

Compressive Fresnel Holography

Yair Rivenson, *Student Member, IEEE*, Adrian Stern, *Member, IEEE*, and Bahram Javidi, *Fellow, IEEE*

Abstract—Compressive sensing is a relatively new measurement paradigm which seeks to capture the “essential” aspects of a high-dimensional object using as few measurements as possible. In this work we demonstrate successful application of compressive sensing framework to digital Fresnel holography. It is shown that when applying compressive sensing approach to Fresnel fields a special sampling scheme should be adopted for improved results.

Index Terms—Compressive imaging, Fresnel digital holography, variable density sampling.

I. INTRODUCTION

THE recently introduced theory of compressive sensing (CS) [1], [2] has attracted the interest of theoreticians and practitioners. CS seeks to minimize the collection of redundant data in the acquisition step, in contrast to common framework of first collecting as much data as possible and then discarding the redundant data by digital compression techniques.

One of the fields that benefits from CS theory is compressive imaging. Compressive imaging is motivated by the fact that most natural images are highly redundant; as experienced with digital compression (e.g., JPEG and JPEG2000). Several works were done already in the field, amongst them we might note [3]–[6].

Recently, CS was introduced in the realm of holography [7]. In [7] reconstruction techniques used in CS were applied on data captured with Gabor holography to reconstruct three-dimensional objects that are sparse in the spatial domain. In this paper, we will demonstrate the application of CS framework with Fresnel digital holography. Objects that are sparse in arbitrary domain (e.g., some wavelet domain) will be considered. In Section III it is explained explain why Fresnel holography fits well to CS framework. However, as shown in Section IV, in order implement optimized compressive digital holography, the sampling scheme needs to be adapted to the special mechanism of Fresnel holography.

II. COMPRESSIVE SENSING

In order to describe CS, let us think of an object \mathbf{f} described by an N -dimensional real valued vector (in case that the object

Manuscript received December 03, 2009; revised January 26, 2010; accepted January 26, 2010. Date of publication May 03, 2010; date of current version September 10, 2010.

Y. Rivenson is with the Department of Electrical and Computer Engineering, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel (email: rivenson@ee.bgu.ac.il).

A. Stern is with the Electro-Optics Engineering Department, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel (email: stern@bgu.ac.il).

B. Javidi is with the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, CT 06269-2157 USA (e-mail: bahram@engr.uconn.edu).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/JDT.2010.2042276

represents an image of N pixels, \mathbf{f} is a one-dimensional vector obtained by rearranging the image in a lexicographic order) being projected (imaged), to \mathbf{g} an M -dimensional vector. One can also think of M as the number of detector pixels. We are interested in the case of $M < N$, meaning that the captured image is undersampled in the conventional sense. Often, the subsampling is done by picking M out of N measurements uniformly at random [1], [2]. The sensing process can be written as (1)

$$\mathbf{g} = \Phi \mathbf{f} \quad (1)$$

where Φ is an M by N matrix. CS relies on two guiding principles. The first one is signal sparsity; we assume that the signal \mathbf{f} can be sparsely represented in some arbitrary orthonormal basis Ψ (e.g., wavelet, or DCT). Thus, α is the K -sparse representation of image \mathbf{f} projected on Ψ^T , meaning, that α has only K nonzero terms.

Another principle that CS requires is the incoherence (dissimilarity) between the sensing and sparsifying operators. This measurement is known as the mutual coherence, μ , and is defined as

$$\mu = \sqrt{N} \max_{ij} |\langle \varphi_i, \psi_j \rangle| \quad (2)$$

where φ_i, ψ_j denote the column vector of Φ and Ψ , respectively, and N is the length of the column vector. The mutual coherence is bounded by $1 \leq \mu \leq \sqrt{N}$ [1]. Many pairs of bases which exhibit low coherence and were reported extensively in the literature [2]. Amongst them we wish to note the Fourier-Delta impulses ($\mu = 1$), Fourier-wavelet ($\mu \approx \sqrt{2}$) and Gaussian random projections, which are universally incoherent with any sparsifying basis (with $\mu \approx \sqrt{2 \log n}$).

CS theory suggests that a signal (image) measured with sensing operator, Φ , can actually be recovered by l_1 -norm minimization; the estimated coefficients vector $\hat{\alpha}$ is the solution of the convex optimization program [2]:

$$\hat{\alpha} = \min_{\alpha} \|\alpha\|_1 \quad \text{subject to} \quad \mathbf{g} = \Phi \Psi \alpha \quad (3)$$

where $\|\hat{\alpha}\|_1 = \sum_i |\alpha_j|$ is the l_1 norm which measures the number of nonzero entries of the coefficient vector α . One way of guaranteeing recovery via the l_1 -norm minimization of a K -sparse signal \mathbf{f} is by taking M measurements satisfying

$$M \geq C \cdot \mu^2(\Phi, \Psi) \cdot K \cdot \log(N) \quad (4)$$

where C is some small positive constant.

