

# A SPICE Compatible Model of High Intensity Discharge Lamps

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**Abstract--A SPICE compatible model of High Intensity Discharge (HID) lamps was developed and verified against experimental results. The model is based on fundamental physical-thermal principles and applies lamp parameters and universal constants. The model was found to faithfully emulate the static and electrical responses of a High Pressure Sodium lamp under low frequency (50Hz) and high frequency (30kHz) excitation.**

## I. INTRODUCTION

High efficacy (Lumen/Watt), long life and good color rendition have made the High Intensity Discharge (HID) lamp one of the most useful light sources [1,2]. The present range of application of HID lamps is broad: from large space lighting such as streets, sports arenas and industrial production halls to specialized applications such as LCD video projectors. The main difficulty with this family of lamps is the need for special electrical circuits (ballasts) to drive them [1,2]. In some applications, electronic ballasts offers important improvements over the traditional electromagnetic ballasts [2]. The main advantages of the electronic ballasts are: higher electrical efficiency and lower weight and size. For this reason there is presently a considerable scientific and engineering interest in the electronic ballasts development.

A SPICE compatible model of HID lamps could be a useful tool in the investigation and design of electronic ballasts for HID lamps. Several studies proposed SPICE compatible models of fluorescent lamps [1,3-10] but the authors are unaware of earlier studies that present a SPICE compatible model of HID lamps. A number of earlier papers presented mathematical models of High Pressure Mercury, Sodium and Metal-Halide Lamps [11-17], but they are not readily adaptable to the environment of common SPICE simulators.

The objective of the present study was to develop a simple SPICE compatible model that emulates the electrical behavior of HID lamps. This model are derived from the arc's

behavior of HID lamps. This model are derived from the arc's physics and therefore should be valid over a wide range of operating conditions. The present investigation did not probe into the process of ignition, warm up and the intricate subject of acoustic resonance [2].

## II. GENERAL

The fundamental approach of the SPICE compatible model developed in this study is to represent the lamp as a behavioral dependent current source (Fig. 1) having the following general definition:

$$i_{lamp} = \frac{v_{lamp}}{R_{lamp}} \quad (1)$$

where  $v_{lamp}$  is the voltage across the lamp and  $R_{lamp} = \frac{1}{\sigma} \frac{L}{S} dV$  is the non-linear resistance of the lamp,

$\sigma$  is the global specific conductance of the plasma,  $V$  is the volume of the plasma,  $L$  is the length of plasma and  $S$  is the cross section.

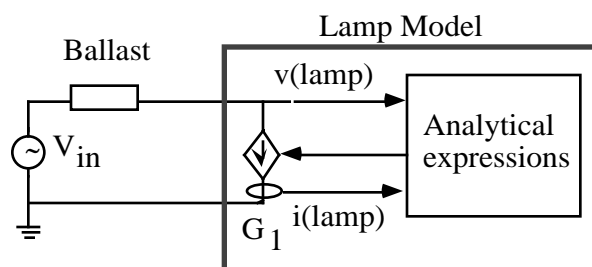


Fig.1. The proposed SPICE compatible model of an HID lamp.  $G_1$  is a behavioral dependent current source.

In general, the global specific conductance equals the sum of all conductances of the plasma's gas components :

$$\sigma = \sum \sigma_i \quad (2)$$

For weakly ionized plasma, the specific conductance  $\sigma_i$  depends on the temperature [18]:

$$\sigma_i = \frac{oi T^{0.75}}{\sqrt{p}} \exp \left( -\frac{E_i}{2kT} \right) \quad (3)$$

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where  $\rho_i$  is a constant of each gas component,  $T$  is the arc's active part (core) temperature,  $p$  is the pressure of the gas,  $E_{ji}$  is the ionization energy of each component  $i$  and  $k$  is Boatsman's constant.

