Current Spectra Translation in Single Phase Rectifiers: Implications to Active Power Factor Corrections

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Abstract - This study examines the input and output current spectra translation of a line commutated single phase full wave rectifier. Analytical expressions for the spectra translation of the current harmonics are derived. The proposed method is then used to quantify the contribution of a current-loop with finite bandwidth to the overall THD budget in average current mode APFC systems. The proposed theory is supported by computer simulation.

I. Introduction

A key design parameter of Active Power Factor Correction (APFC) controllers is the required current-loop bandwidth [1, 2]. One normally assumes that a wide current-loop bandwidth ensures good tracking of the current reference signal and hence results in low THD. In practice, however, wide bandwidth is difficult to achieve while still maintaining dynamic stability. This is especially true in common APFC topologies which include a right half plane zero such as the Boost and Flyback converters.

The APFC is a special case of a dynamic system which emulates a resistive load by forcing a current at the rectified side. Ideally, this current should follow the rectified voltage waveform. In practice, however, the forced current deviates from the required waveform due to the limited bandwidth of the APFC's current loop which omits high harmonics from the forced rectified current. The effect of this current imperfection on the line current is not obvious since the transformation process associated with the rectifier is non linear.

A prerequisite for specifying the current loop bandwidth required to meet a given THD target, is a clear understanding of the rectification process. In particular, the harmonics transformation rule from the rectified side to the line will determine the effect of a limited current loop bandwidth on the line current THD. Consequently, the first question that needs to be investigated in this connection is the non linear effect of the rectifier and in particular, its effect on the harmonics content of the power line current as a function of the harmonic content of the rectified side current. Once the current spectra translation rule between the line side and the rectified side are known, one can apply these to quantify the contribution of the finite current loop bandwidth to the overall THD budget of average current mode APFC systems. This can help to discern the tradeoffs between the current-loop bandwidth and the resulting THD of the APFC systems.

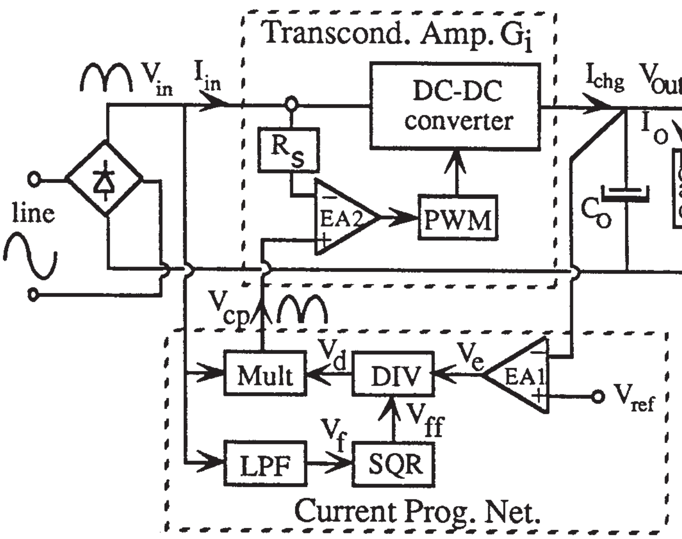

Figure 1: Single phase Active Power Factor Correction system.

II. Methodology

The study includes two parts. First, we develop a general form of the spectra translation rules between the line and rectified currents of a line commutated full wave rectifiers. The proposed theory is demonstrated by examining simple private cases. In the second part of the study, we consider the case of the family of APFC systems shown in Fig. 1 and alike. Since the currrent loop gain is limited, the higher harmonics at the rectified side are suppressed. The effect of this deficiency on the line current THD is then investigated by applying the results of the first part of the study.

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III. Review of the line and the rectified current spectra

We assume that under steady state conditions, the rectified current $i_R(t)$ of a single phase full wave rectifier has a periodic waveform as shown in Fig. 2.