III. COMPRESSIVE FRESNEL HOLOGRAPHY

Let us consider the one-dimensional free-space propagation formula in the Fresnel approximation, which relates the complex values of a propagating wave, measured in a plane perpen-

pendicular to the direction of propagation and separated by a distance z

$$U_z(x) = A \exp \left\{ \frac{j\pi}{\lambda z} x^2 \right\} \cdot \int U_{in}(\xi) \exp \left\{ \frac{j\pi}{\lambda z} \xi^2 \right\} \exp \left\{ \frac{-j2\pi}{\lambda z} x\xi \right\} d\xi, \quad (5)$$

where λ is the wavelength of the light wave and A is a multiplicative constant. If we wish to write the latter equation in discrete form (sampling uniformly on a Cartesian grid) by defining $x = n\Delta_d$ and $\xi = m\Delta_o$, and changing the integral to summation, we get

$$\begin{aligned} U_z(n) &= A \exp \left\{ \frac{j\pi\Delta_d^2}{\lambda z} n^2 \right\} \\ &\cdot \sum_m U_{in}(m) \exp \left\{ \frac{j\pi\Delta_o^2}{\lambda z} m^2 \right\} \exp \left\{ \frac{-j2\pi\Delta_d\Delta_o}{\lambda z} mn \right\} \\ &= A \mathbf{Q}_z \mathbf{F}_{1/\lambda z} \mathbf{Q}_z \mathbf{u}_{in} \end{aligned} \quad (6)$$

where the last equality is the matrix-vector form of the discrete Fresnel transform. \mathbf{u}_{in} is a one-dimensional vector representing the object, \mathbf{Q}_z is a diagonal matrix whose elements are the discrete quadratic phase elements of the Fresnel transform, and $\mathbf{F}_{1/\lambda z}$ is a scaled version of the discrete Fourier transform. It is noted that discrete approximation (6) is not valid for small propagation distances z [8].

The Fresnel diffraction field of the object can be recorded by means of digital holography [9]–[13]. Application of CS scheme on Fresnel digital holograms is attractive for a couple of reasons. The first is that the entire complex field is available. This is in contrast to most imaging applications where we only detect variations in the squared modulus of the propagating optical field. This, in turn, implies that the sensing operator has strictly positive entries. Strictly positive sensing operators are most of the times unsuitable for CS applications, due to their relatively large μ [14]. Second, the Fresnel and Fourier transforms are closely related. As z increases, \mathbf{Q}_z becomes the identity canonical basis, and the Fresnel transform becomes the Fourier transform (yielding the Fraunhofer approximation). If we take a closer look on the quadratic phase factor in (5) we see that the Fresnel approximation behaves like the Fourier transform in a small area by the origin. This infers that more samples should be taken around the origin in order to take full advantage of the incoherence properties of the Fourier-wavelet transform. Our nonuniform (variable density) sampling scheme is described in Section IV.

In this work, we applied the CS on phase-shift digital holography (see, for example, [10]–[12], [15]) which is an efficient holographic method in terms of space-bandwidth utilization [16]. Phase shift digital holography utilizes a Mach–Zehnder setup to interfere the Fresnel diffraction field of the object with reference field impinging on-axis. By taking exposures with multiple phase shifts of the reference beam the entire complex Fresnel field information is captured. A digital hologram was captured at $z = 583.4$ mm, and $\lambda = 514.5$ nm with a 2044×2024 pixel CCD with pixel size $9 \mu\text{m}$. We have downsampled the image using a variable density sampling scheme

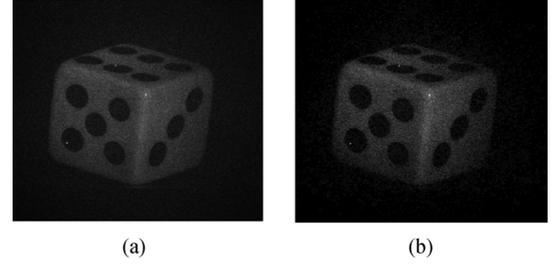


Fig. 1. (a) Original image, reconstructed from 100% samples. (b) The same image reconstructed from 25% of the measurements using variable sampling scheme, with region of interest PSNR of 33.82.

