Modeling the Switch of PWM Converters

The switch model of pulsewidth modulation (PWM) converters proposed by Vorpérian [1, 2] is critically examined. It is suggested that the omission of the inductor from the model has led to errors in the modeling of discontinuous conduction mode (DCM) converters.

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In recent papers [1, 2] Vorpérian described a general approach for modeling pulsewidth modulation (PWM) converters. The basic idea of the proposed model is the replacement of the PWM switch by an equivalent circuit which emulates its average behavior. This average model is claimed to replace the switching part of the converters and to include all the information relevant to the switching nature of PWM converters. That is, it is implicitly assumed that the average voltage across the equivalent circuit and the average currents into its terminals are 1) identical to those of the physical switch, and 2) are sufficient to fully define its operation. These *a priori* assumptions are in conflict with the intuitive feeling that voltages and currents of the switch itself do not contain all the information required to describe all types of PWM converters that have been described hitherto. This underlying feeling is based on the fundamental role of the inductor as an energy storage element in conventional PWM converters. One would therefore expect that the voltage across the inductor will play an important role in an equivalent circuit which is meant to replace the switching portion of the converter. Indeed, the actual switching times of current-mode and discontinuous conduction mode (DCM) converters are a function of the the voltage across the main inductor. These dependencies are built into the SPICE [3] compatible switch inductor model (SIM) that was shown to be applicable to PWM converters operating in voltage and current mode [4, 5], in continuous and discontinuous conduction mode [6] and to quasi-resonant zero-current-switching converters [7].

Considering the fundamental role of the inductor in the basic operation of a PWM converter, it is surprising to learn that the model proposed by Vorpérian [1, 2] does not include the inductor and that its operation is not a function of the voltage across the inductor. The fact that the inductor is not included in the model leads one to suspect that the derivation of the parameters of the model is in error. Consider [2, Fig. 2 and Fig. 3] and [2, eq. (3) and (4)] which are repeated here for the sake of clarity:

$$v_{ac} = L\frac{i_{pk}}{dT_s}$$

where v_{ac} is the average voltage across terminals (a) and (c) of the switch (Fig. 1), i_{pk} is the peak current of the inductor, and dT_s is the time period in which the (a) and (c) terminals of the switch are shorted.

It would seem that the expression should be

$$v_{am} = L\frac{i_{pk}}{dT_s} \quad (1)$$

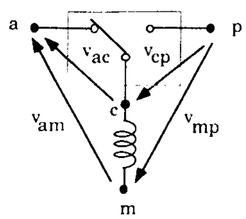

Fig. 1. Definition of average voltages around a PWM switch.

where v_{am} is the average voltage between terminal (a) and (m), the other terminal of the inductor (the one not connected to terminal (c) of the switch, a terminal which is not included in the model). That is, the peak current of the inductor is a function of the voltage across the inductor and not the voltage across the switch.

Similarly, the expression given in [2, eq. (4)] for the period d_2:

$$v_{cp} = L\frac{i_{pk}}{d_2 T_s}$$

where v_{cp} is the average voltage between terminals (c) and (p) of the switch and $d_2 T_s$ is the discontinuous off period.

In this case it would seem that the expression should be (Fig. 1):

$$v_{mp} = L\frac{i_{pk}}{d_2 T_s} \tag{2}$$

where v_{mp} is the average voltage between terminal (p) and (m), the other terminal of the inductor (the one not connected to terminal (c) of the switch, a terminal which is not included in the model).

This observation seems to support the intuitive sense that the voltages across the inductors must be included in models which attempt to express the operation of the switched part of PWM converters and that computer simulations that are based on [2, eq. (3) and (4)] will thus be incorrect.

In the light of the above observation one is puzzled again by the fact that the expression for the dc condition derived in [2] from the proposed model (which could be based on the wrong equations) are found to be compatible with those derived by other investigators. This mystery may be related to another derivation in [2] which raises the same concern as [2, eq. (3) and (4)]. The paper suggests that for the dc condition of a Buck converter [2, eq. (11)]:

$$V_0 = \mu V_{ac} = \mu(V_g - V_0)$$

which implies

$$V_{ac} = V_g - V_0 \tag{3}$$

where V_g is the voltage of the generator and V_0 is the output voltage.

However, $(V_g - V_0)$ is not the dc (nonperturbed) voltage between terminals (a) and (c) of the switch (per the definition of V_{ac}) but rather: the voltage across the inductor when these terminals are a short (during the dT_s period). Namely:

$$V_g - V_0 = V_{am} \tag{4}$$

where V_{am} is the dc (nonperturbed) value of v_{am} (Fig. 1).

This would correct the suspected error of (3) by treating V_{ac} as the voltage across the inductor (V_{am}) and not the voltage across the switch terminals (a) and (c). As a result, the dc expressions are not in error. However, for cases in which the parasitic resistances cannot be ignored, the assumption of (3a) will lead to an erroneous dc expression.

