Analysis and Design of a Piezoelectric Transformer AC/DC Converter in a Low Voltage Application

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Abstract **- steady-state processes in a power PT converter with a halfway two diodes output rectifier are studied. It is shown that such a power converter can operate in two modes: with or without overlap of the conduction intervals of the diodes. The analytical equations and characteristics derived in this study were found to be in good agreement with simulation and experimental results. The study highlights the effect of the load and parasitic resistances on the output voltage and the losses for both modes. Expressions obtained in the paper can be used in engineering designs.**

I. INTRODUCTION

Piezoelectric Transformers (PT) convert electric power into electric power via a mechanical resonant link. PT's have important advantages over magnetic transformers in a few applications that require a low DC output voltage [1-6]. In this case a halfway output rectifier with two diodes is the preferred choice. However, in earlier investigations only one operational mode of this rectifier was studied: when the conducting intervals of both diodes of the rectifier do not overlap. The presented paper shows that this operation mode is not unique. In a wide range of load resistances the rectifier operates in another mode in which the conduction intervals of the diodes overlap. The paper presents a generalized detailed analysis of both operating modes. The results of this analysis reveal new important characteristics of this rectification scheme.

II. PRINCIPLE OF OPERATION OF THE HALF-WAY TWO DIODES RECTIFIER

The conventional equivalent circuit of the PT was used to analyze the low voltage AC/DC converter topology (Fig. 1). It includes $L_r, C_r, R_m, C_{in}, C_O$, a voltage source $\frac{v_{Co}}{n}$ and a current source $\frac{i_r}{n}$ (n is the transformer turn ratio) [7]. A halfway rectifier Rect and an inductor L_o are connected at the output of the PT. The rectifier includes two diodes (D_1, D_2) and L_f, C_f filter. R_L is the load resistance. The function of L_o in this circuit is to provide a path for the DC current and also to cancel the circulating energy caused by C_o [2]. The resonant frequency of the parallel circuit L_o, C_o has to be equal to the series resonant frequency of the PT.

The simulated current and voltage waveforms of the rectifier operating in overlapping and non-overlapping modes are given at Fig. 2 a, b ($\vartheta = \omega t$ is the normalized time, ω is the operating angle frequency). The PT current i_r is assumed to be sinusoidal in both modes due to the high Q of practical PTs. The capacitor C_o voltage is practically sinusoidal in the non-overlapping mode but has a discontinuous nature (intervals of zero voltage) in the overlapping mode. We study here in detail the overlapping mode.

During the interval $\vartheta_1\vartheta_2 = \lambda$ the voltage v_{Co} across the capacitor C_o is positive, therefore the diode D_1 is ON and the diode D_2 – OFF. At ϑ_2, the polarity of the voltage v_{Co} changes and therefore D_2 turns on. If just before ϑ_2, the instant current of D_1 (i_{D1}) is higher than the absolute value of the capacitor C_o current (i_{Co}) the rectifier will operate under overlapping conditions.

In this case the diodes D_1 and D_2 are conducting simultaneously during the interval $\vartheta_2\vartheta_3$, the current i_{D2} is increasing while the current i_{D1} is decreasing (due to changing i_r).

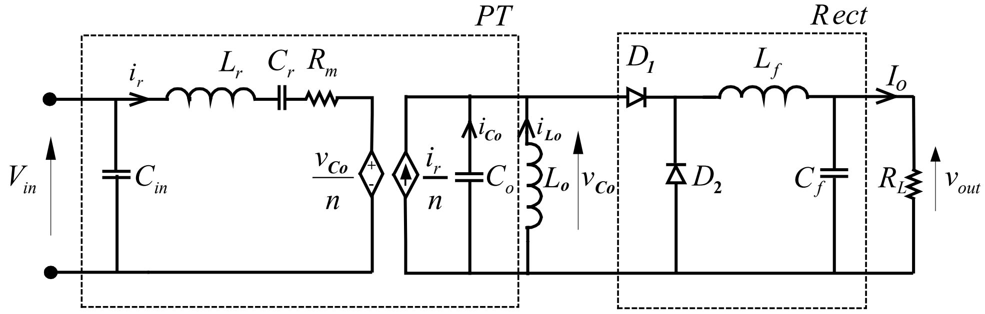

Fig. 1. The PT converter loaded by a halfway two diodes rectifier.

