Implementation of Digital Signal Processing Techniques in the Design of Thermal Pulse Flowmeters

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Abstract—**Various digital signal processing methods, which could be applicable to the design of a microcomputer based thermal pulse flowmeter, were examined. Two excitation methods were investigated: a single thermal pulse and a pseudorandom binary sequence signal (PRBS). The signal recovered downstream was processed by two alternative numerical algorithms to recover the "time of flight", i.e., by peak detection of the signal itself and the peak of the differentiated signal. The recovered thermal pulse and the "time of flight" were then used to test the validity of two models: a diffusion-advection model and a simple time delay model. The delay model was found compatible with the data especially when the peak of the output signal derivative was used as a marker for determining the "time of flight." The single pulse injection method was found, in general, superior to the PRBS cross-correlation technique except for the ability of the latter to provide early indication of flow-rate variations.**

I. Introduction

THERMAL flowmeters have found a rather wide range of applications both in the research laboratory and industry [1]. The classical design of the thermal flow meter is based on the relationship between the electrical heating and flow-induced cooling of an element which is exposed to a liquid or gas flow [2]. A relatively recent approach in the design of thermal flow metering devices is based on the injection of a heat pulse marker and measuring the time it takes it to reach a temperature sensor downstream. Neglecting diffusion and second-order effects, the "time of flight" of the heat pulse marker for a Newtonian flow is given by

$$T_m = L/u \tag{1}$$

where

- u flow rate,
- L pulse traveling distance,
- T_m time delay ("time of flight").

It has been shown though, that the relationship between the "time of flight" and flow is better described by the experimental time delay model

$$T_m = \frac{L}{u} + \tau \tag{2}$$

or

$$T_m = \frac{V}{f} + \tau \tag{3}$$

where

- τ a constant delay
- V effective volume of flowthrough cell,
- f volumetric flow rate.

As is shown later, these equations are only first-order approximation of the theoretical functions and cannot describe accurately the system at low flow rates when heat diffusion is not negligibly small as compared to heat conversion (linear velocity). Aside from the question of the accurate functional relationship between the time delay and flow rate, the thermal pulse method poses a number of theoretical and practical problems when used to determine low flow rates (in the cubic centimeter/minute range). The main problem is the difficulty of accurately measuring the "time of flight" when the arriving heat pulse is smeared by diffusion and has a poor signal-to-noise ratio. Previous workers in this field have attempted to overcome this problem by various analog signal processing techniques [3]. A digital signal processing approach in which pulse arrival time was determined by numerical differentiation of the thermistors' response was also previously described [4].

The purpose of this study was to investigate the possible benefits of advanced digital processing techniques, which are compatible with low cost microcomputers, to the problem on hand. That is, to attempt to improve the accuracy of the thermal pulse flowmeter, at the low flow range, by digital excitation techniques and possibly better digital processing algorithms.

II. Theoretical Considerations

A. The Diffusion-Advection Model

Marshall [5] described a simple model for heat propagation through a stem of a living tree through which the sap is flowing [6]. The assumptions made are that the flow

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rate along a cross section is constant and that the inner volume is homogeneous. Under these conditions, which are similar to the ones for a flowthrough cell (Fig. 1), the thermal diffusion-advection equation becomes

$$\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial z} = k\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2}\right) \quad (4)$$

where

- θ temperature of flowing liquid,
- u linear flow rate along the z axis,
- k thermal diffusivity.

Assuming that no heat is lost to the surfaces and to the temperature sensor and that an infinitely short heat pulse is generated by a line source along the x axis, the solution to (4) is

$$\theta = \frac{C}{t}\exp\left[-\frac{(x-ut)^2 - y^2}{4kt}\right] \quad (5)$$

where the constant C is a function of various parameters such as heat pulse energy and specific heat capacity of the liquid. If the temperature is sensed at the center of the cross section and at a distance L downstream, the expected temperature pulse will be

$$\theta = \frac{C}{t}\exp\left[-\frac{(L-ut)^2}{4kt}\right]. \quad (6)$$

By differentiating (6) one can obtain the basic relationship between the "time of flight" and the flow rate:

$$u = \frac{\sqrt{L^2 - 4kT_m}}{T_m} \quad (7)$$

where T_m is the time delay between heat pulse injection and maximum (peak) temperature rise at a distance L, if

$$K \ll \frac{L^2}{4T_m} \quad (8)$$

then (6) reduces to the ideal relationship of (1). Equation (7), however, is only a first-order model since it neglects a number of factors which are variation of the flow profile along a cross section [7], heat loss to the boundaries, and finite size and heat capacity of the heat source and temperature sensor.

