Maximum Power Tracking of Piezoelectric Transformer HV Converters Under Load Variations

Shmuel (Sam) Ben-Yaakov, *Member, IEEE,* and Simon Lineykin

***Abstract*—The problem of maximum power point tracking of high output dc voltage converters that apply piezoelectric transformers (PT) and voltage doublers was studied theoretically and experimentally. It was shown that the operating frequency of the PT, at which maximum power is reached, is a function of the load. Hence, under load variations, and to overcome parameter instability, there is a need for some frequency tracking mechanism that will help to lock the operating frequency to the optimum one. The proposed method to achieve frequency tracking is based on a phase locked loop (PLL). The PLL inputs are the phase of the input voltage driving the PT and the phase of the current flowing through one of the voltage doubler diodes. Theoretical analysis, verified by experiments, shows that when the phase shift of the diode current relative the phase of the input voltage is zero, the voltage gain of the system is at its maximum. By applying this approach, the system's operation can be made independent of input voltage, load variations, temperature (within a permitted range), and the spread and nonlinearity of the PT parameters, as well as their drift with time.**

***Index Terms*—Phase locked loop (PLL), piezoelectric transformers (PTs).**

I. Introduction

The main advantages of piezoelectric transformers (PTs) are potential low cost, small size, low profile, good insulation capability, and the absense of windings, and hence, magnetic fields. In some specific applications, PTs are superior to electromagnetic transformers, making the PT a good design choice. Among these is the generation of a low power, high dc voltage (HV). Rosen type (see Fig. 1) PTs have a high gain ratio that, when combined with excellent insulation properties of the PT, make it a good candidate for the construction of compact HV converters—up to few kilovolts. Since the PT is a resonant element, its output to input voltage gain is strongly dependent on the operating frequency [1].

For any given load, the problem of maximum power tracking translates into searching and locking to the frequency that provides a maximum voltage gain. Hence, to maintain maximum output voltage under variable operating conditions (load variation, temperature changes and components tolerances) it is necessary to lock the operating frequency to the one that will ensure the highest possible output voltage for a given load.

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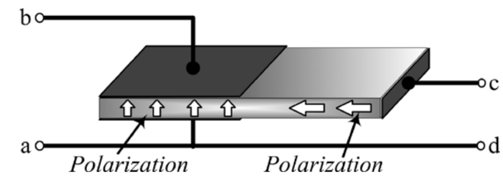

Fig. 1. Rosen type PT.

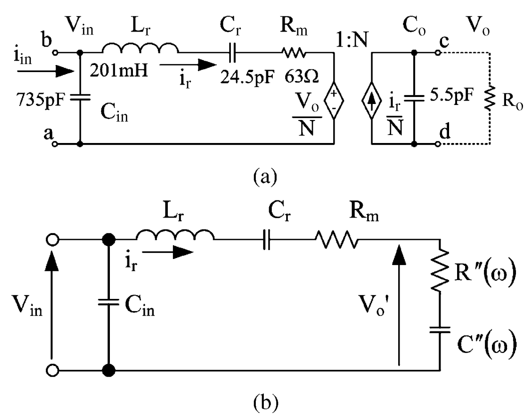

Fig. 2. Equivalent circuits of a PT: (a) original equivalent circuit and (b) simplified equivalent circuit reflecting secondary to the primary side. Values are for PXE43, Philips, operating around 73 kHz.

As a prerequisite for solving the frequency-tracking problem, one needs first to find a parameter that can be used as a measure of the deviation from the desired frequency. Different approaches have been suggested for frequency tracking of PT drivers. In [2], the phase angle between input voltage and input current was used as a criterion while in [3], the phase angle between input and output voltage was used as a measure of the deviation from the frequency of maximum gain. Unfortunately these criteria are load dependent and could be used only over a narrow load resistance range.

The difficulty in locating the optimum tracking parameter can be appreciated by considering the equivalent circuit of a typical PT [Fig. 2(a)] and its series resonance representation [Fig. 2(b)].