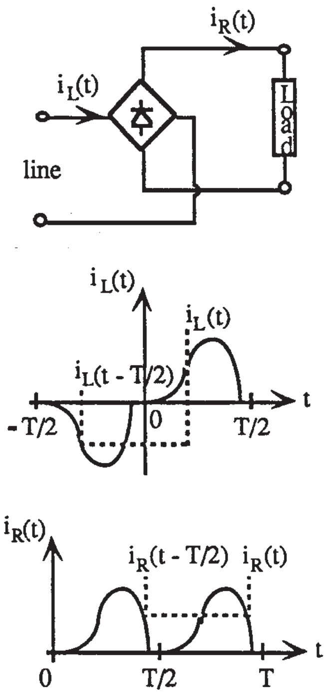

Figure 2: The line and the rectified currents of a single phase full bridge rectifier.

The rectified current is periodic and could be expanded into a Fourier series with complex coefficients:

$$i_R(t) = \sum_{m=-\infty}^{+\infty} C_{Rm} e^{jm\omega t} \qquad (1)$$

where $\omega = \frac{2\pi}{T}$ is the line angular frequency and T is the period. We assume that the rectified current has an identical response during each of the half-cycles of the line frequency, namely, it fulfills the condition:

$$i_R(t) = i_R\left(t - \frac{T}{2}\right) \qquad (2)$$

It is known that for such a case the rectified current spectra consists only of even harmonics of the line frequency, the complex amplitudes of which are given as:

$$C_{Rm} = \frac{\omega}{\pi} \int_0^{\frac{T}{2}} i_R(t) e^{-jm\omega t} dt \qquad m = 0, 2, 4, ... \qquad (3)$$

We further assume that under steady state conditions, the line current $i_L(t)$ of the single phase full bridge rectifier has a periodic waveform as shown at Fig. 2. The current $i_L(t)$ is periodic and could be expanded into a Fourier series with complex coefficients:

$$i_L(t) = \sum_{n=-\infty}^{+\infty} C_{Ln} e^{jn\omega t} \qquad (4)$$

. Assuming that the line current obeys the condition:

$$i_L(t) = - i_L\left(t - \frac{T}{2}\right) \qquad (5)$$

one can recall that the line current spectra consists only of odd harmonics of the line frequency, the complex amplitudes of which are given as:

$$C_{Ln} = \frac{\omega}{\pi} \int_0^{\frac{T}{2}} i_L(t) e^{-jn\omega t} dt \qquad n = 1, 3, 5, ... \qquad (6)$$

IV. The spectra translation

Observing that the line current equals the rectified current in the [0,T/2] interval:

$$i_L(t) = i_R(t) \qquad 0 < t < T/2 \qquad (7)$$

it is possible to derive the line current harmonic coefficients in terms of the rectified current as follows:

$$C_{Ln} = \frac{\omega}{\pi} \int_0^{\frac{T}{2}} i_L(t) e^{-jn\omega t} dt = \frac{\omega}{\pi} \int_0^{\frac{T}{2}} i_R(t) e^{-jn\omega t} dt \qquad (8)$$

By substituting (1) into (8) we obtain:

$$C_{Ln} = \frac{\omega}{\pi} \int_0^{\frac{T}{2}} \left(\sum_{m=-\infty}^{+\infty} C_{Rm} e^{jm\omega t} \right) e^{-jn\omega t} dt =$$

$$= \frac{\omega}{\pi} \sum_{m=-\infty}^{+\infty} C_{Rm} \int_0^{\frac{T}{2}} e^{j(m-n)\omega t} dt \qquad (9)$$

Evaluating the above integral yields:

$$\int_0^{\frac{T}{2}} e^{j(m-n)\omega t}dt = \frac{1}{(m-n)\omega}\left[\sin(m-n)\omega t - j\cos(m-n)\omega t\right]_0^{T/2} \quad (10)$$

Since m is even and n is odd the difference (m-n) is also odd, consequently, the sine terms vanish and each of the cosine terms contributes a unity, yielding:

$$\int_0^{\frac{T}{2}} e^{j(m-n)\omega t}dt = j\frac{2}{(m-n)\,\omega} \quad (11)$$

Substituting this result back into equation (9) above, we get the complex input to output spectra translation rule as:

$$\boxed{C_{Ln} = \frac{2j}{\pi}\sum_{m=-\infty}^{+\infty}\frac{C_{Rm}}{(m-n)}} \quad (12)$$

Equation (12) above relates the coefficients of the complex Fourier series of the line current C_{Ln} to those of the rectified current C_{Rm}. The indexes (m) are assumed to be even while (n) are odd.

