FORBIDDEN STATE SPACE TRAJECTORIES OF PWM CONVERTERS: IMPLICATION TO DIGITAL CONTROL

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Abstract - The possible trajectories of the state vector values ($i_L(t)$ and $v_C(t)$) of Buck and Boost PWM converters under large transients are examined and evaluated. The forbidden trajectories, detected by examining the state space traces, impose sever limitation on the control design. These limitations are best demonstrated by investigation of large signal transient response of conventional linear controllers. Based on this investigation, a simple discrete control rule is proposed. Based on it, a preliminary simple discrete control function for a Boost converter was tested and compared to a linear Peak Current Mode (PCM) control.

I. INTRODUCTION

The prevailing method of stabilizing and controlling PWM converters is by linear controllers that are realized by analog circuits. The design procedure of the feedback loops and phase compensation networks is conveniently done in the frequency domain, based on the small signal responses of the system. This classical method has served designer well and provided a proven tool for handling the various control schemes: Voltage Mode (VM), Peak Current Mode (PCM), Average Current Mode (ACM) [1,2] and H^{∞} control [3]. However, the current need for regulators with extremely fast response on the one hand, and the dramatic advances in digital electronics on the other, call for a fresh look at the control issue. One possible way to tackle the problem is to translate frequency domain design rules into discrete equations implemented by digital controllers [4]. This however, does not improve the control. At best it may approach the performance of the analog circuit.

It would appear that in order to be able improve the control of PWM systems in any substantial way, one has to tackle a major issue: developing non linear and logic based control methods that hinge on time domain design tools. Breaking away from the frequency domain design is a necessity for atleast two reasons. The first one being the fact that the frequency domain design is relevant only to linear control of linear systems. Hence, any attempt to use it in conjunction with digital control is self defeating. Secondly, time domain is more natural to digital control which is based on (time domain) discrete functions. Considering the above, there seem to be a need for re-examination of PWM control from a discrete point of view. This issue was the major objective of the present study.

II. STATE SPACE TRAJECTORIES

The fundamental control issue is the desire to move the power system quickly, with minimum over-shoot and in a stable way, from one state space point to another. Hence, the key to a successful control solution is optimizing the trajectory of the state space vector as it moves from an initial point to the target point. This trajectory in PWM systems has a discrete nature since the duty cycle is set at the beginning of each switching cycle. Assuming an unlimited computer power, one should be able to force the trajectory to an optimum path. However, as it turns out not all paths are possible as there are some forbidden directions. Mapping the possible trajectories and understanding their implication seems therefore a necessary first stage in the development of digital control. The issue was examined in this paper by computer simulations applying linear discrete equations [5] and average behavioral models [6,7]. The first one is very simple to solve and is suitable for calculating the state trajectory points and to develop classical linear control laws. The second approach is applicable to small signal as well as for large signals and can thus be used to compare linear and nonlinear controls.

III. MODELING APPROACH

III.1. Linearized Discrete Equations

Standard procedure is used to develop the state equations for the ON and OFF period:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$

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where x(t) is the state vector ($i_L(t)$ and $v_C(t)$); the matrix A, the vector B, the input u(t) and the initial conditions are different for each stage.
The solution of differential equation (1) is [5]:

$$x(t) = e^{At}x(0) + u(t)A^{-1}(e^{At} - I)B(t) \qquad (2)$$

were x(0) is the initial condition at the beginning of each switching state (ON, OFF or discontinuous) and x(t) is the vector of the state variables within each switching state. The state variables values at the end of each state served as the initial condition for the following state. Equation (1) can be simplified by applying a first order linear approximation [5]:

$$e^{At} \quad I + At \qquad (3)$$

We used MATLAB function "lsim" to solve x(t) more accurately for the general case: Continuous inductor Current Mode (CCM) and Discontinuous inductor Current Mode (DCM). To find the complete one cycle solution, we divided the switching cycle into three stages: ON, OFF and discontinuous conduction. The OFF stage period t_{off} was detected by tracking the instance at which the inductor current reaches zero.
The state vectors at instant (i+1) are thus a function of the solution of the previous cycle (i), the power stage components (L,C,R_C), the voltage source V_S and load resistance R:

$$i_L(i+1) = f_i\left(i_L(i), v_C(i), d_{on}(i), V_S, R, R_C, L, C\right) \qquad (4)$$

$$v_C(i+1) = f_v\left(i_L(i), v_C(i), d_{on}(i), V_S, R, R_C, L, C\right) \qquad (5)$$