As can be easily observed from Fig. 2(a) [1], the frequency of the maximum output to input voltage ratio will be between f_{oc} for high resistance loads (close to the open circuit situation, load is negligible) and f_{sc} for low resistance loads, when C_{o} is practically shorted

$$f_{\mathrm{oc}} = \frac{1}{2\pi}\sqrt{\frac{(C_r + N^2 C_{\mathrm{o}})}{(L_r C_r N^2 C_{\mathrm{o}})}} \tag{1}$$

$$f_{\mathrm{sc}} = \frac{1}{2\pi\sqrt{L_r C_r}}. \tag{2}$$

Thus, the frequency giving the maximum output to input voltage ratio is a strong function of the load resistance. However, since the series resonant branch [Fig. 2(b)] is responsible for the input to output power transfer, it stands to reason that maximum voltage gain will be obtained when the operating frequency is locked to this series resonance. In this study we explored this possible criterion for frequency locking to maximum power and propose a novel method for obtaining a reliable bipolar signal that is a measure of the deviation from the optimal frequency. It is then demonstrated how this signal can be used to lock the frequency to the optimal one.

II. Rosen Type PT in HV Applications

The Rosen-type piezoelectric transformer 48×12 × 2 mm (Phillips [4]) was chosen for the present research (see Fig. 1). This unit has a large output to input voltage ratio at the frequency of the maximum output voltage, relatively high power (up to 5 W), and high input to output insulation (tens of kilovolts). The equivalent circuit of a PT operating near its resonance point [Fig. 2(a)] includes a resonant network (L_r, C_r, $R_{\rm m}$) that emulates the effect of the mechanical vibration and dependent sources that express the mechanical to electrical energy transformation [1]. The model also comprises the physical dielectric capacitors ($C_{\rm in}$, $C_{\rm o}$) that are formed by the input and output electrodes. Since the network is highly selective it will pass, with reasonable gain, only frequencies that are in the vicinity of the resonant frequency.

To simplify the analysis some equivalent transformations of the equivalent circuit can be applied. First, the output part [right-hand side of Fig. 2(a)] of the equivalent circuit is reflected to the input side. $R_{\rm o}$ and $C_{\rm o}$ of the equivalent circuit are transformed to $R'_{\rm o}$ and $C'_{\rm o}$. Output voltage $\rm V_o$ is transformed to $\rm V'_o$

$$R'_{\rm o} = \frac{R_{\rm o}}{N^2} \quad (3)$$

$$C'_{\rm o} = N^2 C_{\rm o} \quad (4)$$

$$\rm V'_o = \frac{V_o}{\mathit{N}}. \quad (5)$$

Second, the parallel network $R'_{\rm o}, C'_{\rm o}$ is transformed to a series frequency dependent network $R''(\omega), C''(\omega)$ [Fig. 2(b)], where

$$R''(\omega) = \frac{R_{\rm o}}{N^2\left(1 + C_{\rm o}^2 R_{\rm o}^2 \omega^2\right)} \quad (6)$$

$$C''(\omega) = \frac{N^2\left(1 + C_{\rm o}^2 R_{\rm o}^2 \omega^2\right)}{C_{\rm o} R_{\rm o}^2 \omega}. \quad (7)$$

The voltage transfer function of a PT (in terms of equivalent circuit parameters) is

$$\frac{\rm V_o}{\rm V_{in}}(\rm s) = \frac{{\rm N}C_r R_{\rm o}{\rm s}}{C_r R_{\rm o}{\rm s} + N^2(1 + C_{\rm o}R_{\rm o}{\rm s})\left(1 + C_r(R_{\rm m} + L_r{\rm s}){\rm s}\right)} \quad (8)$$