Using only positive values of m this result may be written as:

$$C_{Ln} = -\frac{2j}{\pi}\frac{C_{R0}}{n} + \frac{2j}{\pi}\sum_{m=2}^{+\infty}\left(\frac{C_{Rm}}{(m-n)} + \frac{C_{R-m}}{(-m-n)}\right)$$

(13)

Substituting $C_{Rm} = |C_{Rm}|e^{-j\phi m}$ and $C_{R-m} = |C_{Rm}|e^{j\phi m}$ and manipulating the expression (13) we obtain:

$$C_{Ln} = -\frac{2j}{\pi}\frac{C_{R0}}{n} + $$
$$+\frac{2j}{\pi}\sum_{m=2}^{+\infty}\frac{|C_{Rm}|}{(m^2-n^2)}\left((m+n)e^{-j\phi m} - (m-n)e^{j\phi m}\right)$$
$$= -\frac{2j}{\pi}\frac{C_{R0}}{n} +$$
$$\frac{2j}{\pi}\sum_{m=2}^{+\infty}\frac{|C_{Rm}|}{(m^2-n^2)}(2n\cos\phi_m - j2m\sin\phi_m) =$$
$$= -j\frac{a_{R0}}{\pi\, n} + \frac{2}{\pi}\sum_{m=2}^{+\infty}\left(\frac{mb_{Rm}}{(m^2-n^2)} + j\frac{na_{Rm}}{(m^2-n^2)}\right) \quad (14)$$

where $a_{R0} = 2C_{R0}$, $a_{Rm} = 2|C_{Rm}|\cos\phi_m$ and $b_{Rm} = 2|C_{Rm}|\sin\phi_m$ are the cosine and sine coefficients of the real valued Fourier expansion of the rectified current respectively. Comparing the real and imaginary parts of equation (14) with those of the general form of the complex Fourier coefficients:

$$C_{Ln} = \frac{1}{2}(a_{Ln} - jb_{Ln}) \quad (15)$$

we can write the Fourier coefficients for the line current as follows:

$$\boxed{\begin{aligned} a_{Ln} &= \frac{4}{\pi}\sum_{m=2}^{+\infty}\frac{mb_{Rm}}{m^2-n^2} \\ b_{Ln} &= \frac{2}{\pi}\frac{a_{R0}}{n} - \frac{4}{\pi}\sum_{m=2}^{+\infty}\frac{na_{Rm}}{m^2-n^2} \end{aligned}} \quad (16)$$

Equation (16) above relates the Fourier coefficients of the line current a_{Ln}, b_{Ln} to those of the rectified current a_{Rm}, b_{Rm}.

Following the same reasoning we can find the inverse spectra translation of the rectified current in terms of the coefficients of the line current. This results in a similar expressions but with interchanged indexes. The formulas below relate the complex (C_{Rm}), sine (a_{Rm}) and cosine (b_{Rm}) coefficients of the Fourier series of the rectified current to those of the line current. Again, (m) is assumed to be even while (n) is odd:

$$\boxed{\begin{aligned} C_{Rm} &= \frac{2j}{\pi}\sum_{n=-\infty}^{+\infty}\frac{C_{Ln}}{(n-m)} \\ a_{Rm} &= \frac{4}{\pi}\sum_{n=1}^{+\infty}\frac{nb_{Ln}}{n^2-m^2} \\ b_{Rm} &= -\frac{4}{\pi}\sum_{n=1}^{+\infty}\frac{ma_{Ln}}{n^2-m^2} \end{aligned}} \quad (17)$$

The spectra translation (12) involves a 90 degrees phase shift of the complex Fourier coefficients and couses an interchange of the sine and cosine coefficients on the line and rectified sides. This is clearly demonstrated by (16). It shows that the sine and the cosine coefficients on the line side are a function of the cosine and sine coefficients on the rectified side respectively. This interesting property holds also for the inverse spectra translation (17).

V. Applications of the spectra translation rules

As will be shown next, the proposed theory checks well against known Fourier series expansions and provides an easy to apply tool to derive the harmonic content of signals.