B. Excitation Alternatives

If the heat pulse duration is much shorter than T_m, the heat pulse sensed downstream is, in effect, the system's impulse responses. Hence, if the system is statistically stationary and linear with respect to temperature, the impulse response can be measured by any one of the system identification methods. In this study we chose to test and compare two methods:

1) injection of a single short heat pulse (an approximate Dirac function) and directly determining the impulse response by measuring the temperature variation downstream,

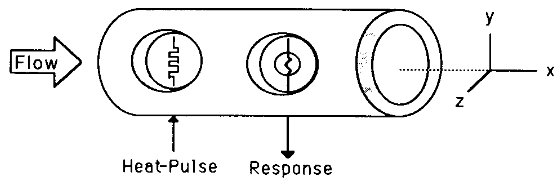

Fig. 1. Schematic representation of the thermal pulse flowmeter.

2) injection of pseudorandom binary pulse sequence (PRBS) [8] followed by cross-correlation of excitation and response [9] using a recursive algorithm.

C. Data Processing

Once the system's impulse response is determined, it has to be further processed to obtain the flow rate. In this study we tested and compared two signal processing approaches based on the two approximate models (2) and (7) described above. In the first case we attempted to extract the flow rate by fitting the impulse response data (obtained by either excitation method), to the diffusion-convection model. This was accomplished by least square fitting of the impulse response data to the logarithmically transformed model (7)

$$4t\ln(\theta t) = -\frac{L^2}{K} + \left(4\ln C + \frac{2Lu}{K}\right)t - \frac{u^2}{K}t^2 \quad (9)$$

which can be represented as a polynomial in t:

$$y = a + bt + ct^2 \quad (10)$$

where

$$y = 4t\ln(\theta t)$$

$$a = -\frac{L^2}{K}$$

$$b = \frac{2Lu}{K} + 4\ln(C)$$

$$c = -\frac{u^2}{K}.$$

If θ_i represent the temperature measured by the sensor down stream at t_i increments then

$$y_i = a + bt_i + ct_i^2 \quad (11)$$

where y_i is a function of θ_i and t_i. Using linear least square fitting, the coefficients can be estimated from the sampled data and u can then be calculated from the relationship

$$u = L\sqrt{c/a}. \quad (12)$$

Determination of the flow rate through the model of (3) requires an *a priori* knowledge of τ and V. The two can be estimated by the least-square fitting method from a set of experimental data in which T_m and f (the volumetric flow rate) have been measured:

$$T_m = V(1/f) + \tau. \quad (13)$$

By this procedure both V (which might differ from the physical value) and τ can be estimated. T_m is determined by using the response of the temperature sensor downstream as a marker. However, since the signal is digitized, the resolution that can be obtained by a direct peak search will heavily depend on the sampling rate. As already shown [10], better peak determination can be achieved by fitting the peak signal to a known function. Peak position can then be more precisely evaluated from the analytical expression of the fitted function. The procedure adopted here was as follows: The sampling time t_k corresponding to the neighborhood of peak position was first detected by a simple Euler derivative. Using the t_k sample and four other samples (two on each side), the data were then fitted to a second-order polynomial of the form

$$\theta_i = a_0 + a_1 t_1 + a_2 t_1^2. \tag{14}$$

The relevant coefficients are obtained from the difference equations [11]

$$a_1 = \tfrac{1}{2}[-2\theta_{k-2} - \theta_{k-1} + \theta_{k+1} + \theta_{k+2}] \tag{15}$$

$$a_2 = \tfrac{1}{14}[2\theta_{k-2} - \theta_{k-1} - 2\theta_k - \theta_{k+1} + 2\theta_{k+2}]. \tag{16}$$

T_m (referred to heat pulse onset) can now be estimated from

$$T_m = t_k - (a_1/a_2)\Delta t \tag{17}$$

where Δt is the sampling time increment.

III. Experimental

A general description of the experimental set up is depicted in Fig. 2. The flow cell was made out of Plexiglass with an inner bore of 2.5 mm. This diameter was chosen small enough to obtain high linear velocities but large enough as compared to the size of the thermistor.

