A BEHAVIORAL AVERAGE MODEL OF SEPIC CONVERTERS WITH COUPLED INDUCTORS

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Abstract

An average model of SEPIC converters with coupled inductors was developed and verified against cycle-by-cycle simulations. The model can be used as-is by any modern circuit simulator to run steady state (DC), large signal (Transient) and small signal (AC) analyses. The inductors coupling coefficient, incorporated as a parameter in the model, can be varied from zero to almost unity. Zero coupling coefficient represents the case of a SEPIC converter with uncoupled inductors.

Introduction

Practical SEPIC converters are usually built with coupled inductors (L_s and L_p, Fig. 1) to lower production costs. Yet, the SEPIC dynamic behavior is still poorly understood and the single simulation model proposed hitherto for this topology is valid only for the uncoupled inductors situation [1]. The coupled inductors case appears to represent a still open simulation issue. It involves two coupled inductors acting as a transformer, in the sense that current flows on both sides at the same time, while both sides are loaded by capacitors (C_{fi}, C_s and C_p, Fig. 1). Assuming that the capacitors are large enough so that there voltages do not vary appreciably during one cycle, they can be replaced by voltage sources. Therefore, we have in fact a transformer connected to voltage sources at the input and output at the same time (Fig. 2). Consequently, the leakage inductances (L_{sp} and L_{pp}, Fig. 2) cannot be neglected in this case. The voltages of the two terminals (V_1 and V_2, Fig. 2) have different values during the 'on' and 'off' times, but can be considered almost constant within the 'on' and 'off' intervals. The magnitude of the voltages, assuming ideal diode and switch, are as follows:

$$V_1 = \begin{cases} V_{C_{fi}} & ; \ t_{on} \\ V_{C_{fi}} - V_{C_s} - V_{C_p} & ; \ t_{off} \end{cases} \quad (1)$$

$$V_2 = \begin{cases} V_{C_s} & ; \ t_{on} \\ -V_{C_p} & ; \ t_{off} \end{cases} \quad (2)$$

Model Derivation

The proposed average model hinges on the observation [2] that by evaluating the average voltage across a switched inductor one can easily derive the average current flowing through it. The same reasoning applies to switched capacitors. Namely, by injecting the average current into a capacitor one obtains the average voltage across it.

In the case of the SEPIC converter with coupled inductors, the leakage inductances (L_{sp} and L_{pp}) are switched inductors. Thus, in order to derive the average voltages across them, one has to evaluate the internal voltage (V_m) across the mutual inductance (L_m, Fig. 2).

A simple way to derive the expression for V_m is to start with the currents equation at the primary of the coupled inductors model of Fig. 2 :

$$I_{L_m} = I_{L_s} + I_{L_p} \quad (3)$$

where I_{L_m}, I_{L_s} and I_{L_p} are per the notations of Fig. 2.

Taking the derivative of both sides implies:

$$\frac{dI_{L_m}}{dt} = \frac{dI_{L_s}}{dt} + \frac{dI_{L_p}}{dt} \quad (4)$$

Assuming constant voltages over one switching cycle and substituting the current derivatives by the voltage to inductance ratio for each inductor, we obtain (see notations in Fig. 2):

$$\frac{V_m}{L_m} = \frac{V_1 - V_m}{L_{sp}} + \frac{V_2 - V_m}{L_{pp}} \quad (5)$$

which yields an explicit expression for V_m:

$$V_m = \frac{\frac{V_1}{L_{sp}} + \frac{V_2}{L_{pp}}}{\frac{1}{L_m} + \frac{1}{L_{sp}} + \frac{1}{L_{pp}}} \quad (6)$$

Notice that the voltage V_m is an algebraic function of the voltages V_1 and V_2. This implies that it does not change significantly within the 'on' or the 'off' time.

Based on (6), a behavioral average model [3] can be easily developed for the general SEPIC converter. The complete average model of the SEPIC converters is shown in Fig. 3. In this model all the time dependent variables are represented by voltages or currents [2, 3].