Example 1.

A single phase full bridge rectifier with a pure resistive load R_0 is fed by a sinusoidal line voltage $v_L(t) = V_{max}\sin\omega t$ as shown at Fig. 3. Find the rectified current spectra.

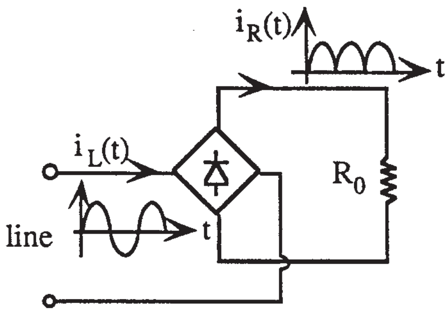

Figure 3: Single phase full bridge rectifier with resistive load R_0.

Solution: neglecting the rectifiers voltage drop, the line current is:

$$i_L(t) = I_{max}\sin\omega t$$

where $I_{max} = V_{max}/R_0$. All the Fourier coefficients are zero eccept the $b_{L1} = I_{max}$. Applying equation (17) yields the general expression for the rectifier's output current spectra:

$$a_{Rm} = \frac{4}{\pi}\frac{I_{max}}{1-m^2} \qquad m = 0, 2, 4,...$$

$$b_{Rm} = 0 \qquad m = 0, 2, 4,...$$

Thus the expression of the rectified current is:

$$i_R(t) = \frac{2}{\pi} I_{max} - \frac{4}{3\pi} I_{max}\cos2\omega t -$$

$$- \frac{4}{15\pi} I_{max}\cos4\omega t - \frac{4}{35\pi} I_{max}\cos6\omega t - ...$$

This result checks with the well known Fourier expansion.

Example 2.

A single phase full bridge rectifier is driven by a sinusoidal line voltage $v_L(t) = V_{max}\sin\omega t$ and loaded by a constant current sink I_0 as shown at Fig. 4. Find the line current spectra.

Solution: in this case all the Fourier coefficients of the rectified current are zero except its DC term:

$$a_{R0} = 2I_0$$

Applying equation (16) yields the general expression for the line current spectra:

$$a_{Ln} = 0 \qquad n = 1, 3, 5,...$$

$$b_{Ln} = \frac{4}{\pi}\frac{I_0}{n} \qquad n = 1, 3, 5,...$$

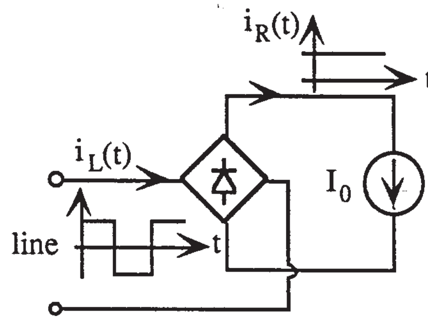

Figure 4: Single phase full bridge rectifier loaded with a current sink I_0.

Thus the expression of the line current is:

$$i_L(t) = \frac{4}{\pi} I_0\sin\omega t + \frac{4}{3\pi} I_0\sin3\omega t + \frac{4}{5\pi} I_0\sin5\omega t \ ...$$

This result checks with the well known Fourier expansion of the function:

$$i_L(t) = I_0\text{sign}[\sin\omega t].$$

VI. Estimation of the THD of APFC systems

We can use the rectifier current spectra translation theory developed above, to explain the appearance of the line current harmonics due to the limited bandwidth of the inner loop in the APFC systems.

To investigate the effect of the current loop response, we consider here only the inner loop of the APFC system of Fig. 1 and alike. We model the APFC's power stage under the closed inner loop condition as a linear transconductance amplifier, fed by the current programming signal as shown in Fig. 5. The transconductance G_i is generally of a low pass type. We further assume that the current programming voltage $v_{cp}(t)$ of the outer feedback and feedforward loops comprising the current programming network is an ideal voltage source of a pure 'rectified sine' shape [3]:

$$v_{cp}(t) = V_{max}\left|\sin\omega t\right| \qquad (18)$$

The $v_{cp}(t)$ signal is composed of infinite number of harmonics $v_{cp}(t) = \sum_m V_{cp_m}(j\omega_m)$ and can be presented by the well known Fourier series:

$$v_{cp}(t) = \frac{V_0}{2} + \sum_{m=2}^{+\infty} V_m \cos m\omega t \qquad (19)$$

where $V_0 = \frac{4V_{max}}{\pi}$ and the successive V_m coefficients are given by:

$$V_m = \begin{cases} \frac{4}{\pi}\frac{V_{max}}{1-m^2} & m = 2, 4, \ldots \\ 0 & m = 1, 3, \ldots \end{cases} \qquad (20)$$