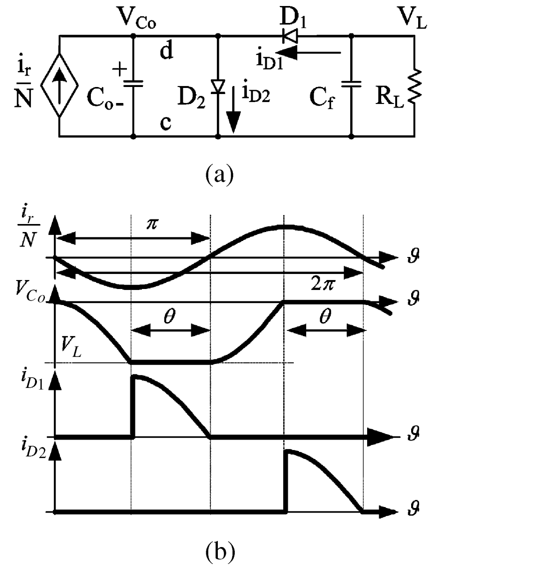

Fig. 3. Output stage of PT with voltage-doubler rectifier: (a) topology and (b) voltage-doubler waveforms versus angle $\vartheta = \omega$t.

As one can see, the system is of third order. An analysis of this system is given in [1]. In terms of angular frequency $\omega = 2\pi f$, the transfer function will be (9), shown at the bottom of the page.

In a typical HV application, one would use a voltage doubler to boost the output voltage. Hence, the output section will be constructed as shown in Fig. 3(a), where C_f is the output filter capacitor and R_L is the load resistance.

Assuming that the quality factor (Q) of the PT is high, the current through the primary series resonant circuit i_r will be sinusoidal. This current, after being transferred to the secondary, charges and discharges capacitor $C_{\rm o}$ of the output section [Fig. 3(b)].

As shown in [5], the voltage doubler and load section can be represented as an equivalent reactive load (a resistor in parallel to a capacitor). Consequently the gain function (8) is valid for this case too, except that the load resistance $R_{\rm o}$ and output capacitance $C_{\rm o}$ in Fig. 2(a) need to be replaced by an equivalent resistor and capacitor, respectively.

III. Criteria of Maximum Power Point Frequency

One can obtain the analytical expression for the frequency of the maximum output to input voltage ratio by taking the derivative of (9) with respect to ω and equating it to zero (10). This results in a third-order equation, which can be solved, for example, by using Cardan's method. Since the analytical expression was found to be too cumbersome, we have solved (10) numerically

$$2C_r^2 N^6 R_{\rm o}^2 - \left(2C_r^4 L_r^2 N^6 R_{\rm o}^2 + 4C_{\rm o} C_r^4 L_r N^4 R_{\rm o}^4 + 4C_{\rm o}^2 C_r^3 L_r N^6 R_{\rm o}^4 - 2C_{\rm o}^2 C_r^4 N^6 R_{\rm m}^2 R_{\rm o}^4\right) {\rm x}^2 - 4C_{\rm o}^2 C_r^4 L_r^2 N^6 R_{\rm o}^4 {\rm x}^3 = 0 \quad (10)$$

$$\frac{\rm V_o}{\rm V_{in}}(\omega) = \frac{C_r R_{\rm o} N \omega}{\sqrt{N^4\left(\omega^2 C_r (L_r + C_{\rm o} R_{\rm o} R_{\rm m}) - 1\right)^2 + \omega^2\left(N^2 C_{\rm o} R_{\rm o} + C_r\left(R_{\rm o} + N^2(R_{\rm m} - \omega^2 C_{\rm o} L_r R_{\rm o})\right)\right)^2}} \quad (9)$$

where: $x = (2\pi f_{\max})^2$, $f_{\max}$—frequency of the maximum of PT's transfer function.

The proposed criterion for maximum voltage gain is zero phase-shift between the input voltage $\mathrm{V_{in}}$ and the virtual current i_r of the resonant branch of the equivalent circuit [Fig. 2(b)]. The basis of this hypothesis is that maximum output power will be obtained when the PT is driven at the series resonance frequency. This is further supported by the observation made in [1] that the behavior of transfer function (9) is like that of a second order R–L–C band-pass filter, because within the narrow band around the resonant frequency, $R''(\omega)$ and $C''(\omega)$ change slowly.