The major assertion of the above discussion is that models that attempt to replace the switching part of PWM converters must include information on the voltage across the terminals of the inductor. When the inductor is included in the model, the model can be represented as SPICE (or other circuit simulator) subcircuit whose dependent sources are a function of the average voltages and currents of its terminals. This is the approach adopted by us [4–7].

Whereas the inclusion of the voltage of the inductor in PWM models is essential for DCM and current mode converters (in which the switching periods are a function of the voltage of the inductor), it may not be important in other modes of operation. For example, in the case of a voltage mode PWM converter operating in the continuous conduction mode (CCM), the operation of the switched part can be shown to be independent of the voltage across the terminals of the inductor. This is a result of the fact that in this simple case, the switching periods (d) and (d_2) are a function of the primary duty cycle only and are independent of the voltage of the inductor. This might explain the observation made in [2, p. 497]: "Whereas, use of the model of the PWM switch in CCM yields the *same results* as those given by the method of state-space averaging, in DCM the model of the PWM switch yields results which are *different* than those of the state-space averaging."

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Author's Reply

The questions raised by Professor Ben-Yaakov in his correspondence are understandable, and they probably have occurred to other readers who have intuitive conflicts with the average switch model. Such conflicts can easily arise if the expressions and derivations are examined outside the context and approximations of average modeling. The following is an explanation of the points raised by Professor Ben-Yaakov.

1) It is important to distinguish between *average* (continuous) and *instantaneous* (discrete) quantities. Since it is the accuracy of [4, eqs. (3) and (4)] which is being contested, I use one of them to explain the distinction between average and instantaneous quantities. All the quantities in the equation

$$i_{pk} = \frac{v_{ac}}{L} dT_s \qquad (1)$$

are average quantities and i_{pk} does not stand for the *instantaneous* peak value of the inductor current in a particular interval as this is *not* a sampled-data equation. Hence, the pulsewidth modulation (PWM) switch model does not predict instantaneous peak currents with great accuracy although it can be used to estimate them reasonably correctly if need be.

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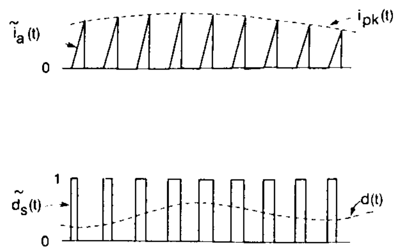

Fig. 1.

This is not too hard to see if one realizes that in discontinuous conduction mode (DCM), the average voltage *per cycle* across the inductor is always zero, even under a transient (this does not mean that, under sinusoidal perturbations, there is no voltage signal at the frequency of perturbation across the inductor). Hence, the actual voltage across the inductor *during the one-time* of a particular switching cycle, v_{am} as Dr. Ben-Yaakov observes in (1) of his correspondence, can be replaced with the average voltage across the switch in that entire switching cycle. This is particularly simple to see under dc conditions. Hence, in reference to the buck converter, the observation $V_{am} = V_g - V_0$ made by Dr. Ben-Yaakov in (4) is not any different than $V_{ac} = V_g - V_0$ given in my paper (eq. (4)) in Dr. Ben-Yaakov's correspondence) since the dc voltage across the inductor is zero. Therefore, the model of the PWM switch does not suffer from the syndrome of two wrongs making one right as one may infer from reading the paragraph containing (4).

(Note that $i_{pk}(t)$ and the *duty-ratio function* $d(t)$ are continuous-time functions which can be visualized with the help of Fig. 1. These functions must be distinguished from the instantaneous inductor current $\tilde{i}_L(t)$ and the *switching function* $\tilde{d}_s(t)$ in order to avoid any confusion. It is clear that neither $d(t)$ nor $i_{pk}(t)$ have any physical meaning at every instant of time—they are simply continuous-time abstractions of the actual waveforms and cannot be interpreted as exact sampled data or discrete quantities.)

What happens then in the case of a parasitic resistance in series with the inductor? Will the PWM switch model yield erroneous results? It is shown next that in the linear ripple approximation, the parasitic resistance of the inductor has negligible effect on the average voltages and currents of a PWM converter.