III.2. Average Modeling

The average modeling approach [6,7] implementation in SIMULINK (The MathWorks Inc.) for a Boost converter, operating under CCM conditions, is shown in Fig. 1.
The blocks of Fig. 1 are defined as follows:

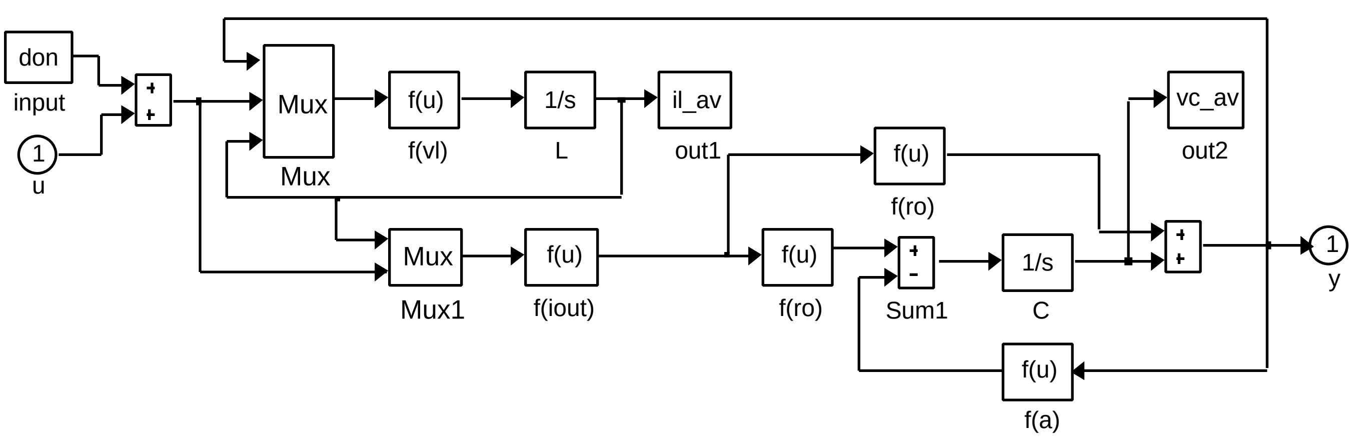

Fig. 1. SIMULINK based average model of a Boost power stage operating under CCM conditions.

$$f(vl) = [V_S - (i_L R_{swon} d_{on} + (V_D + v_{out})(1 - d_{on}))]/\ L \qquad (6)$$

where R_{swon} is the 'on' resistance of the switch and V_D is the 'on' voltage of the diode.

$$f(iout) = (1 - d_{on})i_L \qquad (7)$$

$$f(a) = i_{out}R_C R/\ (R_C + R) \qquad (8)$$

$$f(b) = v_{out}C/\ (R_C + R) \qquad (9)$$

$$f(ro) = i_{out}RC/\ (R_C + R) \qquad (10)$$

f(vl) in equation (6) generates the average voltage across the inductor, this voltage is integrated to produce the inductor's average current. The output current, f(iout) in equation (7) generates the load section average current. The input signal to the model is d_{on} while the load and the input voltage source are parameters. By a slight modification of the model, the load and input voltage can also be made time dependent variables. Equations (8)-(10) represent the load R and the output capacitor with it's resistance R_C. The load section L, C, R_C and R was built around an integrator. The initial conditions can be set by forcing an initial voltage on the capacitor as for the case of the integrator representing the inductor.

IV. Trajectory Analyses

The discrete equations (4) and (5) were used to probe into the state space trajectories issue. One such study on a Buck power stage is depicted in Fig. 2. The plot (Fig. 2a) represents a situation in which the converter was subjected to a 50% step in the load. The "start" point represents the initial stage just before the change was made. The "end" point is the target state. The computed points represent the possible states that the system can reach after one switching cycle for the duty cycle range d_{on}=0.02 to d_{on}= 0.98 (computed for 0.04 step resolution). The state vector $\{i_L(i+1),\ v_C(i+1)\}$