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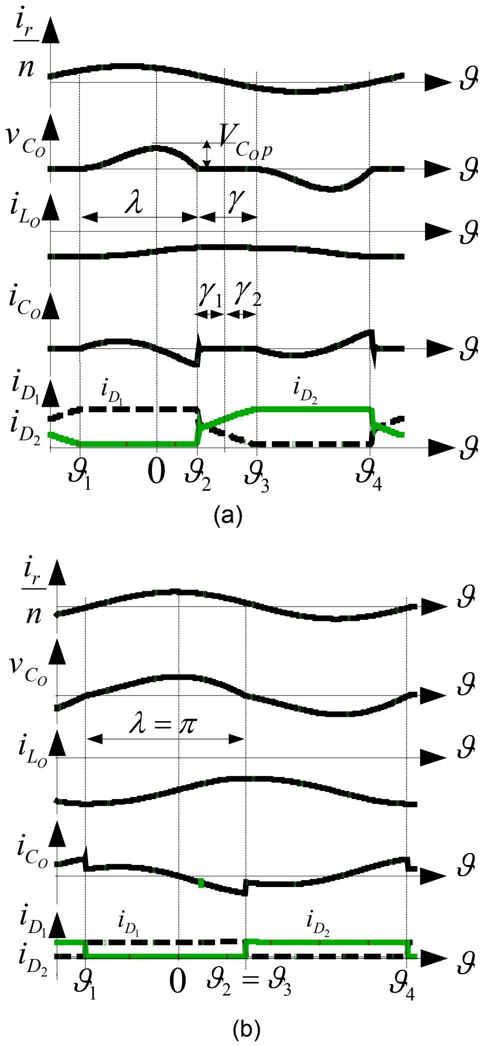

(a)

(b)

Fig. 2. The rectifier's current and voltage waveforms in the overlapping (a) and non-overlapping (b) modes.

During this overlapping interval, the current $i_{Co}=0$, because diodes short the capacitor. At ϑ_3, $i_{D1}=0$ and D_1 turns off. During $\vartheta_3\vartheta_4$, only D_2 is ON and $v_{Co}<0$. Inductance L_o charges by the input supply and it's current increases. At ϑ_4, v_{Co} changes polarity and an overlapping interval begins again, but now i_{D1} is increasing while i_{D2} is decreasing, etc.

III. THE UNIFIED ANALYSIS OF THE OVERLAPPING AND NON-OVERLAPPING MODES

Analysis is carried out under the following assumptions:

a) Diodes and reactive elements are ideal;

b) The output filter is ideal ($L_f=\infty$, $C_f=\infty$);

c) The current i_r has a sinusoidal waveform.

d) The operation frequency ω is equal to the resonant frequency: $\omega=\frac{1}{\sqrt{L_rC_r}}=\frac{1}{\sqrt{L_oC_o}}$.

A. Lossless PT ($R_m=0$)

We assume that the impulses of v_{Co} have a sinusoidal shape and that the duration of each impulse is approximately λ (Fig. 2). According to this assumption:

$$v_{Co}(\vartheta)=V_{Cop}\cos\left(\frac{\pi}{\lambda}\vartheta\right)\tag{1}$$

where V_{Cop} is the peak of the voltage across the capacitor C_o.