2) In normal DCM operation, the parasitic resistance in the inductor is small enough that the current in the inductor satisfies the *linear-ripple approximation*, i.e., the inductor, or in the common terminal, current looks like a nearly perfect triangle. This automatically implies that the average voltage drop across this parasitic resistor is much smaller than the average terminal voltages of the switch or the input or output voltage of any of the two-switch PWM

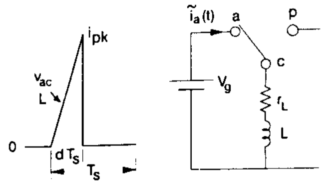

Fig. 2.

converters. Here is the proof.

$$\bar{\imath}_a(t) = \frac{V_g}{r_L}(1 - e^{-tr_L/L}), \qquad 0 < t < dT_s \tag{2}$$

$$\bar{\imath}_a(t) \simeq \frac{V_g}{r_L}\left(1 - \left(1 - t\frac{r_L}{L}\right)\right) = \frac{V_g}{L}t \tag{3}$$

where the various quantities are shown in Fig. 2. The second equation is based on the linear ripple approximation which assumes that the parasitic resistance r_L is small so that:

$$t\frac{r_L}{L} \ll 1 \Rightarrow dT_s\frac{r_L}{L} \ll 1. \tag{4}$$

When this equation is substituted in (1) above, or (3) of my paper, we get

$$\frac{i_{pk}r_L}{v_{ac}} \ll 1. \tag{5}$$

This equation states that the *peak* voltage drop across the parasitic resistance is much smaller than the average voltage across terminals *ac* of the PWM switch. It follows that the *average* voltage drop across the parasitic resistance is even much smaller than v_{ac}. This in turn implies that the average voltage drop across the parasitic resistance is very much smaller than the average terminal voltages of the PWM switch which are the same as the input or the output voltage of any two-switch PWM converter. Hence, even in the presence of a parasitic resistance in series with the inductor, use of the PWM switch model does not yield erroneous expressions of the dc conversion ratio as Professor Ben-Yakov claims in his correspondence.

3) In light of the above it can be seen that the average model of the PWM switch is not flawed when used to predict the dynamics of the average voltages and currents. Does this mean that the average model of the PWM switch cannot be used in modeling the dynamics of a PWM converter in current-mode control as Professor Ben-Yaakov suggests in his correspondence? The main point of concern here is that in current-mode control, the instantaneous peak current and not the average current, is used to control the operation of the switch. Hence, it may seem that the average model of the PWM switch cannot be used in this case. That this is not the case was shown for the first time by Ridley [1] who derived a simple, accurate, and continuous-time model of current-mode controlled PWM converters. In his work, Ridley obtained an accurate continuous-time model of the sample-and-hold action of the *current loop only* while allowing the power stage to be modeled as usual with the average model of the PWM switch in continuous or discontinuous conduction mode. Hence, he was able to demonstrate, without recourse to a discrete model of the entire converter, that the continuous dynamics of current mode controlled converters had a complex pole-pair at half the switching frequency which became unstable for duty cycles greater than fifty percent in the absence of an external stabilizing ramp. The small-signal transfer functions which he obtained were in excellent agreement with experimental measurements. Subsequently, I was able to show another way of arriving at the same conclusions by introducing the model of the current-controlled PWM (CC-PWM) switch [2] which describes the behavior of the average terminal voltages and currents of the PWM switch in response to the action of the current loop *including* the sample-and-hold effect.

Hence, the model of the PWM switch has no inherent limitations which prevents it from being used in the analysis of current-mode controlled converters. I would like to point that the current-mode model given by Kimhi and Ben-Yaakov [5], which Professor Ben-Yaakov references in his correspondence, is only an average low-frequency model which does not model the sample-and-hold action of the current loop and cannot predict the instability of current-mode control mentioned above. For example, their current-mode model predicts a second-order transfer function for the buck converter instead of a third-order transfer function (a dominant pole and a second-order resonance at half the switching frequency whose damping depends on the duty ratio and the magnitude of the external ramp) as explained in [1] and [2] and supported by plenty of experimental evidence. It is only at low frequencies that the magnitude of the transfer functions given in [5] agree with those of [1] and [2]. Hence, the switched inductor model (SIM) proposed by Professor Ben-Yaakov does not necessarily lead to more accurate modeling of current-mode control as one may infer from his correspondence.

4) The model of the PWM switch can be easily simulated as an invariant subcircuit on circuit analysis programs without an inductor. Requiring that the inductor be part of the switch as in the switched inductor model (SIM) adds no benefits neither to the simulation nor to the accuracy of the model, if anything, it diminishes the flexibility and versatility of the modeling technique. This is best demonstrated in the examples of the Cuk converter worked out in Parts I and II of my paper, [3 and 4], using the model of the PWM switch. Since the Cuk converter has two inductors connected to the common terminal *c*, the SIM, which has a single inductor connected to terminal *c*, is not compatible with the Cuk converter. The SIM model is also incompatible with other converters with two switched inductors such as the sepic and

zeta converters. In order for the SIM model to be applicable, each of these converters must first be decomposed into a cascade of converters with a single switched inductor—a fact which suggests that the SIM model is not as flexible as the PWM switch model.

I would like to conclude by reminding the reader that the primary purpose of the PWM switch model, as stated in [3, Part I, Introduction], was not simulation but education.

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