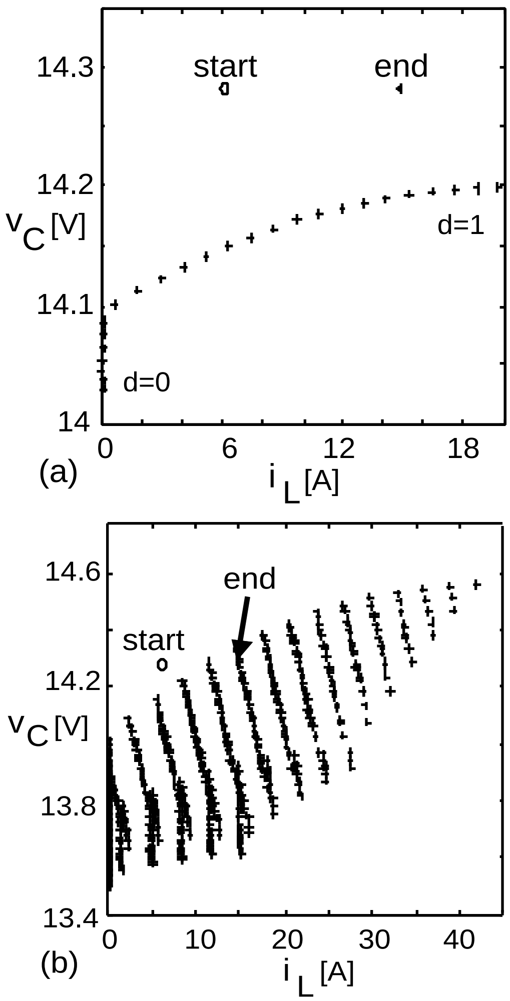

Fig. 2. State plane plot (i_L versus v_C) of a Buck converter state vector obtained by discrete control function. Possible state points after one (a) and after three (b) switching cycles.

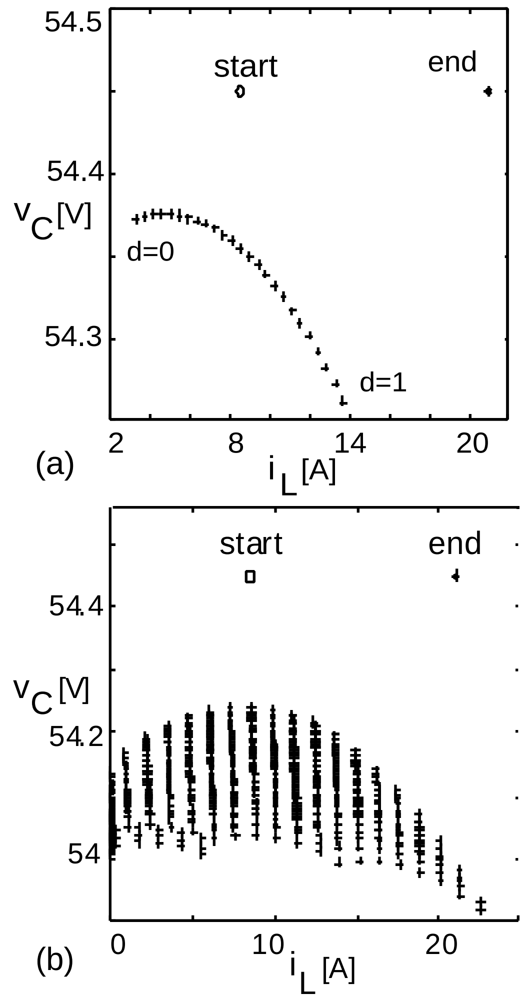

Fig. 3. Response of a Boost converter. (a) and (b), correspond to those of Fig. 2.

was calculated using MATLAB function "lsim" to solve the linear differential equations, while taking into account the discontinues conduction of the inductor current. In Fig. 2b the mapping process was advanced two steps further for a total of three switching cycles. The calculated dots represent all possible state points that the system can reach after three switching cycles, scanning all possible d_{on} (some points were eliminated from Fig. 2b for the sake of clarity). This extended examination shows that for the Buck power stage under the specific disturbance, a 'smart' controller should be able to bring the system to the target neighborhood within three switching cycles.

When repeating the same mapping process on a Boost converter (Fig. 3) one realizes that the behavior of Boost is markedly different. It appears that for the given conditions, there is no way to bring the Boost system to the target in three switching cycles. This is due to the fact that some space trajectories are just impossible (or forbidden). A smart controller must be clever enough to follow permissible paths and still bring the system to the target in a minimum number of switching cycles. Conventional linear controllers are not that smart, but yet they do bring the system to the desired target. This can be judged by examining the state space trajectory of a Boost system controlled by three linear controllers: PCM, VM and the H^{∞} [3]. The plots of Fig. 4 describe the performance of the three controllers, in closed loop, as obtained by detailed cycle by cycle simulation. The PSPICE simulation results were fed to MATLAB for plotting. The points of Fig. 4 represent the values of the state vector at the beginning of each switching cycle. The details of the controllers were as follows:

for PCM control we implemented the exact controller of [2], for VM control we applied the function C(s) that was derived by standard design techniques:

$$C(s) = \frac{-96\ 10^3(s+667)(s+677)}{(s+0.026)(s+13\ 10^3)(s+50.6\ 10^3)} \qquad (11)$$