The peak of the first harmonic of this voltage:

$$V_{Co(1)p}=\frac{4}{\lambda}V_{Cop}\frac{\cos\left(\frac{\lambda}{2}\right)}{\left(\frac{\pi}{\lambda}\right)^2-1}\tag{2}$$

The average output voltage V_{out} is equal to the DC component of the positive value of v_{Co}:

$$V_{out}=V_{Cop}\frac{\lambda}{\pi^2}\tag{3}$$

Combining (3) in (2) we obtain:

$$V_{Co(1)p}=4V_{out}\frac{\cos\left(\frac{\lambda}{2}\right)}{1-\left(\frac{\lambda}{\pi}\right)^2}\tag{4}$$

Since the operating frequency is equal to the resonant frequency of the series circuit C_r, L_r:

$$\frac{V_{Co(1)p}}{n}=V_{in.p}\tag{5}$$

The transfer ratio of the output voltage of the rectifier V_{out} to the peak of the input voltage $V_{in.p}$ for the ideal PT is:

$$ko_{ideal}=\frac{V_{out}}{nV_{in.p}}=\frac{1-\left(\frac{\lambda}{\pi}\right)^2}{4\cos\left(\frac{\lambda}{2}\right)}\tag{6}$$

In the non-overlapping mode:

$$ko_{ideal}^{NOM}=\lim_{\lambda\to\pi}(ko_{ideal})=\frac{1}{\pi}\tag{7}$$

At the instant ϑ_2 the diode D_2 carries a current that was flowing via capacitor C_o. This current that starts as a step can be found from (1):

$$\left|I_{ch}\right|=\omega C_o\left.\frac{dv_{Co}}{d\vartheta}\right|_{\vartheta_2}=\frac{\pi}{\lambda}\omega C_oV_{Cop}\tag{8}$$

or applying (3):

$$\left|I_{ch}\right|=\frac{\pi^3}{\lambda^2}V_{out}\omega C_o\tag{9}$$

The output current I_O at ϑ_2 may be written as

$$I_O = \left|I_{ch}\right| + \frac{I_m}{n}\sin(\gamma_1) + i_{L_O} \qquad (10)$$

where: $\frac{I_m}{n}\sin\gamma_1$ is the current $\frac{i_r}{n}$ (Fig.1) at the instant ϑ_2, I_m is the peak of the input current, γ_1 - see Fig. 2 (a), and i_{L_O} is the inductor L_O current. The inductor L_O is shorted by the diodes D_1 and D_2 during the whole interval $\vartheta_2\vartheta_3$. That is why its voltage is practically zero and its current i_{L_O} has a constant value up to the instant ϑ_3 when D_1 turns off. Taking into account this condition, we use ϑ_3 to derive i_{L_O}. At ϑ_3 $i_{C_O} = i_{D_1} = 0$ and therefore:

$$i_{L_O} = \frac{i_r}{n} = \frac{I_m}{n}\sin(\gamma_2) \qquad (11)$$

where γ_2 - see Fig. 2 (a).

Consequently, the current balance at ϑ_2 (10) may be written as:

$$I_O = \frac{\pi^3}{\lambda^2}V_{out}\omega C_o + 2\frac{I_m}{n}\sin\left(\frac{\gamma}{2}\right)\cos\left(\frac{\gamma_1-\gamma_2}{2}\right) \qquad (12)$$

where $\gamma = \gamma_1 + \gamma_2$ is the overlapping angle:

$$\gamma = \pi - \lambda \qquad (13)$$

Taking into account that $\sin\left(\frac{\gamma}{2}\right) = \cos\left(\frac{\lambda}{2}\right)$ and assuming that $\cos\left(\frac{\gamma_1-\gamma_2}{2}\right) \approx 1$ equation (12) can be simplified to:

$$I_O = \frac{V_{out}}{R_L} = \frac{\pi^3}{\lambda^2}V_{out}\omega C_o + 2\frac{I_m}{n}\cos\left(\frac{\lambda}{2}\right) \qquad (14)$$

From the energy balance it follows that:

$$\frac{V_{in.p}I_m}{2} = \frac{V_{out}^{\ 2}}{R_L} \qquad (15)$$

Solving (6), (14) and (15) together, we obtain the equation for duration of the impulses of the voltage v_{Co} in the overlapping mode:

$$\lambda = \sqrt[4]{\pi^5\omega C_o R_L} \qquad (16)$$

In the non-overlapping mode $\lambda = \pi$.