The expressions for the dependent sources are as follows:

$$E_{L_{sp}} = V_{(mon)} * V_{(don)} + [V_{(Cs)} + V_{(Cp)} + V_{(moff)}] * V_{(doff)} \tag{7}$$

$$E_{L_{pp}} = [V_{(Cs)} - V_{(mon)}] * V_{(don)} - [V_{(Cp)} + V_{(moff)}] * V_{(doff)} \tag{8}$$

$$G_{C_s} = -I_{L_{pp}} * V_{(don)} + I_{L_{sp}} * V_{(doff)} \tag{9}$$

$$G_{Cp} = (I_{Lsp} + I_{Lpp}) * V_{(doff)} \tag{10}$$

All voltages are node voltages referred to 'ground' (Fig. 3).

$$E_{mon} = \frac{\left\{ \frac{V_{(Cfi)}}{L_{sp}} + \frac{V_{(Cs)}}{L_{pp}} \right\}}{\left(\frac{1}{L_m} + \frac{1}{L_{sp}} + \frac{1}{L_{pp}} \right)} \tag{11}$$

$$E_{moff} = \frac{\left\{ \frac{V_{(Cfi)} - V_{(Cs)} - V_{(Cp)}}{L_{sp}} - \frac{V_{(Cp)}}{L_{pp}} \right\}}{\left(\frac{1}{L_m} + \frac{1}{L_{sp}} + \frac{1}{L_{pp}} \right)} \tag{12}$$

$$E_{doff} = 1 - V_{(don)} \tag{13}$$

The voltage dependent sources E_{mon} and E_{moff} (eq. 11, 12) generate V_m of the 'on' and 'off' intervals respectively. The resulting voltages ($V_{(mon)}$ and $V_{(moff)}$) are applied in the expressions of $E_{L_{sp}}$ and $E_{L_{pp}}$ (eq. 7-8) which emulate the average voltages across the switched inductors (L_{sp} and L_{pp}). The resulting average currents of the inductors are then used to define the dependent current sources G_{C_s} and G_{C_p} (eq. 9-10), which inject the average currents into capacitor C_s and to the output section (C_p and output filter).

The dependent voltage source E_{doff} generates a voltage analog of the 'off' time ratio (D_{off}) by eq. (13) assuming Continuous Conduction Mode (CCM).

The independent voltage source V_{don} (Fig. 3) emulates the duty cycle (D_{on}) in open loop simulations. This source can be replaced by a dependent voltage source along with the corresponding control circuitry for closed loop simulations.

Results and Discussion

The average model was verified against a cycle-by-cycle PSPICE (V. 6.2, MicroSim Co.) simulation of the switched SEPIC converter. The parameters of the converter were as follows (see Fig. 1 for notations):

V_s = 36V, L_{fi} = 2.75µH, C_{fi} = 0.2µF, L_s = L_p = 9.75µH,

C_s = 0.3µF, C_p = 0.44µF, L_{fo} = 3.8µH,

C_{fo} = 940µF, R_{Cfo} = 90 mΩ, $R_{o(nominal)}$ = 5 Ω , F_s = 1MHz (switching frequency)

Excellent agreement was obtained for steady state (DC), large signal (Transient) and small signal (AC) responses for the full range of coupling coefficient values. For the sake of brevity only the small signal comparison is demonstrated here (Fig. 4). The cycle-by-cycle values were collected tediously one by one. Each point was evaluated by running a transient simulation of the straightforward switched circuit while modulating the control voltage of the pulse generator (PWM) by a constant frequency sine wave. The average model results were obtained by one AC analysis sweep, in which an AC signal was superimposed on the duty cycle voltage source (V_{don}, Fig. 3)).

Conclusions

The behavioral average model presented here seems to be an excellent tool for the analysis and design of SEPIC converters. The model is compatible with any modern circuit simulator and can be used to run DC, AC and transient analysis. In AC analysis the task of linearization is left for the simulator.

References

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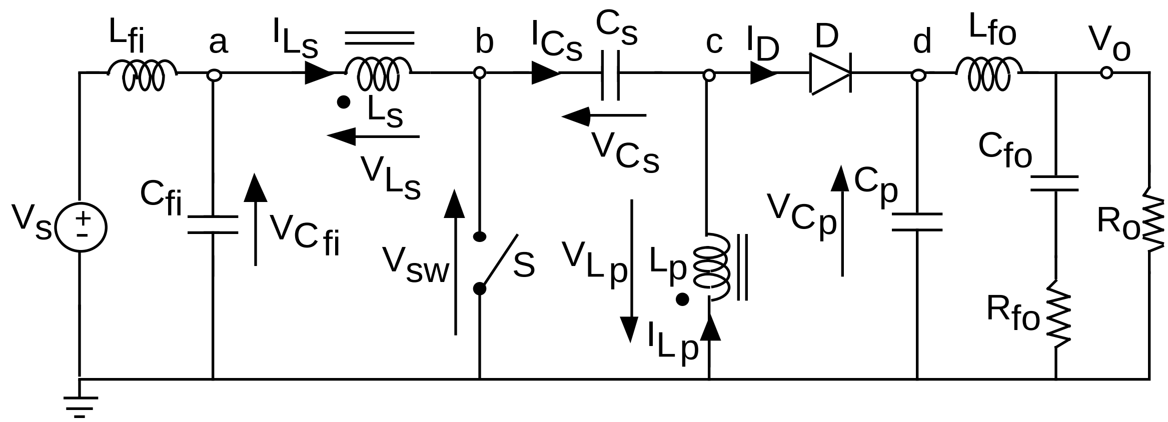