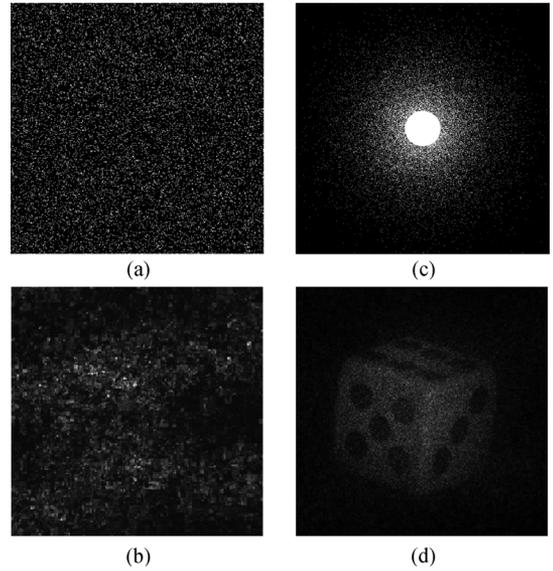


Fig. 2. (a) 8% uniformly random Sampling scheme. (b) Image reconstructed from 8% uniformly random subsampling shown in (a). (c) Variable sampling scheme with 8% sub-sampling. (d) Image reconstructed from 8% scheme shown in (c), with region of interest PSNR of 23.78.

described in next section. Basically, more samples are taken at the center and gradually less towards the margins of the hologram. Due to the very intense speckle noise the reconstruction was carried using total variation (TV) optimization

$$\min \|\Psi \mathbf{u}_{in}\|_1 + \alpha TV(\mathbf{u}_{in}) \text{ s.t. } \|\mathbf{y} - \mathbf{Q}_z \mathbf{F}_{1/\lambda z} \mathbf{Q}_z \mathbf{u}_{in}\|_2 < \varepsilon. \quad (7)$$

where

$$TV(x) = \sum_{i,j} \sqrt{(x_{i+1,j} - x_{i,j})^2 + (x_{i,j+1} - x_{i,j})^2}. \quad (8)$$

The minimization of the ℓ_1 norm tends to find a sparse solution, while the ℓ_2 norm insures data consistency. The measured signal is denoted by \mathbf{y} , ε sets the estimation of the noise level, and Ψ , is the sparsifying operator. Usinage of the TV functional is well-known in signal and image processing for its tendency to suppress spurious high-frequency features [17]. Fig. 1, shows the reconstruction results from only 25% of the hologram points with the nonuniform random sampling scheme. Fig. 2 demonstrates the advantage of the variable density sub-sampling scheme over the uniformly one, commonly used in CS [1]–[4].

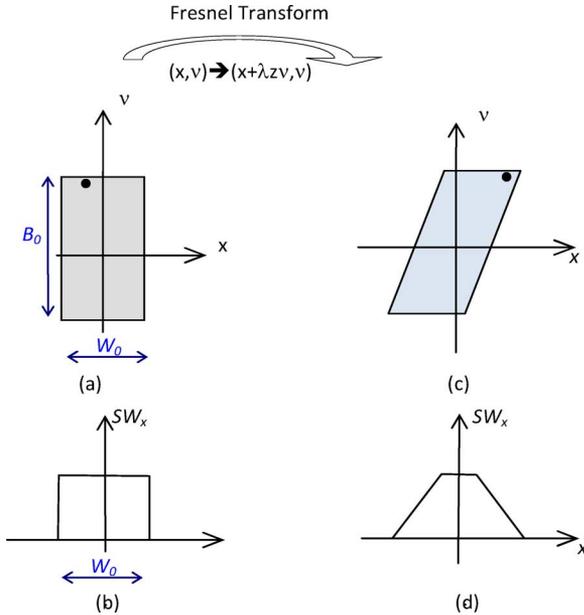


Fig. 3. (a) Support of the object field in the joint spatial (x)—spatial frequency domain (ν), e.g., Wigner domain; (b) SW_x spatial information density along the x (object) dimension; (c) the support of the Fresnel transformed field in the joint spatial-spatial frequency domain; and (d) SW_x spatial information density along the x (measurement) dimension.

IV. VARIABLE DENSITY SUB-SAMPLING OF THE FRESNEL TRANSFORM

The motivation for the variable sampling density scheme appears evident from the examination of the effect of the Fresnel transform on the joint spatial-spatial frequency distribution (phase-space distribution) of the object field. Spatial-spatial frequency representations, such as Wigner–Ville distribution (WVD), of the optical fields were found particularly useful to holography exploration [16], [18].