Duration of the capacitance C_o voltage impulses λ as a function of the load factor $\omega C_o R_L$ based on (16) is depicted in Fig. 3. The rectifier operates in overlapping mode when $\omega C_o R_L < \frac{1}{\pi}$.

B. *Real PT (R_m>0).*

We assume the followings:

a) The voltage across the resonant circuit L_r, C_r is zero.

b) The voltage across R_m and the first harmonic of the input voltage of the rectifier are in phase with the input voltage of the PT.

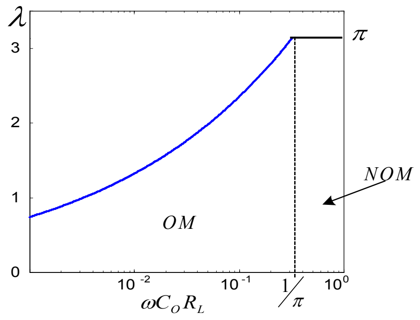

Fig. 3. Duration λ as a function of the normalized load factor $\omega C_o R_L$. OM - overlapping mode, NOM- non-overlapping mode.

Under these assumptions:

$$V_{in.p} - I_m R_m = \frac{V_{C_O(1)p}}{n} \qquad (17)$$

or, according to (4):

$$V_{in.p} - I_m R_m = \frac{4V_{out}}{n}\frac{\cos\left(\frac{\lambda}{2}\right)}{1-\left(\frac{\lambda}{\pi}\right)^2} \qquad (18)$$

It can be shown that equation (14) derived above for the ideal PT, is also correct for the lossy PT. Taking $\cos\left(\frac{\lambda}{2}\right)$ from (14) and inserting it to (18) we obtain:

$$\frac{V_{in.p}I_m}{2} - \frac{I_m^{\ 2}R_m}{2} = \frac{\frac{V_{out}^{\ 2}}{R_L} - V_{out}^{\ 2}\omega C_O\frac{\pi^3}{\lambda^2}}{1-\left(\frac{\lambda}{\pi}\right)^2} \qquad (19)$$

The left side of this equation is the output power, hence (19) may be rewritten as:

$$\frac{V_{out}^{\ 2}}{R_L} = \frac{\frac{V_{out}^{\ 2}}{R_L} - V_{out}^{\ 2}\omega C_o\frac{\pi^3}{\lambda^2}}{1-\left(\frac{\lambda}{\pi}\right)^2} \qquad (20)$$

Equation (20) can now be used to obtain the angle λ. The solution is identical to (16). We find here that even in the lossy case, λ is independent on the equivalent losses resistance of the PT - R_m.

C. *The Diodes Losses*

In low output voltage rectifiers the voltage drops of the diodes are significant and therefore should be taken into account in the converter's design.

The power losses P_D in each diode are calculated from equation given in [2]

$$P_D = V_F I_{D(ave)} + R_F I_{D(rms)}^2 \qquad (21)$$

where: $I_{D(ave)}$ and $I_{D(rms)}$ are the average and the rms diode currents, and R_F and V_F are the diode parameters: R_F is the equivalent forward resistance and V_F is the forward voltage:

According to Kirchoff's law, the diodes' currents i_{D_1} and i_{D_2} during the overlapping intervals can be described by the following equations:

$$i_{D_1} = i_{L_O} + \frac{i_r}{n} \qquad (22)$$

$$i_{D_2} = I_O - i_{L_O} - \frac{i_r}{n} \qquad (23)$$

During the non-overlapping intervals, the current of one of the diodes is I_O and the current of the other diode is zero.

The average diode current $I_{D(ave)}$ is approximately equal to the half output DC current $I_{D(ave)} = 0.5 I_O$. The rms diode current $I_{D(rms)}$ was determined using the assumption that during the overlapping intervals the diode currents are changing linearly.