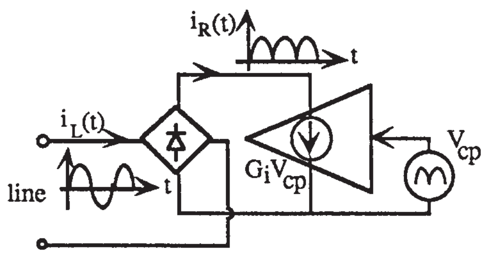

Figure 5: Simplified linear model of the inner loop of a APFC.

The current of the power stage is the response of the transconductance $G_i(j\omega)$ to the current programming signal:

$$I_R(j\omega) = G_i(j\omega) \sum_m V_{cp_m}(j\omega_m) \qquad (21)$$

and as far as the rectifier is concerned, this is the rectified current.

We note that the sine coefficients of the expansion (19) are equal to zero and according to (21) the sine coefficients of the APFC's current on the rectified side vanish: $b_{Rm} = 0$. Applying the spectra translation rule (16) we conclude that the cosine coefficients of the line current must vanish also: $a_{Ln}=0$.

We are left to analyze the effect of the cosine coefficients of the rectified current a_{Rm}. The spectra translation rules (16) imply that the high harmonics at the line side will vanish if the DC term exactly balances out the sum of the high harmonics of the rectified side. That is, $b_{Ln} = 0$ if:

$$\frac{2}{\pi}\frac{a_{R0}}{n} = \frac{4}{\pi}\sum_{m=2}^{+\infty} \frac{n a_{Rm}}{m^2-n^2} \qquad n > 1 \qquad (22)$$

If equation (22) holds for all $n > 1$, the line current of the APFC is harmonic free and the THD equals zero.

However, the rectified current $I_R(j\omega)$ tracks the programming voltage only approximately. Since the current programming signal is composed of infinite spectra components and the transconductance $G_i(j\omega)$ of the current loop has a limited bandwidth, the resulting rectified current lacks some of the higher harmonics. Perfect balance of equation (22) is violated. Consequently, the right hand side of equation (16) fails to converge to zero. The residual, b_{Ln}, is interpreted as high harmonics on the line side and contribute to the distortion of the line current. Next, we turn to investigate this phenomena quantitatively.

As stated by (21), the input current $i_R(t)$ of the DC-DC converter (Fig. 5) is the response of the transconductance of the inner loop $G_i(j\omega) = G_i(\omega)e^{j\theta(\omega)}$ to the current programming signal (19):

$$i_R(t) = \frac{V_0}{2} G_i(0) + \sum_{m=2}^{+\infty} V_m G_i(m\omega) \cos(m\omega t + \theta(m\omega)) \qquad m = 2, 4, \ldots \quad (23)$$

We approximate the transconductance of the inner loop $G_i(j\omega)$ to that of an ideal low pass function of constant gain and no phase shift within its bandwidth and zero gain otherwise:

$$G_i(j\omega) = \begin{cases} G_o & f \le f_c \\ 0 & f > f_c \end{cases} \qquad (24)$$

The highest current harmonic that could be found in the rectified current is lower or equal to the corner frequency f_c of the current loop. The infinite series (23) is then truncated accordingly. Given the line frequency f_L and taking into account that only the even harmonics of the line frequency are present, we can find the number of harmonics which are present in the rectified current as:

$$M_{max} = \frac{f_c}{f_L} \qquad (25)$$

These assumptions further simplify equation (23) to that of:

$$i_R(t) = G_o V_{max}\left(\frac{2}{\pi} + \frac{4}{\pi}\sum_{m=2}^{M_{max}} \frac{1}{1-m^2}\cos(m\omega t)\right) \qquad m = 2, 4, \ldots \quad (26)$$