To describe the electrical processes of the lamp one has to describe its thermodynamic state. This description of the discharge lamp state is based on the power balance equation. At any given time, the incremental increase in the arc's heat ( $dQ$ ) can be described by the following equation :

$$dQ=(P_{in}-P_{out})dt$$

where:  $P_{in}$  - input power in the lamp;  $P_{out}$ - power removed from the lamp (radiation plus heat conduction);  $dt$  - time increment.

The present study was conducted under the some simplifying assumptions of a channel model:

1. The arc is in the High Pressure Lamp is confined to a plasma cylinder.
2. The plasma is in a Local Thermal Equilibrium (LTE) state.

The next conventional simplification step is the assumption that some arc's parameters are constant. This study was conducted under the assumptions:

3. The arc is composed of a two-layer cylinder:
  - the radius of the internal layer is equal to the electrode's radius,
  - the external radius of the external layer is equal to the arc's tube internal radius,
  - the arc's length is equal to the gap between the electrodes.
4. All physical properties in each layer are constant.
5. Electrical current run only in the internal layer - core.
6. The losses are due to radiation and heat conduction.
7. The energy losses arc's active part run only in the radial direction.
8. The influence of the arc's tube, electrodes and bulb is neglected.

Additional assumptions are stated in Appendix A in conjunction with other aspects of the model.

Under those assumptions, the incremental increase in heat for a unit length of the arc can be expressed by :

$$dQ = [c_{arc} r_{arc} R_{arc}^2 + k_{Tred} c_{per} r_{per} (R_{tube}^2 - R_{el}^2)] \quad (5)$$

where:  $c_{arc}, c_{per}$  - heat capacity of the internal and external layers respectively;  $\rho_{arc}, \rho_{per}$  - density of the internal and external layers' medium, consequently;  $R_{tube}, R_{el}$  - arc cavity and electrodes radius, respectively,  $k_{Tred}$ - factor keeping the temperature reduction in the arc's periphery.

The input power to the lamp per unit length ( $P_{ind}$ ) is:

$$P_{ind} = \frac{v_{lamp} i_{lamp}}{L_g} \quad (6)$$

where:  $L_g$  - distance between electrodes.

The formula for the radiated energy  $P_{rad}$  :

$$P_{rad} = 2 R_{el} (T) \quad (7)$$

where:  $\epsilon(T)$  - arc's surface emissivity, defined in Appendix A. The heat conduction losses  $P_{con}$  are described by the law:

$$P_{con} = (T - T_{amb}) * 2 R_{el}, \quad (8)$$

where:  $h = (T) -$  coefficient of heat transfer defined in Appendix A.

By replacing the energy components of (4) by expressions (5)-(8) we obtain a fundamental differential equation that describes the arc's state:

$$\frac{dT}{dt} = \frac{\frac{v_{lamp} i_{lamp}}{L_g} - 2 R_{el} [ \epsilon(T) + h(T - T_{amb}) ]}{[ c_{arc} r_{arc} R_{arc}^2 + k_{Tred} c_{per} r_{per} (R_{tube}^2 - R_{el}^2) ]} \quad (9)$$

### III. STRUCTURE of SOLUTION

To solve (9) one has to evaluate the numerical values of the medium parameters  $c_{arc}, c_{per}, \rho_{arc}, \rho_{per}$  and  $\epsilon$  in the numerator and denominator. The procedure adopted in the study is summarized in Fig. 2.

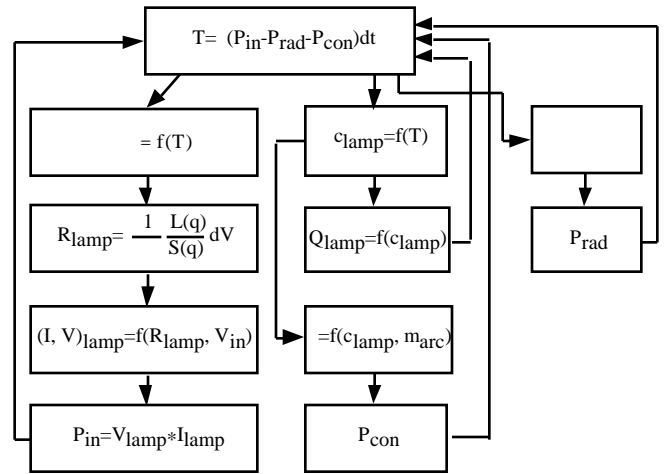


Fig.2. Solution structure of lamp state equation.