Based on the simplified equivalent circuit of Fig. 2 the transfer function $(i_r/\mathrm{V_{in}})(\mathrm{j}\omega)$ is

$$\frac{\mathrm{i}_r}{\mathrm{V_{in}}}(\mathrm{j}\omega) = \frac{\mathrm{j}\omega C_{\mathrm{eq}}(\omega)}{1 - L_r C_{\mathrm{eq}}(\omega)\omega^2 + \mathrm{j}\omega R_{\mathrm{eq}}(\omega) C_{\mathrm{eq}}(\omega)} \tag{11}$$

where $R_{\mathrm{eq}}(\omega)$ and $C_{\mathrm{eq}}(\omega)$ are expressed in (12) and (13) as a function of the parameters shown in Fig. 2

$$R_{\mathrm{eq}}(\omega) = R_{\mathrm{m}} + R''(\omega) \tag{12}$$

$$C_{\mathrm{eq}}(\omega) = \frac{C_r C''(\omega)}{C_r + C''(\omega)}. \tag{13}$$

From (11), one can see that zero phase-shift between i_r and $\mathrm{V_{in}}$ occurs when the imaginary part is zero, i.e.

$$1 - L_r C_{\mathrm{eq}}(\omega_{\mathrm{opt}})\omega_{\mathrm{opt}}^2 = 0 \tag{14}$$

or

$$\omega_{\mathrm{opt}} = \frac{1}{\sqrt{L_r C_{\mathrm{eq}}(\omega_{\mathrm{opt}})}}. \tag{15}$$

Applying (6), (7), (12), (13), and (15) the frequency of zero phase-shift f_{opt} gives (16), shown at the bottom of the page.

This frequency of series resonance of the equivalent circuit is not identical to $f_{\max}$ but is very close to it. For the experimental high voltage PT used in this study, the maximum difference between $f_{\max}$ and f_{opt} is less than 50 Hz (less than 0.1% of the operating frequency) and the phase-shift of i_r at $f_{\max}$ is less than 1.2° (Fig. 4). Fig. 5 shows the ratio of the PT's output power at $f_{\max}$ to the output power at f_{opt}. These numerical values clearly point out to the fact that f_{opt} is a very good approximation of $f_{\max}$. Hence, the phase shift of i_r can be used for all practical purposes as a sensing signal for the deviation from $f_{\max}$. The objective of the tracking system will thus to be zero this phase shift.

IV. Proposed Tracking Method

The results of Section III suggest that the maximum gain is reached when the phase angle between the input voltage $\mathrm{V_{in}}$ and i_r is zero.

Unfortunately, there is no direct way to measure this current or its phase since there is no physical access to the (virtual) series branch. This is overcome in the proposed method by an indirect measurement that makes use of the observation that the current of diode D_2 is in fact a sample of the current i_r [Fig. 3(b)]. Note, in particular, that D_2 will stop conducting when the polarity of the current i_r is reversed. Hence, this polarity reversal instant can be used as an indicator for the phase of the series current. This proposed sensing method is demonstrated in Fig. 6. Comp1 and Comp2 are used to generate two square waves; one is synchronized to $\mathrm{V_{in}}$ while the second one is synchronized with i_r. The phase angle between $\mathrm{V_{in}}$ and i_r can thus be measured by feeding these two signals to a phase detector.

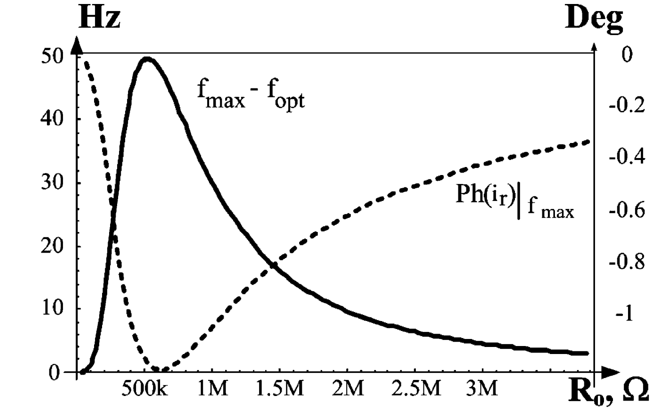

Fig. 4. Phase of i_r when $f = f_{\max}$ (dashed-line), and difference between $f_{\max}$ and f_{opt} (solid line).