Fig. 1 Basic topology of SEPIC converter with coupled inductors L_s and L_p.

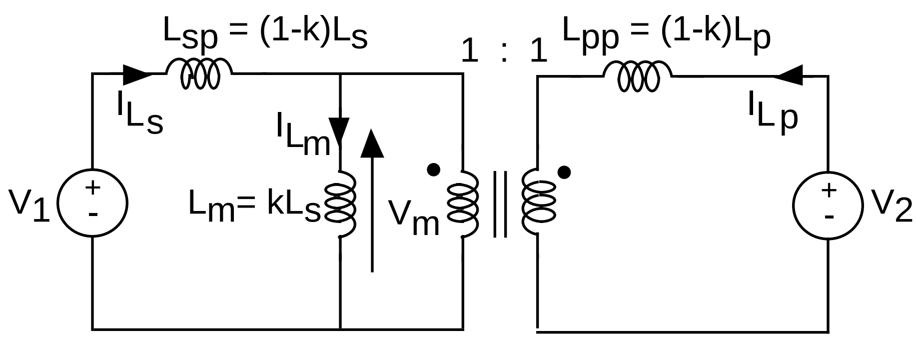

Fig. 2 The coupled inductors model with voltage sources representing the capacitors' voltages. k is the coupling coefficient between L_s and L_p.