Applying (9), (13) and (20), following expression of $I_{D(rms)}$ was obtained:

$$I_{D(rms)} = \frac{I_O}{\sqrt{2}} \varphi(\lambda) \qquad (24)$$

where:

$$\varphi(\lambda) = \sqrt{\frac{1}{\pi}\left\{\left[\frac{\lambda^2}{\pi^2} + \frac{2}{3}\left(1 - \frac{\lambda^2}{\pi^2}\right)^2\right](\pi - \lambda) + \lambda\right\}} \qquad (25)$$

Hence, the general equation of the power losses in each diode P_D (21) can be rewritten in the following form:

$$P_D = \frac{1}{2} I_O V_F + \frac{1}{2} I_O^2 R_F (\varphi(\lambda))^2 \qquad (26)$$

In non-overlapping mode this equation is simplified:

$$P_D^{NOM} = \frac{1}{2} I_O V_F + \frac{I_O^2 R_F}{2} \qquad (27)$$

Assuming that the diode's power dissipations is the major power loss, the rectifier efficiency is expressed as:

$$\eta_{rect} = \frac{P_{out}}{P_{out} + 2P_D} \qquad (28)$$

where:

$$P_{out} = V_{out} I_O \qquad (29)$$

is the output power.

Applying (26) we transform (28) into:

$$\eta_{rect} = \frac{1}{1 + \frac{V_F}{V_{out}} + \frac{R_F (\varphi(\lambda))^2}{R_L}} \qquad (30)$$

The value of R_F in modern diodes is usually small and in most practical cases it is much lower than the load resistance of the rectifier R_L. This and the fact that the function $\varphi(\lambda) \le 1$ for every λ suggest that the term $R_F(\varphi(\lambda))^2/R_L$ in the last equation can be neglected. Hence,

$$\eta_{rect} \approx \frac{1}{1 + \frac{V_F}{V_{out}}} \qquad (31)$$

As this equation shows, the rectifier efficiency η_{rect} decreases as the output voltage V_{out} gets lower. It is especially noticeable when V_{out} is getting closer to the diodes forward voltage V_F.

D. Voltage Range and Power Characteristics of a Real Converter

Applying the power balance condition we can replace the loaded output rectifier by an equivalent resistance R_{eq} connected to the output of the PT (Fig. 4).

The power balance condition including the rectifier losses can be expressed as:

$$\frac{V_{C_O(1)p}^2}{2R_{eq}} = \frac{V_{out}^2}{R_L} + V_F I_O = \frac{V_{out}^2}{R_L \eta_{rect}} \qquad (32)$$

from which:

$$R_{eq} = \left(\frac{V_{C_O(1)p}}{V_{out}}\right)^2 \frac{R_L \eta_{rect}}{2} \qquad (33)$$

It was found that the voltage ratio of the last equation could be calculated by an expression, which is similar to (4), but takes into account the rectifier's efficiency:

$$\frac{V_{C_O(1)p}}{V_{out}} = \frac{4}{\eta_{rect}}\left[\frac{\cos\frac{\lambda^*}{2}}{1 - \left(\frac{\lambda^*}{\pi}\right)^2}\right] \qquad (34)$$

The angle λ^* is defined by an expression similar to (16):

$$\lambda^* = \sqrt[4]{\frac{\pi^5 \omega C_o R_L}{\eta_{rect}}} \qquad (35)$$

In non-overlapping mode, expressions (33) and (34) converges to:

$$R_{eq}^{NOM} = \frac{\pi^2}{2\eta_{rect}} R_L \qquad (36)$$

Applying (31)-(34) we obtain the output to input voltage ratio of a converter with losses in the rectifier and the PT:

$$ko = \frac{0.25\eta_{rect}\left[1 - \left(\frac{\lambda^*}{\pi}\right)^2\right]}{\left(1 + \frac{n^2 R_m}{R_{eq}}\right)\cos\left(\frac{\lambda^*}{2}\right)} \qquad (37)$$

The dependences of ko_{ideal} (6) and ko (37) on the normalized load factor ($\omega C_o R_L$) are depicted in Fig. 5 a, b.

