$ denotes the correlation operation and (x_0, y_0) are the coordinates of the output plane P_3.

It is evident from Eq. (4) that three spatially separated diffraction orders can be observed. The first term is the diffraction zero-order at the vicinity of the origin of the output plane. This term represents the autocorrelation of the two original complex functions. The second and the third terms at the points $(\pm 2a, \pm 2b)$ correspond to the cross-correlations between the two complex functions s and r. According to the output results of the POCS algorithm, these cross-correlations approximately produce the desired image. Therefore we can retrieve the coded image by reading it from the vicinity of the point $(2a, 2b)$, or $(-2a, -2b)$.

Note that although in our system we use conventional Fourier transforms between the input to output planes, the output image is obtained as a cross-correlation between two holograms. Thus, the DCH is not any familiar hologram but a combination of two holograms, one is used as the kernel and the other is the input mask of the correlator. Also note that our JTC is not the conventional one used for many schemes of pattern recognition [9]. In the present JTC we mask the joint spectral plane, and process only the first diffraction order on this plane. This procedure enables us to receive a cross-correlation between effectively two phase functions, although there are actually two real positive functions on the input plane.

3. Experimental results

To demonstrate the proposed technique, we experimentally performed the three stages of synthesizing and testing the proposed DCH. In the first stage the desired image (containing the letters DCH) was encrypted into two phase functions by the iterative JTC-based POCS algorithm [6]. Since the SLM used in this study (CRL, Model XGA3) can modulate the transferred light with positive gray-tones only, the complex functions were coded into a positive real transparency according to Eq. (1). The obtained double-sections hologram was displayed on the input plane of the JTC. Each of the two sub-holograms covered only 110×225 pixels out of 600×600 pixels, as shown

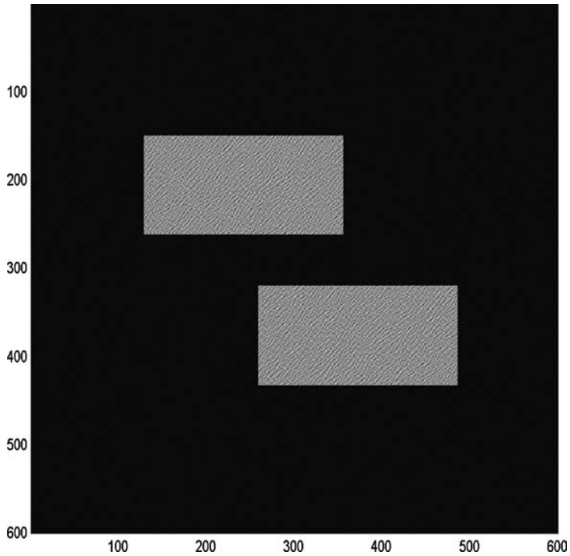

Fig. 2. Two sub-holograms on the JTC input plane.

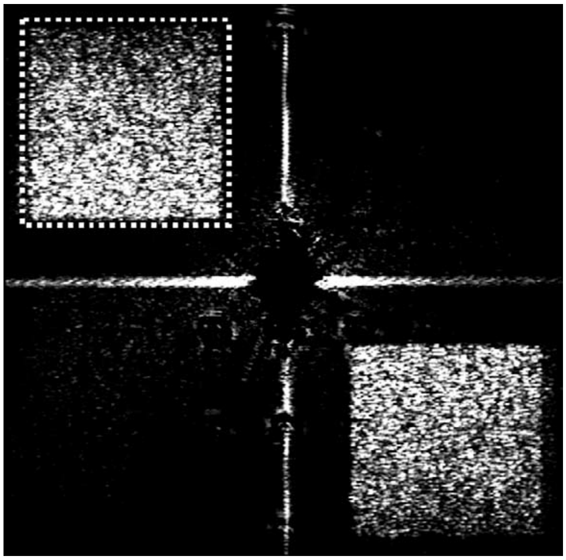

Fig. 3. Three diffraction orders on the joint power spectrum plane.

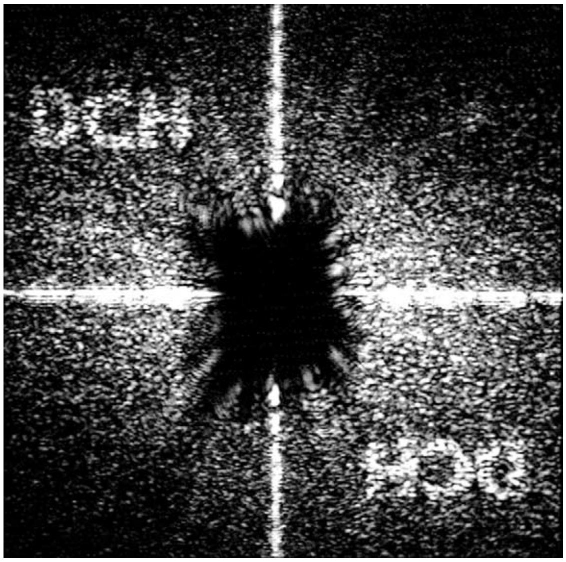

Fig. 4. Experimental result of the desired image constructed on the JTC correlation plane.

in Fig. 2. A collimated beam from He–Ne laser with $\lambda = 633$ nm illuminated SLM1 and created a diffraction pattern of the joint transform power spectrum on the back focal plane of lens L_1 ($f = 400$ mm). The entire intensity of the joint spectrum on plane P_2 is shown in Fig. 3. Two first diffraction orders on either diagonal side of the zero order can be observed. The center of the zero order is blocked in Fig. 3 for a clearer visualization. Since only one of these orders is needed, we recorded only the intensity distribution inside the frame denoted by the dashed line. The size of this frame was 576×768 pixels, and the pattern inside this frame was displayed on SLM2. Finally, after another Fourier transform by lens L_2 ($f = 400$ mm), the correlation plane was obtained as shown in Fig. 4. The center of the zero order is also blocked in Fig. 4 for a clearer visualization. The three orders of the correlation plane and the two images of the letters DCH in the first diffraction orders can be clearly seen, demonstrating that the proposed method has reached its goal.