For the H^{∞} control we implemented the method given in [3] to derive:

$$C(s) = \left[\frac{-18.17(s+310.7)(s+8.725 \; 10^{3})}{(s+3.14 \; 10^{3})(s+41.67 \; 10^{3})} \quad -45.7 \; 10^{-3} \right] \tag{12}$$

The weight function for the H^{∞} control is the same as in [3].

For the VM and H^{∞} controllers we used a 0 to 2.5V ramp to generate the duty ratio from the error signal.

It is evident from Fig. 4 that the linear controllers indeed avoid the forbidden region but apparently at the expense of speed. Notice in particular the very slow rate at which the system approached the neighborhood of the target point. A closer examination revealed that the three linear controllers behave quite the same at the first switching cycles and the main different is their approach to the target neighborhood.

V. Proposed Discrete Control Law

Based on the above trajectory mapping study, we propose a non linear control rule that seems to have the potential of improving the closed loop performance of PWM converters. It is assumed at this stage that there are no computer power limitation to carry out the suggested algorithm.

The control law is defined as follows:

Find $d_{on}(i)$ to minimize $f_{control}$

Where $f_{control}$ is defined by:

$$f_{control} = f(i_L(i), v_C(i), d_{on}(i)) \tag{13}$$

Base on (13) we propose a preliminary control function:

$$f_{control} = \left| \left[V_F - v_C(i+1) \right] + \left[I_F - i_L(i+1) \right] k_{weight} \right| \tag{14}$$

Where k_{weigh} is a weighing constant calculated by:

$$k_{weigh} = \frac{v_C(0)}{i_L(0)} \tag{15}$$

The inputs to (14) are the state vectors ($i_L(i+1)$ and $v_C(i+1)$) and the computed state values at the target steady state (V_F and I_F). $v_C(0)$ and $i_L(0)$ of (14) are the state variable values at the "start" point.

The transient responses of the linear controller in [5] and the proposed controller were compared (Figs. 5 and 6). Both converter systems were implemented in SIMULINK using the average model [7]. A 50% load step was forced on the converter and the values of i_L and v_C (marked as '+',) were plotted after each switching cycle, as the systems moved from the "start" state to the "end" state. The proposed algorithm improves the transient response by a factor of about 40 (2,500 cycles were needed in Fig. 5 and only 60 cycles were needed in Fig. 6 to reach the "end" point).