Fig. 5 shows that a decrease in the load factor $\omega C_o R_L$ causes a decrease of the voltage ratio ko in the overlapping mode even in ideal case of a lossless PT ($R_m = 0$), but this decrease is much more pronounced in the real case ($R_m > 0$, $P_D > 0$).

Fig. 6 depicts the normalized output power $P_O^* = \frac{V_{out}^2}{R_L}\frac{2\sqrt{\frac{L_r}{C_r}}}{V_{in.p}^2}$ as a function of the normalized load factor $\omega C_o R_L$.

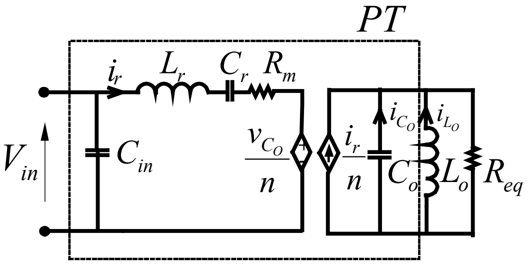

Fig. 4. A simplified equivalent circuit of the converter.

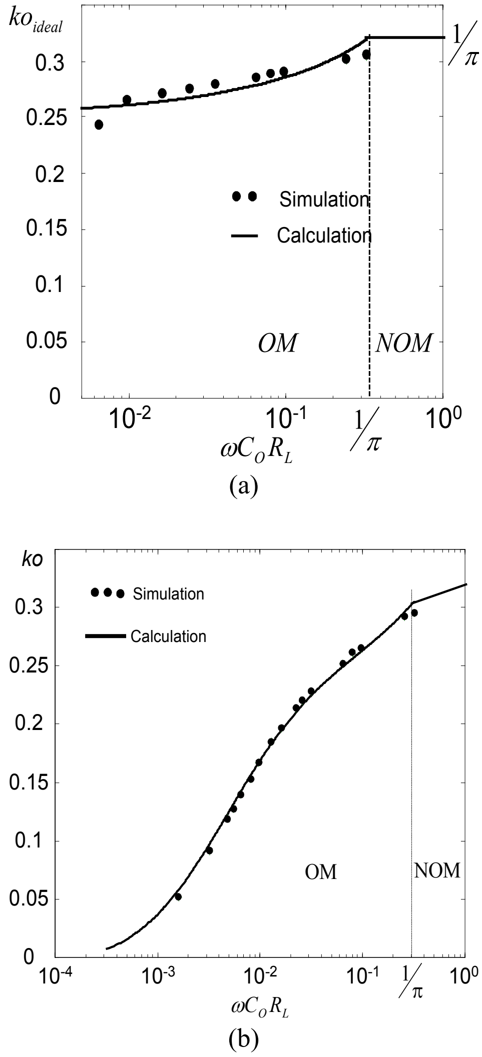

Fig. 5. The voltage ratio as a function of the normalized load factor $\omega C_O R_L$: (a) Lossless PT, ideal diodes, (b) Real PT ($\omega C_o R_m$=0.034), real diodes (MBR160P). The peak input voltage $V_{in.p}$=30V. OM – overlapping mode, NOM – non-overlapping mode.

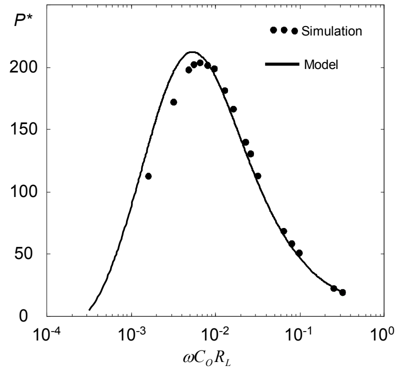

Fig. 6. The normalized output power P* as a function of the normalized load factor $\omega C_O R_L$. Real PT and real diodes (the same parameters as in Fig. 5 b).