There are two main sources for the speckle noise shown clearly on the constructed image. One reason for the noise is because of the POCS algorithm. This iterative optimization algorithm reaches to a suboptimal solution in the sense that the final hologram does not yield an image closest to the desired image. These days, we are in a process of examining alternative algorithms for the same task. Of course, in any algorithm, increasing the holograms' number of pixels increases the degrees of freedom, and thus the constructed image's quality is improved. However our available SLM

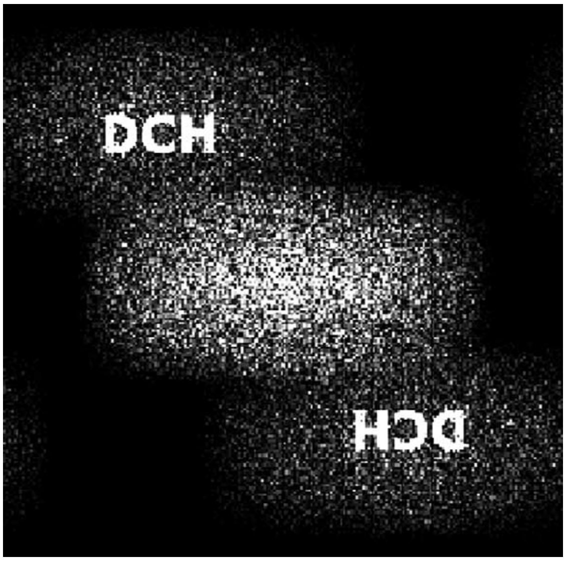

Fig. 5. Simulation result of the desired image constructed on the JTC correlation plane.

size limits us with the maximum pixels that we can use.

The second source of noise comes from the optical construction of the DCH. There are several points along the optical JTC in which the noise is added. The input holograms are both displayed on SLM that suffers from non-uniformity, brightness quantization and structure of partially blocked pixels. Since the spatial spectrum is displayed on the same type of SLM, the distortions due to the non-ideal nature of the SLM are introduced twice along the process. In addition, the CCD records the spatial spectrum in a non-ideal way. All these distortion sources are accumulated to the noisy result shown in Fig. 4. For comparison, Fig. 5 shows the same constructed image obtained from the digital simulation of the process. Although, there is some noise on this last image due to the sub-optimality of the POCS, the quality of the constructed letters is better then the optical result in Fig. 4.

4. Discussion and conclusions

We have proposed and demonstrated a system of computer-generated holograms termed DCH. The main purpose of the method is to construct on the output plane a complex function in a conditional way. This means that two holograms are cross-correlated with each other in order to create the complex function. Although there are infinite pairs of holograms that can generate the same correlation function, one needs to know a priori the distribution of one of the holograms and the correlation function in order to compute the correct solution of the other hologram. Of course one can compute a single hologram which under a proper illumination can construct the same output correlation. However, the use of two holograms enables several applications that cannot be achieved by a single hologram. Some of these applications are discussed in the next paragraphs. In principle the idea of DCH can be extended to more than two holograms, but such an idea will complicate the computing process and its usefulness is unclear.

As an encryption system, the image in the correlation plane is considered as the information that we wish to encrypt. The sub-hologram, modified during the POCS algorithm, is the encrypted data, whereas the other unchanged sub-hologram is employed as the decoder for this encrypted data. When this same decoder is used in several encryption procedures, it can become a general decoder for many encrypted holograms. Placing the two sub-holograms together in the input plane of the JTC is the only way to reconstruct the original image. However, note that the present work is not about encryption but about an unconventional kind of holograms. Encryption can be one possible application for this hologram. Other applications maybe suitable as well, and some are mentioned in the next paragraphs. Therefore, it is out of the scope of this article to analyze the proposed encryption and to compare it with other encryption methods.

As a security system the two sub-holograms are considered as a key–lock pair. The POCS-generated sub-hologram is a kind of a key, say one of many, whereas the other sub-hologram is used as a lock, which is suitable for many possible keys. The predefined image is built up on the correlation plane only if the true key appears in the input. The form of the constructed image provides the required information on the specific key among the entire population of keys. Moreover, knowing the desired output pattern and the distribution of one hologram is not enough to reveal the values of the other hologram. The phase distribution of the constructed image (which cannot be recorded by any ordinary intensity detector) is also needed in order to expose the unknown hologram. Hence, the degree of security of this system is higher than similar types of security systems [11]. However, since we use here real positive masks instead of phase masks, the degree of security is harmed in comparison with system of two phase holograms proposed in [5,6].

Another application for the DCH can be as a tool to control any displayed information. Only users with the appropriate private key (one of the sub-holograms) can view the broadcasted public data (the other sub-hologram). We realize that the success of this application is conditioned by a significant improvement in the quality of the constructed image.

Finally, we have initial indications that the present concept can be combined with the recently invented hiding information method called Concealogram [12,13]. The difference between the DCH and the Concealogram is that in the latter, the binary mask is a figurative meaningful picture instead of a meaningless collection of dots as the DCH. Thus, the JTC described above can be used to reveal a watermark or steganography data embedded in the Concealogram. In this case, the Concealogram is displayed on the input plane instead of one of the sub-holograms.

In conclusion, the proposed method provides the advantages of simple design and easy alignment with a reasonable degree of security, and therefore it has a promising future in its practical applications.

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