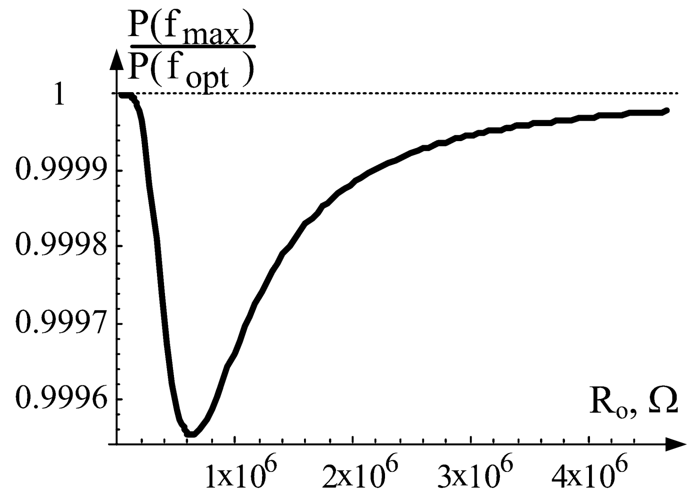

Fig. 5. The ratio between the output power of PT driven at f_{opt} to the output power at $f_{\max}$ as a function of R_{o}.

V. Experimental Results

The experimental circuit (Fig. 7) included a phase locked loop (PLL) fed by the two phase signals. The digital frequency

$$f_{\mathrm{opt}} = \frac{\sqrt{C_{\mathrm{o}} C_r R_{\mathrm{o}}^2 + C_{\mathrm{o}}^2 N^2 R_{\mathrm{o}}^2 - C_r L_r N^2 + \sqrt{4C_{\mathrm{o}}^2 C_r L_r N^4 R_{\mathrm{o}}^2 + \left(C_r L_r N^2 - C_{\mathrm{o}} C_r R_{\mathrm{o}}^2 - C_{\mathrm{o}}^2 N^2 R_{\mathrm{o}}^2\right)^2}}}{\pi C_{\mathrm{o}} \mathrm{N} \mathrm{R_o} \sqrt{8 C_r L_r}} \tag{16}$$

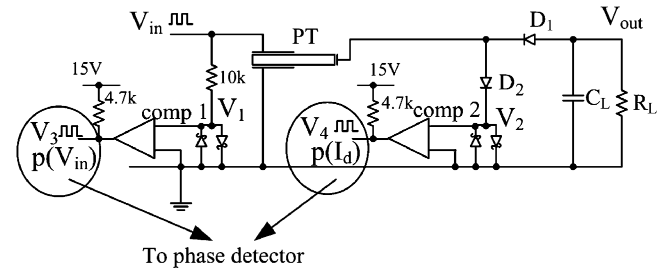

Fig. 6. Proposed method for extracting the phase signals of $\mathrm{V_{in}}$ and i_r.

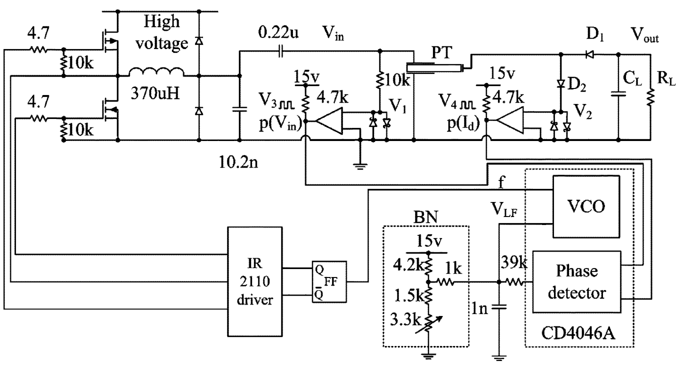

Fig. 7. Experimental setup of the proposed frequency tracking to maximum output voltage.