Efficiency of the PT η_{PT} and efficiency of the whole converter η can be also easily calculated if we know R_{eq}:

$$\eta_{PT} = \frac{R_{eq}}{n^2 R_m + R_{eq}} \tag{38}$$

$$\eta = \frac{P_{out}}{P_{in}} = \eta_{PT}\eta_{rect} \tag{39}$$

IV. EXPERIMENTAL RESULTS

Philips PT (RT 35x8x2 PXE43-S) with thickness polarization mode was used in the experiment. Its main parameters (Fig. 1) were measured to be L_r=165mH, C_r=15.1pF, C_{in}=C_o=510pF, R_m=105Ω, n=1. Operation frequency was equal to the resonant frequency. The parallel resonant inductance was $L_o = 4.88\text{mH}$ and the diodes were a Schottky diode (MBR160P). The experiments were carried out for a load resistance R_L range of 30Ω to 1kΩ. The range of the peak of the input voltage was $V_{in.p} = 5 - 30\text{V}$. The experimental and the theoretical voltage ratios as a function of the load resistance are shown in Fig. 7. It is evident that the analytical equations and characteristics, derived in this study, are in excellent agreement with simulation and experimental results

V. DISCUSSION AND CONCLUSIONS

It was shown that the PT power converter with a halfway two diodes rectifier can operate in two modes: NOM - the operational mode described earlier in which there is no overlap between the conduction intervals of the diodes, and OM – the newly uncovered mode with overlap between the conduction intervals of the diodes. It was found that the operation mode is depending on the normalized load factor $\omega C_o R_L$. The load factor is a function of the PT parameters and the load resistance.

The study highlights the effect of the load and parasitic resistances on the output voltage and the losses for both modes. The analytical investigations were supported by simulation and experiment results, and found to be in excellent agreement.

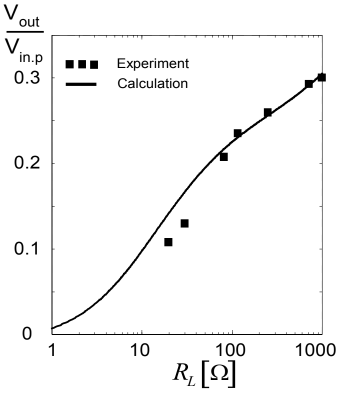

Fig. .7 Experimental and theoretical voltage ratio $V_{out}/V_{in,p}$ as a function of the load resistance R_L. $V_{in,p}$=30V, $\omega C_o R_m$=0.034.

It is important to note that a design that is based on the known NOM may be in a considerable error if the rectifier operates in OM.

The analytical equations derived in this study can be used in engineering design. An example of a design procedure is given in the Appendix. In this design example we assume that the PT is given and calculate the required input voltage to meet the out specifications (P_{out}, V_{out}). If the calculated input voltage is not equal to the available input source, a trial and error procedure need to be following until the correct PT is found. Alternatively, the PT parameters (e.g. n) can be adjusted to meet the requirement and by this define the set of parameters of the PT that will meet the given specifications.

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APPENDIX

Design Guidelines

Given: the AC/DC PT converter output requirements (V_{out}, P_{out}), the PT parameters ($L_r, C_r, C_{in}, C_o, R_m, n$), and the diodes to be used in converter.

Variables to be evaluated: the input voltage V_{in} that ensures the required output attributes of the power converter and the input power P_{in}.

Design steps:

1) Define the resonant frequency ω_r, the load resistance R_L, the resonant inductance $L_O = \frac{L_r C_r}{C_o}$ and the load factor $\omega C_o R_L$ from the output specifications and the PT parameters.
2) Find the forward diode voltage V_F from data sheet and evaluate the rectifier efficiency η_{rect} (31).
3) Calculate the parameter λ^* (35). The rectifier is operating in non-overlapping mode when $\lambda^* = \pi$ and in overlapping mode when $\lambda^* < \pi$.
4) Define the equivalent load resistance of the converter R_{eq} (33) and (34).
5) Obtain the voltage ratio ko of the real converter (37).
6) Calculate the peak input voltage as $V_{in(p)} = \frac{V_{out}}{ko}$.
7) Find the PT and the whole converter efficiencies η_{PT} and η (38), (39).
8) Determine the input power $P_{in} = \frac{P_{out}}{\eta}$.