Let us consider a finite object with width W_0 and essential bandwidth B_0 . Its support in the WVD domain is a rectangle as depicted in Fig. 3(a). The area of the support of the WVD, denoted by SW , is proportional to the number of spatial degrees-of-freedom [19]. In the case of Fig. 3(a), $SW = W_0 \cdot B_0$ which is the value predicted by the classical space-bandwidth product definition, representing the total spatial information of the object field. The number of degrees of freedom per dimension unit dx , denoted SW_x , is constant in Fig. 3(a), meaning that the spatial information is spread uniformly along the object x axis [see Fig. 3(b)]. Now let us examine the WVD of the Fresnel transform of the object field. The Fresnel transform is expressed in the WVD by a shearing operation as shown in Fig. 3(b) [16], [18], [19]. It is evident from Fig. 3(c) that the area is not uniformly spread along the measurement x dimension, rather SW_x in the transformed plane has the distribution shown in Fig. 3(c). It can be seen that more spatial information is concentrated at the center and that the average spatial information density decreases as moving away from the origin. This implies higher sampling rate should be used in this region.

In our experiments we have used the Haar–Wavelet transform as the sparsifying basis, Ψ . Randomly sampling with sampling density of the shape in Fig. 3(d) has the advantages of the low coherence of the Fourier-wavelet basis at coarse scales, where

the signal tends to be less sparse, and the universality of the Gaussian random projections sensing [17] in fine scales, where the signal tends to be more sparse.

V. CONCLUSION

We have demonstrated that Fresnel DH suits well in the framework of compressive sensing. However, conventional CS uniform random sampling approach needs to be fitted to the Fresnel holographic model by applying variable density sub-sampling scheme. Using this scheme we have demonstrated satisfying reconstruction from only 25% and 8% samples of the Fresnel hologram.

ACKNOWLEDGMENT

The authors wish to thank E. Tajahuerce for the use of his recorded Fresnel field data.

REFERENCES

- [1] E. Candès and J. Romberg, "Sparsity and incoherence in compressive sampling," *Inverse Problems*, vol. 23, no. 3, pp. 969–985, 2007.
- [2] E. Candès and M. Wakin, "An introduction to compressive sampling," *IEEE Signal Process. Mag.*, vol. 25, no. 2, pp. 21–30, Mar. 2008.
- [3] A. Stern and B. Javidi, "Random projections imaging with extended space-bandwidth product," *J. Display Technol.*, vol. 3, no. 3, pp. 315–320, Sep. 2007.
- [4] R. Fergus, A. Torralba, and W. T. Freeman, "Random lens imaging," Computer Science and Artificial Intell. Lab., MIT, Cambridge, MA, Tech. Rep. MIT-CSAIL-TR-2006-058, 2006.
- [5] A. Stern, "Compressed imaging with linear sensors," *Opt. Lett.*, vol. 32, no. 21, pp. 3077–2079, Nov. 2007.
- [6] R. Marcia and R. Willett, "Compressive coded aperture superresolution image reconstruction," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, ICASSP, Mar. 31–Apr. 4, 2008.
- [7] D. Brady, K. Choi, D. Marks, R. Horisaki, and S. Lim, "Compressive holography," *Opt. Express*, vol. 17, pp. 13040–13049, 2009.
- [8] D. Mas, J. Garcia, C. Ferreira, L. M. Bernardo, and F. Marinho, "Fast algorithms for free-space diffraction patterns calculation," *Opt. Commun.*, vol. 164, pp. 233–245, Jun. 1999.
- [9] F. Dubois, L. Joannes, and J.-C. Legros, "Improved three-dimensional imaging with a digital holography microscope with a source of partial spatial coherence," *Appl. Opt.*, vol. 38, no. 34, pp. 7085–7094, 1999.
- [10] Y. Zhang, G. Pedrini, W. Osten, and H. J. Tiziani, "Reconstruction of in-line digital holograms from two intensity measurements," *Opt. Lett.*, vol. 29, pp. 1787–1789, 2004.
- [11] P. Ferraro, S. De Nicola, G. Coppola, A. Finizio, D. Alfieri, and G. Pierattini, "Controlling image size as a function of distance and wavelength in Fresnel-transform reconstruction of digital holograms," *Opt. Lett.*, vol. 29, pp. 854–856, 2004.
- [12] E. Cuche, F. Bevilacqua, and C. Depeursinge, "Digital holography for quantitative phase-contrast imaging," *Opt. Lett.*, vol. 24, pp. 291–293, 1999.
- [13] Y. Frauel, T. Naughton, O. Matoba, E. Tajahuerce, and B. Javidi, "Three dimensional imaging and display using computational holographic imaging," *Proc. IEEE*, vol. 94, no. . pp. 636–654, 2006.
- [14] A. M. Bruckstein, M. Elad, and M. Zibulevsky, "A non-negative and sparse enough solution of an underdetermined linear system of equations is unique," *IEEE Trans. Inf. Theory*, vol. 54, no. 11, pp. 4813–4820, Nov. 2008.
- [15] I. Yamaguchi and T. Zhang, "Phase-shifting digital holography," *Opt. Lett.*, vol. 22, pp. 1268–1270, 1997.
- [16] I. Moon, M. Daneshpanah, B. Javidi, and A. Stern, "Automated three dimensional identification and tracking of micro/nanobiological organisms by computational holographic microscopy," *Proc. IEEE*, vol. 97, no. 6, pp. 990–1010, Jun. 2009.
- [17] M. Lustig, "Sparse MRI," Ph.D. dissertation, Dept. Elect. Eng., Stanford Univ., Palo Alto, CA, 2008.
- [18] A. Lohmann, M. E. Testorf, and J. Ojeda-Castaneda, "Holography and the Wigner function," in *The Art and Science of Holography, A Tribute to Elmhett Leith and Yuri Denisyuk*, J. H. Caulfield, Ed., WA: SPIE Press, 2004, ch. 8, pp. 129–144.
- [19] Z. Zalevsky, D. Mendlovic, and A. W. Lohmann, "Understanding superresolution in Wigner space," *J. Opt. Soc. Amer. A*, vol. 17, no. 12, pp. 2422–2430, Dec. 2000.