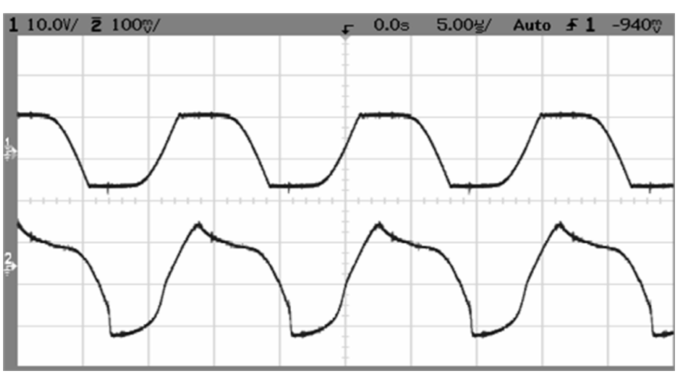

Fig. 8. $\mathrm{V_{in}}$ (upper trace) and $\mathrm{V_2}$ (lower trace) (see Fig. 6) at the series resonant frequency.

phase detector of the PLL (CD4046A) compares these rectangular waveforms and feeds the VCO with the phase error signal. The PLL was designed to have a lock-in range $f_L < (f_{\max} - f_{\min})/2$ that covers the frequency range $f_{\rm sc}$ to $f_{\rm oc}$ (1), (2). Typical experimental waveforms are shown in Figs. 8–10. A bias network (BN) was used to slightly shift the zero point such that frequency locking is obtained when there is a small phase shift between the Comp1 and Comp2 signal. This was found necessary for the compensation of the phase shift caused by the parasitic capacitances of the diodes used as clamps (Fig. 7). This phase offset was found to be constant and independent of load resistance. Practical designs of this tracking system should attempt to minimize this parasitic effect by choosing low capacitance diodes.

The objective of the experimental work was to validate the ability of the proposed method to track the frequency of maximum output voltage and to ascertain that the operation is stable. These goals were achieved by conducting two different experiments. In the first one, a comparison was made between the

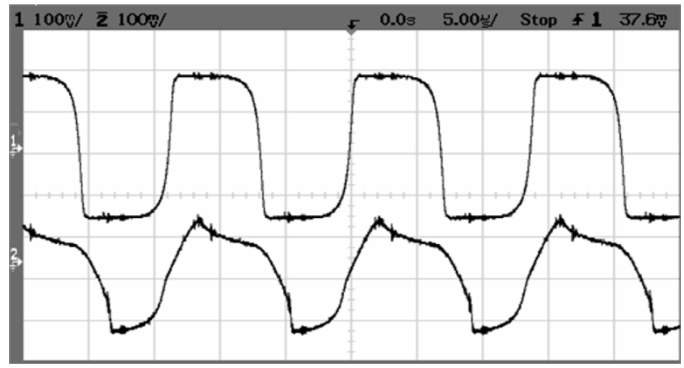

Fig. 9. $\mathrm{V_1}$ (upper trace) and $\mathrm{V_2}$ (lower trace) at the series resonant frequency. (See Fig. 6).

Fig. 10. $\mathrm{V_3}$ (upper trace) and $\mathrm{V_4}$ (lower trace) of the comparators outputs (see Fig. 6).

output voltage measured with and without frequency tracking, while the PT was subjected to a temperature rise. In the second set of experiments the output voltage was measured, under open and closed loop conditions, while the load was switched from one resistance value to another.

A. Frequency Tracking While the PT is Exposed to a Temperature Rise

The output voltage of the system was measured while the PT was heated. Since the parameters of the PT are temperature dependent, the frequency of maximum output voltage is expected to change as the temperature of the PT is varied. Fig. 11 summarized the measured output voltage when the PT was exposed to a temperature change of 30 °C to 65 °C. With frequency tracking the voltage drop at 65 °C was only 2% while with no tracking the output voltage dropped by 34%. It should be pointed out that the small 2% variation could be due to less efficient operation at the elevated temperature and not necessarily improper frequency tracking.