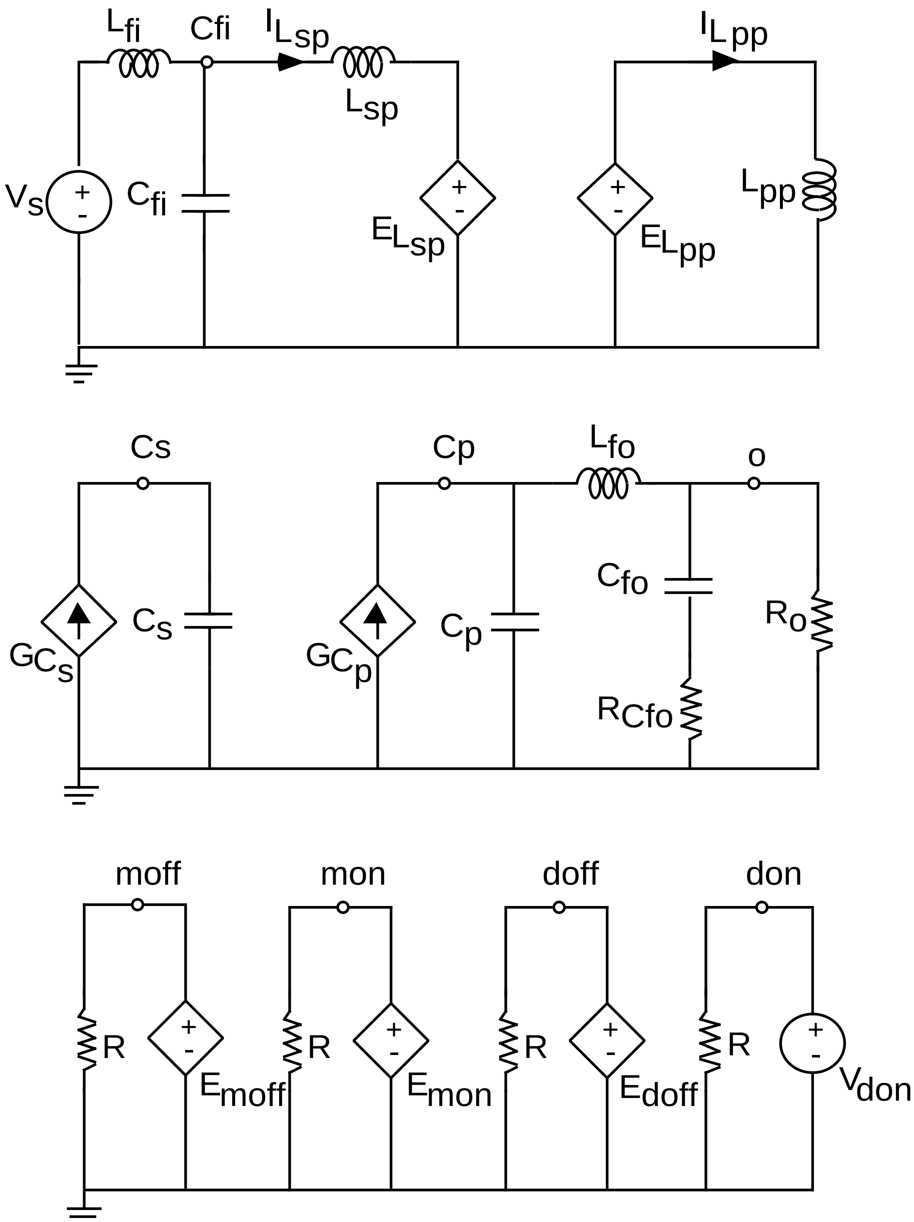

Fig. 3 Proposed behavioral average Model for SEPIC converters with coupled inductors. Nodes names are marked by -o-.

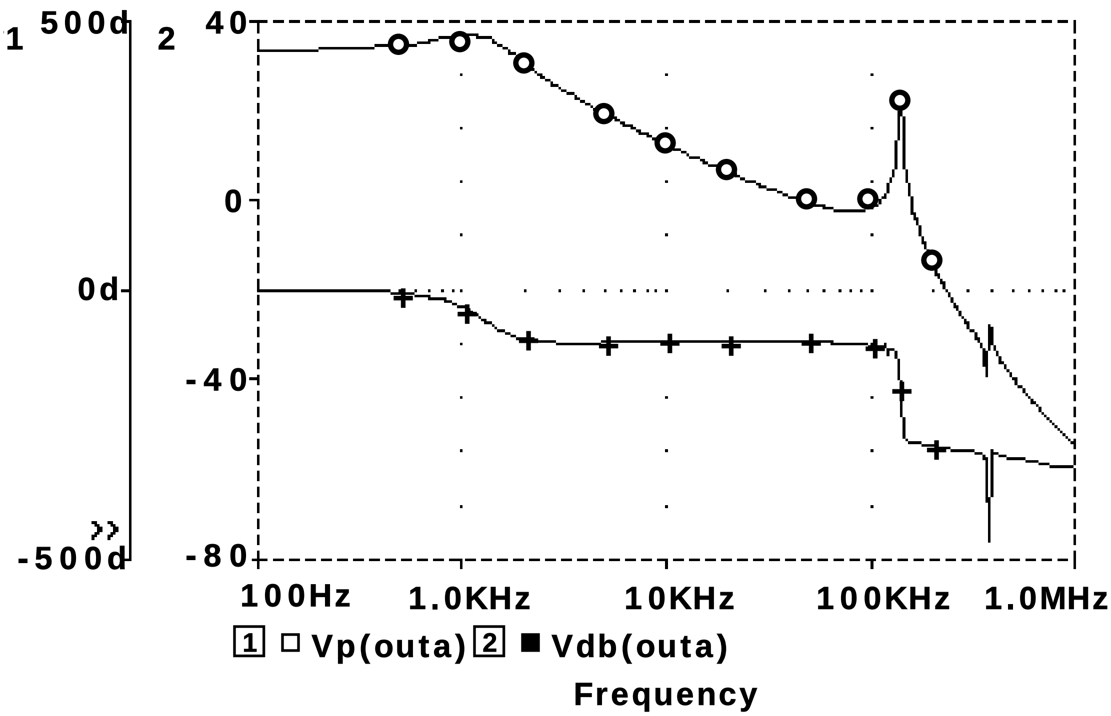

Fig. 4 AC small signal control-to-output $\left(\frac{V_o}{V_{don}}(f)\right)$ response of SEPIC converter with coupled inductors. Cycle-by-cycle simulations (**o** for Magnitude[db], + for Phase[deg]) vs. average model simulations (continuous line). Operating point: $D_{on}=0.124$, $R_o=5\Omega$, $k=0.9$.