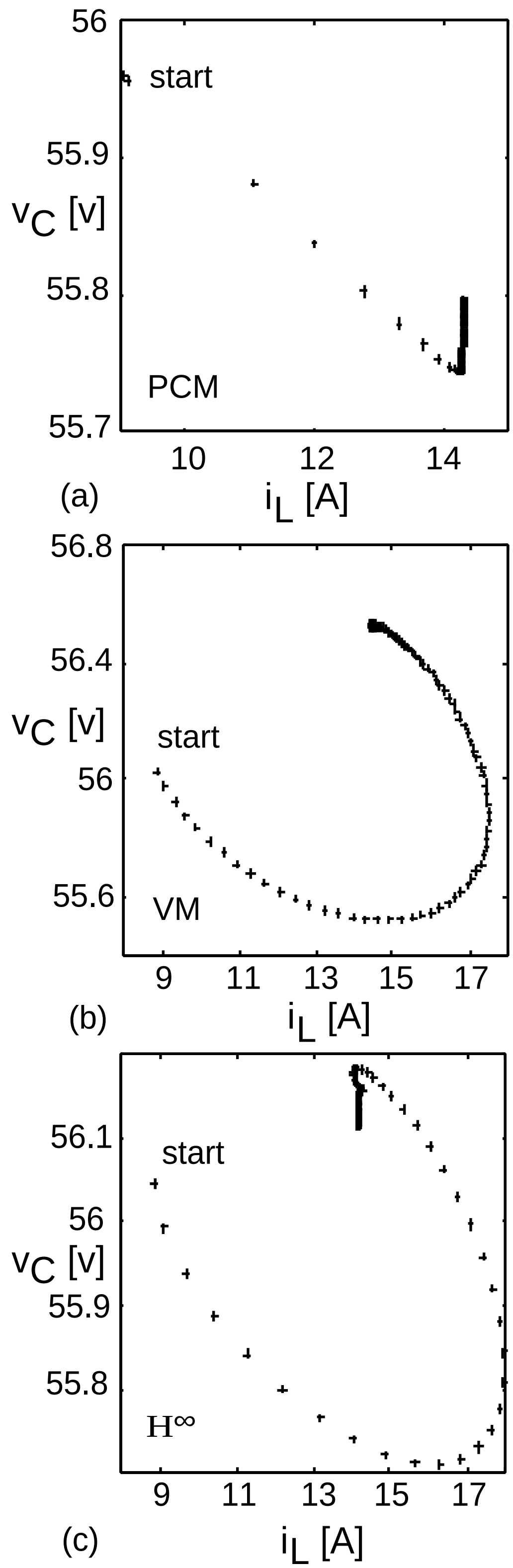

Fig. 4. State plane plot (i_L versus v_C) of a Boost converter describing the closed loop response to a 33.34% load step when controlled by linear controllers. (a) PCM, (b) VM and (c) H^{∞}. The records include each 100 cycles after the step change.

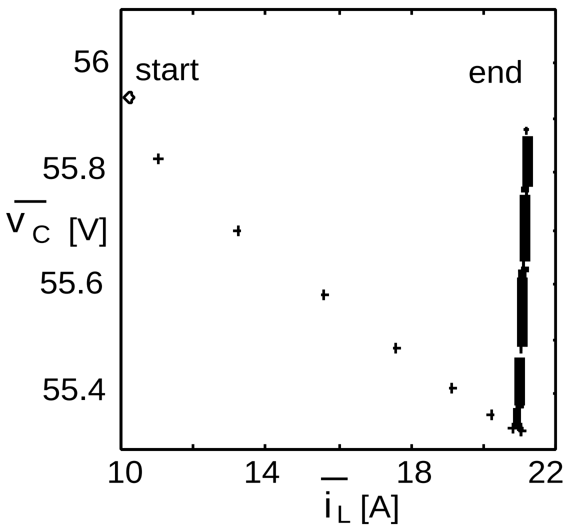

Fig. 5. State plane (Average values, $\overline{i}_L$ versus $\overline{v}_C$) representation of Boost converter response to a 50% load step when controlled by linear PCM controller.

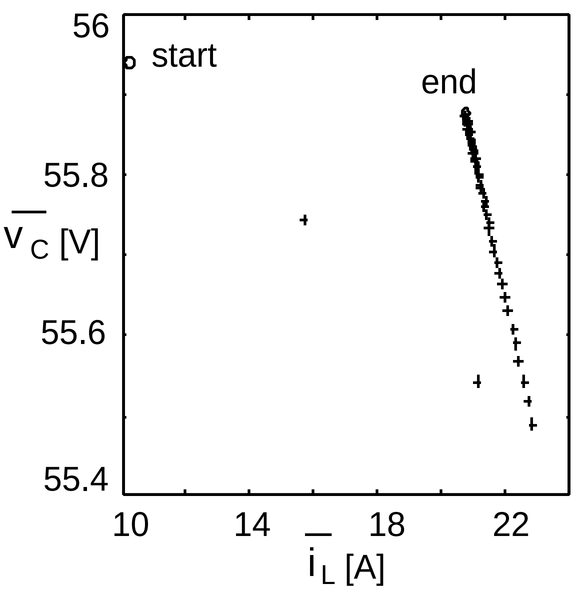

Fig. 6. State plane (Average values, $\overline{i}_L$ Versus $\overline{v}_C$) representation of Boost converter response to a 50% load step when controlled by the proposed discrete controller

VI. CONCLUSIONS

By investigating the State space diagrams of PWM converts for various large-signal transient disturbances conditions, the possible state space trajectories were examined. It was found that some trajectories (such as a direct path from "start" to "end" points) are impossible for a Boost converter. Based on this study, a discrete control rule (13) is proposed. A preliminary controller based upon this rule (14) was found to speed up the transient response of a Boost converter by a factor of 40 as compared to a PCM controller.

The present study reveals the main strength and drawback of linear controllers. Their advantage is there ability to overcome the problem of forbidden trajectories and to land safely at the target state. The main handicap of these controllers is the rather slow pace at which the target is reached. It would appear that non-linear control combined with logic decision could improve the response. The heuristic control algorithm described here shows that substantial improvement could be achieved with smarter controllers.

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