B. Response to a Load Step

The frequency tracking system was also tested under static and dynamic conditions. In the dynamic response tests, the system was subjected to a load variation by an auxiliary switch (see Fig. 12). First, the open loop output voltage as a function of the drive frequency was measured for each load (see Fig. 13). The plot clearly shows that maximum output voltage is obtained at different frequencies f_1 and f_2, corresponding to $R_L = 1.2$ MΩ and $R_L = 1.76$ MΩ, respectively. Three

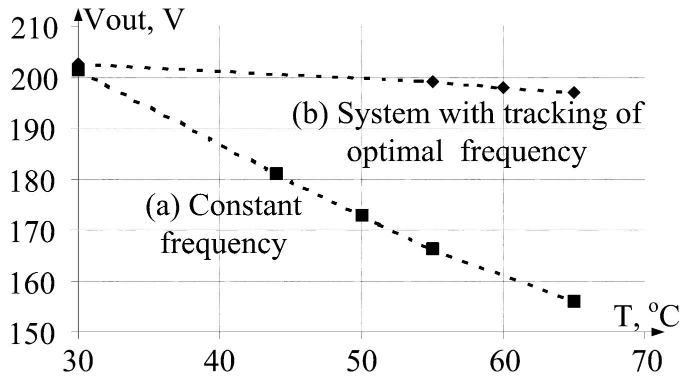

Fig. 11. Output voltage versus PT temperature. (a) Constant frequency operation. (b) With proposed frequency tracking scheme.

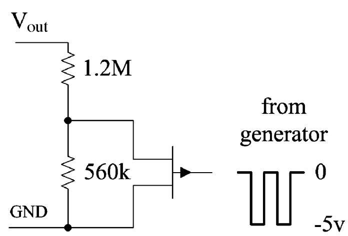

Fig. 12. Variable load.

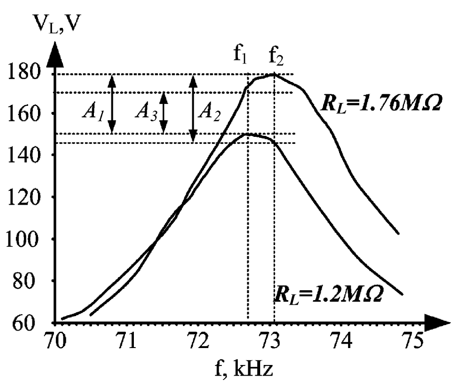

Fig. 13. Steady state output voltage as a function of the operation frequency f, for two values of the load resistance R_L. (V_{in} = 20.3 V).

experiments were conducted: (a) operating the system under closed loop conditions, (b) operating the system under open loop condition with drive frequency f_1, and (c) operating the system under open loop condition with drive frequency f_2. It is expected that under a switched load situation, each one of these conditions will produce a different output voltage step per Fig. 13. In case (a) the step should be A_1 (from 150.5 to 178.5 V); in case (b) it should be A_3 and in case (c) it should be A_2. Fig. 14 shows the operation under closed loop conditions as per case (a). The output voltage variations as well as the frequency hopping matched the expected ones (A_1 in Fig. 13).

Fig. 15 shows the results for case (b) while Fig. 16 depicts the results for case (c). The amplitudes of the output voltage steps were found to match to the expected ones: A_2 and A_3 (see Fig. 13), respectively. The results of these tests suggest that the tracking circuitry of the experimental setup is functioning as expected.