Yair Rivenson received the B.Sc. and the M.Sc. degrees from the Department of Electrical and Computer Engineering, Ben-Gurion University of the Negev, Israel, in 2003 and 2009, respectively. He is currently working toward the Ph.D. degree with the Electrical and Computer Engineering Department of the Ben-Gurion University of the Negev.

His research interests are Compressive Sensing, Compressive Imaging, Computational Optics, Holography, multi-dimensional imaging, optical super-resolution, and image reconstruction.



Adrian Stern (M'09) received the B.Sc., M. Sc. (cum laude) and Ph.D. degrees from Ben-Gurion University of the Negev, Israel, in 1988, 1997, and 2003 respectively, all in electrical and computer engineering.

He was a Post-Doctoral Fellow at the University of Connecticut, Storrs, and Ben-Gurion University of the Negev, Israel during 2004–2007. During 2007–2008 he served as senior research and algorithm specialist for GE Molecular Imaging.

Currently he is with department of Electro-Optical Engineering at Ben-Gurion University of the Negev, Israel. His current research interests include compressed imaging, computational imaging, 3D imaging, bio-medical imaging, sequences of images processing, image restoration and reconstruction.

Dr. Stern is a member of the Optical Society of America (OSA), and the International Society for Optical Engineering (SPIE). He is an associate editor for *Optics Express*.



Bahram Javidi (S'83–M'83–SM'96–F'98) received the B.S. degree from George Washington University, Washington, DC, and the M.S. and Ph.D. degrees from the Pennsylvania State University, University Park, all in electrical engineering.

He is the Board of Trustees Distinguished Professor at the University of Connecticut which is the highest rank and honor bestowed on a faculty member based on research, teaching, and service. He has over 630 publications. He has completed 8 books and 44 book chapters. He has published over

250 technical articles in major peer reviewed journals. He has published over 330 conference proceedings, including over 110 Plenary Addresses, Keynote Addresses, and invited conference papers. His papers have been cited over 5500 times according to the citation index of WEB of Science.

Dr. Javidi is Fellow of seven scientific societies, including Institute of Electrical and Electronics Engineers (IEEE), American Institute for Medical and Biological Engineering, Optical Society of America, and Institute of Physics. In 2008, he received a Fellow award by John Simon Guggenheim Foundation. In 2010, he was the recipient of George Washington University's Distinguished Alumni Scholar Award, University's highest honor for its alumni in all disciplines. He received the 2008 IEEE Donald G. Fink prized paper award among all (over 180) IEEE Transactions/Journals, and Magazines. In 2007, The Alexander von Humboldt Foundation awarded him the Humboldt Prize for outstanding US scholars. In 2005, he received the Dennis Gabor Award in Diffractive Wave Technologies from the International Society for Optical Engineering (SPIE). Early in his career, the National Science Foundation named him a Presidential Young Investigator, and he received The Engineering Foundation and the Institute of Electrical and Electronics Engineers Faculty Initiation Award. He was selected in 2003 as one of the nation's top 160 engineers between the ages of 30–45 by the National Academy of Engineering to be an invited speaker at The Frontiers of Engineering Conference.