The heating element was a Nichrome wire, which was placed along the diameter of the bore and hence perpendicular to the flow. The pulse excitation current range was 3–9 A. A model 35A36 (VECO, USA) thermistor was placed 10 mm downstream from the heating element. A similar thermistor was placed 50 mm upstream from the heating element and was used to balance out the ambient temperature signal. The two thermistors were incorporated in a bridge circuit whose output was only a function of the heat pulse sensed downstream. Since the maximum temperature rise was in the order of 2°C, the output signal of the bridge was, to a first approximation, linear with temperature variations. The bridge output signal was amplified by a difference amplifier and fed to a personal computer via an A/D D/A board DAS − 16F (Metrabyte, USA). This board was also used to excite the heating element (via a power amplifier). The A/D resolution was 12 b. The pulse injection frequency and signal sampling rate, in the single thermal pulse method, were adaptively selected to match the flow rate. The thermal pulse injection rate must be low enough to ensure complete settling at the sensing point before the next pulse arrives. Otherwise, pulse piling will occur, resulting in a distorted output signal. Therefore, for optimum operation, i.e., fasted response, the injection rate must be adjusted to the flow rate. The desired relationship was determined experimentally and was used by the signal processing program for automatically adjusting the injection rates. For the present experimental range of 26–0.1 cc/min (a linear velocity range of 8.6–310.2 cm/s) the delay between injected pulses was 2.3–255 s. The output signal sampling rate was also adjusted adaptively from 65 samples per second for the highest flow rate to 2 samples per second for the low flow rates. The adaptive approach was also used in the PRBS excitation and sampling. Here one must make sure that the PRBS cycle is longer than the impulse response transient. In these experiments, pulse injection and sampling rates must be equal. Each rate was adaptively selected by the signal processing program over the frequency range of 55–2 Hz. The PRBS was synthesized by a simulated shift register [8] with 7–9 cells for the experimental flow range. This corresponds to PRBS cycles of 121–511, respectively. The calculated flow rates were calibrated gravitometrically [12].

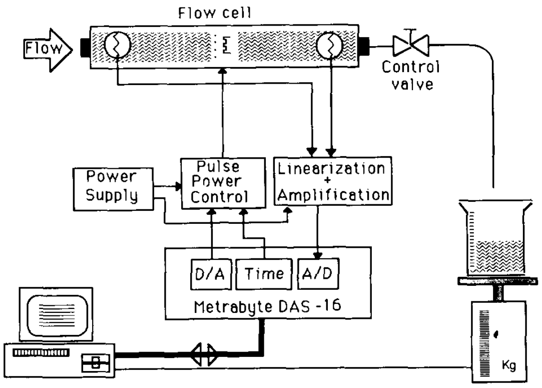

Fig. 2. Block diagram of experimental setup.

IV. Results and Discussion

A. Excitation Alternatives

The system's impulse responses obtained by a single heat pulse injection and by PRBS injection (after cross correlation) were found to be in agreement (Fig. 3). However, the signal-to-noise ratio of the recovered PRBS injection was much poorer than the one obtained by the single injection. In fact, the signal-to-noise ratio deterioration below about 1 cc/min flow rendered the data useless. As the signal is recovered by a rather lengthy and involved calculation [11], small inaccuracies seem to significantly affect the cross-correlation results.

B. Estimation of "Time of Flight"

The proposed method for the estimation of T_m (previously mentioned) was tested for both the single and PRBS pulse injection against the model of (2). Both methods