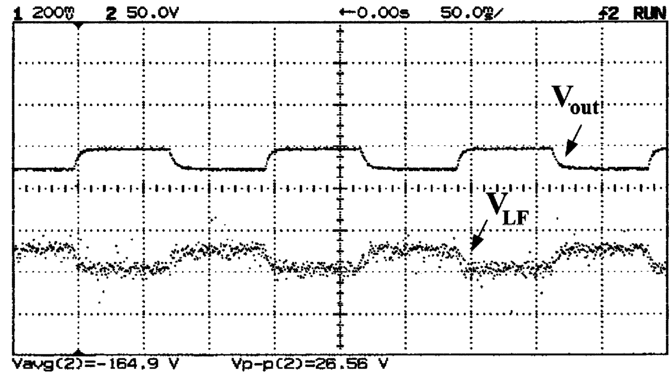

Fig. 14. PLL response under closed loop conditions. Amplitude of V_{out} is equal to A_1 of Fig. 13. V_{LF} is the output voltage of the loop filter, (VCO input, Fig. 7).

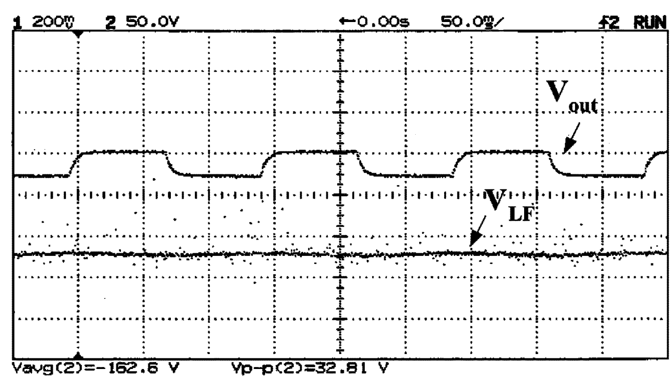

Fig. 15. Open loop response when the drive frequency is f_1—the frequency of resonance when R_L = 1.2 MΩ. Amplitude of V_{out} is equal to A_2 of Fig. 13. V_{LF} is constant voltage to the VCO input (Fig. 7).

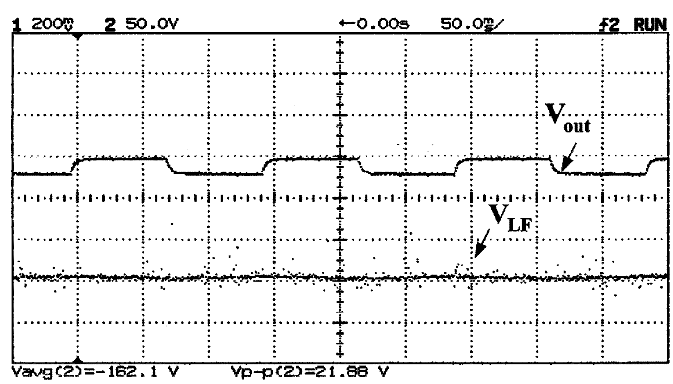

Fig. 16. System without PLL control. Drive frequency is f_2—the frequency of resonance with R_L = 1.76 MΩ. Amplitude of V_{out} is equal to A_3 of Fig. 13. V_{LF} is constant voltage to the VCO input (Fig. 7).

VI. Conclusion

The proposed tracking method offers a way to lock to the frequency that provides the maximum output power for any load. This could be useful in various applications that need to generate high output voltage (e.g., ionization equipment, sparkers and the like). By applying the proposed approach, the system's operation can be made independent of input voltage, load variations, temperature (within the permitted range), and the spread and nonlinearity of the PT parameters, as well their drift with time.

The phase detection method proposed here can also be used in cases that call for output voltage regulation. In such cases, a simple feedback loop via a voltage controlled oscillator (VCO) would be ambiguous. If, say, the output voltage is too low, should the frequency be increased or decreased? This ambiguity can be resolved by applying the phase detection method proposed here that generates a clear unequivocal bipolar signal.

The proposed method was verified experimentally and it was demonstrated that the control circuitry needed for the implementation is simple and can be easily constructed from off-the-shelf components.

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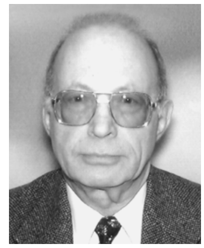

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