The line current harmonics of the APFC system I_{Ln} (only the odd ones exist) can now be found according to the current spectra translation rules obtained earlier in the paper. Applying equation (16) we find:

$$I_{Ln} = \frac{16}{\pi^2} G_o V_{max}\left(\frac{1}{4n} - \sum_{m=2}^{M_{max}} \frac{n}{(m^2-n^2)(1-m^2)}\right) \qquad m = 2, 4, \ldots;\ n = 1, 3 \ldots \quad (27)$$

The current harmonics (27) can now be normalized to the base quantity:

$$I_{base} = \frac{16}{\pi^2} G_o V_{max} \qquad (28)$$

This simplifies the calculation of the resulting total harmonic distortion (THD) which is done by following its definition:

$$THD = \frac{\sqrt{\sum_{n=3}^{+\infty} I_n^2}}{I_1} 100\% \qquad n = 3, 5 \ldots \quad (29)$$

The theoretical result of (28) was evaluated by a MATLAB software package and presented below in Fig. 6. The bar plot shows the total harmonic distortion (THD) of the line current versus the normalized current loop bandwidth BW_{norm} of the APFC. The normalized bandwidth is defined relatively to the line frequency f_L:

$$BW_{norm} = M_{max} = \frac{f_c}{f_L} \qquad (30)$$

The barplot also compares the calculated results to a PSPICE simulation of an ideal rectifier followed by an ideal low pass transconductance amplifier as defined by equations (23) and (30). Good agreement of the theoretical and simulated results is found.

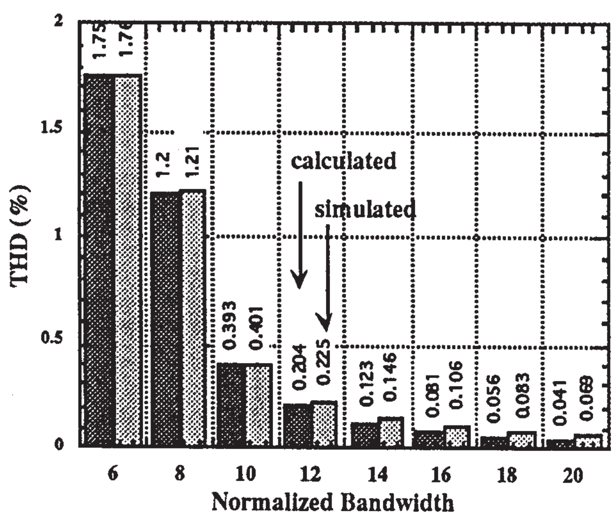

Figure 6: THD as a function of the of APFC's current loop normalized bandwidth. (Calculations done by MATLAB and simulation by PSPICE packages.)

Examining the presented results, we are able to make an important general conclusion about the current loop bandwidth of the averaged current mode APFC systems. To ensure low contribution to the THD budget, the current loop should be designed with a crossover frequency about 20 times the line frequency. For the European or North-American utility lines, bandwidth within the 1-1.2kHz range is sufficient, while for a 400Hz power systems the required bandwidth should be about 8kHz.

VII. Conclusions

The objectives of this paper were to link the current spectra of the line and load sides of the full wave rectifier and to establish the tradeoffs between the current loop bandwidth and the resulting THD of the average current mode APFC systems. First we developed the spectra translation rules for the line and rectified currents of the line commutated full bridge rectifier. Then, assuming that the current loop is represented by a low pass network, we applied the theory to estimate the line current harmonics and the steady state THD of average current mode controlled APFC system of Fig. 1 and alike. The results were used to calculate the total harmonic distortion (THD) of the line current as a function of the normalized current loop bandwidth of the APFC. The practical implication of this study is that the current loop bandwidth of the averaged current mode APFC systems should span 20 times the line frequency to ensure low contribution to the total harmonic distortion (THD). For the European or North-American utility lines, bandwidth within the 1-1.2kHz range is sufficient while for a 400Hz power systems the required bandwidth is about 8kHz.

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