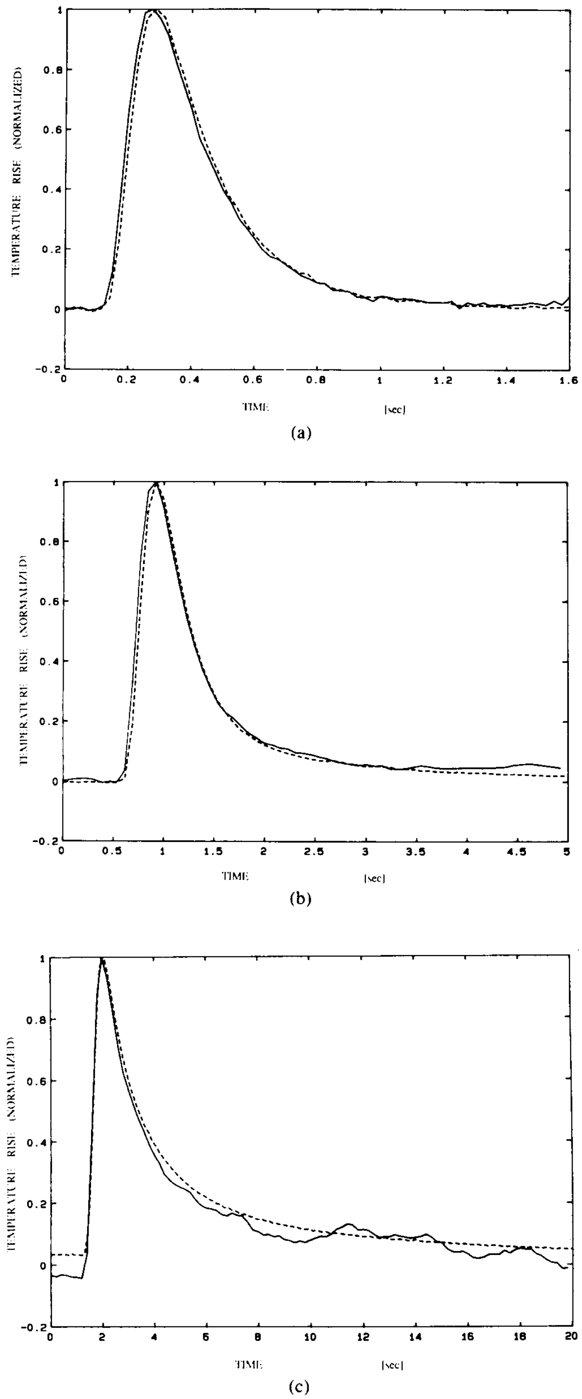

Fig. 3. Recovered heat pulse by direct response to a single pulse (broken line) and after cross correlation of PRBS signal (solid line) for (a) 25.8 cc/min; (b) 3.17 cc/min; and (c) 1.26 cc/min.

yielded good fit (Fig. 4) but with somewhat different coefficients. The latter is of no consequence, however, since in both cases the calibration curve must be derived experimentally.

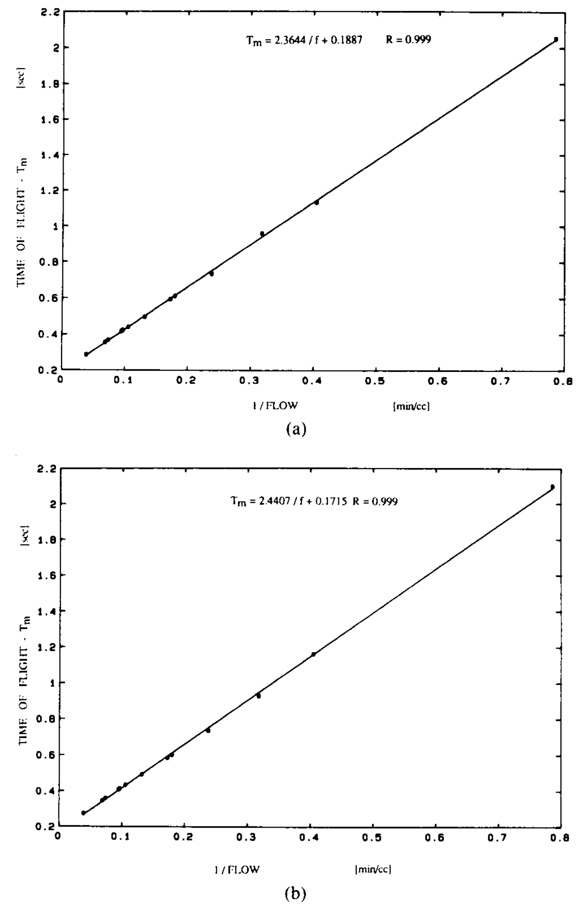

Fig. 4. Calibration curve for (a) single pulse method and for (b) the PRBS-cross-correlation method.

A better agreement between the experimental data and the model of (2) and (3) was obtained when T_m was measured to the peak of the response signal derivative [2]. In this case, the heat pulse response was first subjected to the numerical differentiation algorithm and then to the peak fitting as discussed above. It was found that in this case, one calibration curve (according to (13)) covers a much wider dynamic range, down to approximately 0.1 cc/min (Fig. 5), at least an order of magnitude larger than was obtained for the original peak picking procedure. No explanation has been found for the improvement.

C. Conformity to the Diffusion-Advection Model

Attempts to fit the experimental data to the model of (7) were unsuccessful. Our conjecture is that this discrepancy is due to the failure of this model to take into account the responses from the electric pulse to the thermal pulse and that of the thermistor used to detect the heat pulse downstream. The latter might be the dominant interfering cause in this particular case. As a first approximation one might assume that the thermistor's response is that of a first order system and one can attempt to correct the corrupted

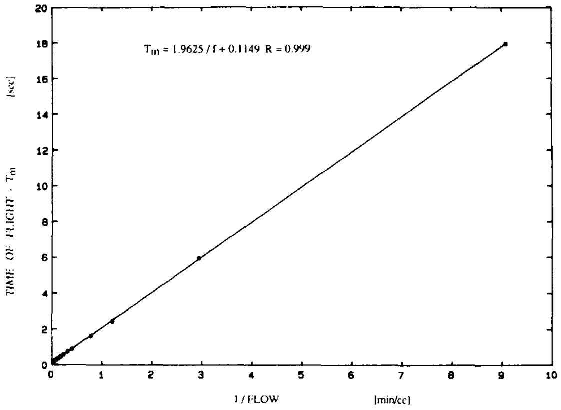

Fig. 5. Calibration curve for the single pulse injection method when the peak of signal derivative is used as time marker.

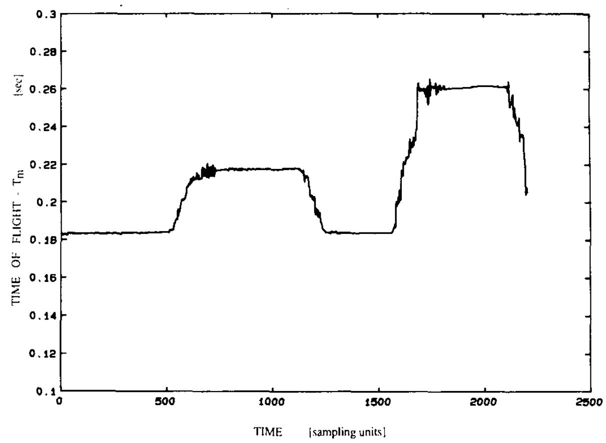

Fig. 6. Output responses of the cross-correlation flowmeter to step changes in flow.

signal by the reverse function. This approach has been previously suggested as a method to make a temperature sensing system "transparent" to the sensor's response [13], [14]. We have found, however, the thermistor's response is highly nonlinear and that a different representation of the time constants T had to be used for each flow. Consequently, this correction method is impractical in real applications. It would appear that the delay due the thermistor's response is somehow already incorporated in the experimental equations (2) and (3). The delay τ is apparently a lumped representation of all delays of the system as well as of other nonidealities such as the nonconstant flow profile along a cross section [7].

D. Flow-Rate Tracking

One of the assumptions made when estimating the impulse response by the cross-correlation technique is that the system is linear and statistically stationary. This implies that accurate measurements of flow rate by the injection of thermal PRBS followed by cross-correlation is possible only for constant flow rates. That is, although the cross-correlation output can be obtained continuously (using a recursive algorithm [15]), one should not expect an immediate response if the flow rate is changing stepwise. The physical explanation for the expected delay is the inherent lag between the time a heat pulse is injected until it reaches the thermistor. To test this issue, we have subjected the flow rate to a stepwise change while continuously computing the cross correlation by a recursive algorithm and subsequently estimating T_m (Fig. 6). The response was not immediate, as expected, but fortunately the continuous output did not show any irregularities. Rather, the average response is a gradual change from one level to the other. A full recovery was obtained after one complete cycle of the PRBS which was chosen to be compatible with the impulse response of the system. This delay corresponds to the maximum rate at which pulses can be injected in the single pulse technique, to avoid the piling up of thermal pulses and hence distortion of the recovered heat pulse.

V. Conclusions

The results of the present study suggest that both the single heat pulse and PRBS methods can be implemented in the design of a microcomputer base liquid flowmeter. Our data does not support the conjecture that the PRBS is a better or more sensitive technique. The PRBS cross-correlation method could be useful for higher flow rates when a better signal-to-noise ratio is available. This technique has some advantage in following changes in flow rates as it provides a continuous, rather smooth, estimate of the transition.

The results of our investigations further suggest that a more precise flow-rate estimate is obtained when the "time of flight" is estimated from the time delay between pulse injection and the *peak of the response derivative*. The calibration curve obtained by this method covered a wider dynamic range than the one obtained by measuring the time delay to the peak of the response itself. The simple algorithm suggested for estimating peak time (of the heat pulse or its derivative) was found effective and easy to implement.

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