The required parameters of gas mixture, partial pressures, gas temperature at the axis and wall temperature, radiant power efficiency and nominal voltage and current values were estimated from the data given in [2] for 400-W Sodium-lamps. Some geometric sizes: arc tube's length, diameter, wall thickness, electrode's diameter and etc. were determined by direct measurements (see Appendix A).

### IV. SPICE ADAPTATION

The thermodynamic phenomena in the arc are described by the differential equation (9). This expression has the appearance of a first order differential equation but it is, in fact, a non linear differential equation. This is due to the fact that the values of all equation coefficients are a functions of temperature ( $T$ ) which, according to the above assumptions, are associated with the average core temperature. The approach taken in this study is to emulate the thermodynamic phenomena by an equivalent circuit and to apply an electronic circuit simulator to solve this equation numerically. We have used PSPICE

(MicroSim Inc., Irvine, USA), but any other simulator that includes behavioral dependent source is equally applicable. To solve (9) one has to obtain the running value of the temperature and apply it to calculate all temperature dependent parameters. This can be done by using the right side of (9) to define a behavioral dependent current source(DCS). The output of this current source ( $I_{th}$ ), which equals to  $(\frac{dT}{dt})$ , can now be integrated by feeding it to a capacitor  $C_{th}$ . Hence, the voltage across the capacitor ( $V_C$ ) represents the arc's temperature at any given time:

$$V_C = \frac{1}{C_{th}} \int I_{th} dt \quad T \quad (10)$$

If we select  $C_{th} = IF$  than there is no need for scaling. The SPICE adaptation of (10) is depicted in Fig. 3.

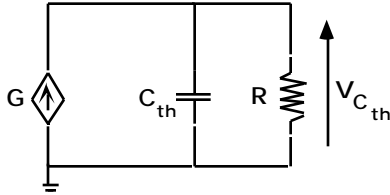


Fig.3. Method used to solve (9). G is a behavioral dependent current source defined by the right side of (9).  $V_{Cth}$  represents the (voltage coded) temperature of the arc.

Resistor  $R$  is needed for technical reason, to provide a galvanic path to ground as required by some simulators. The value of  $R$  should be chosen such that:

$$R \gg \frac{1}{C_{th}} \quad (11)$$

where  $\omega$  is the angular frequency of the phenomena to be investigated by the model. It should be pointed out that the maximum value of  $R$  is limited only by the numerical range allowed by the simulator.

The calculation of all parameters as shown schematically in Fig. 2 can be carried out by interconnection behavioral dependent sources (DS). That is, the variables are coded into voltage or currents that can be handled by the simulators and the equations are evaluated by behavioral expressions. A general description of the SPICE compatible equivalent circuit is given in Fig. 4 that is PSPICE Version 7 (MicroSim Inc.) compatible.

The values of the parameters are given in Appendix B. The definition of the devices and dependent sources are as follows:

$V_{in}$  - VSIN-type independent voltage source (IVS), that powers the ballast/lamp system. The values of the attributes:  $VAMPLitude = \{V_{in}\}$ ,  $FREQUENCY = \{F_{in}\}$  are given in the "PARAMETERS" box (see Appendix B).

$L_{bal}$  - Ballast inductor.

The combination  $V_{in} - L_{bal}$  represents a particular case of driver (source plus ballast inductor). Any other sources and ballasts are of course possible.

$G_{lamp} = V(lamp)/(R_{arco} - V(DRarc))$ :

$G_{value}$ -type dependent current source that emulates the lamp corresponding to (1), except that it includes an additional term in the in denominator to avoid division to zero in the first

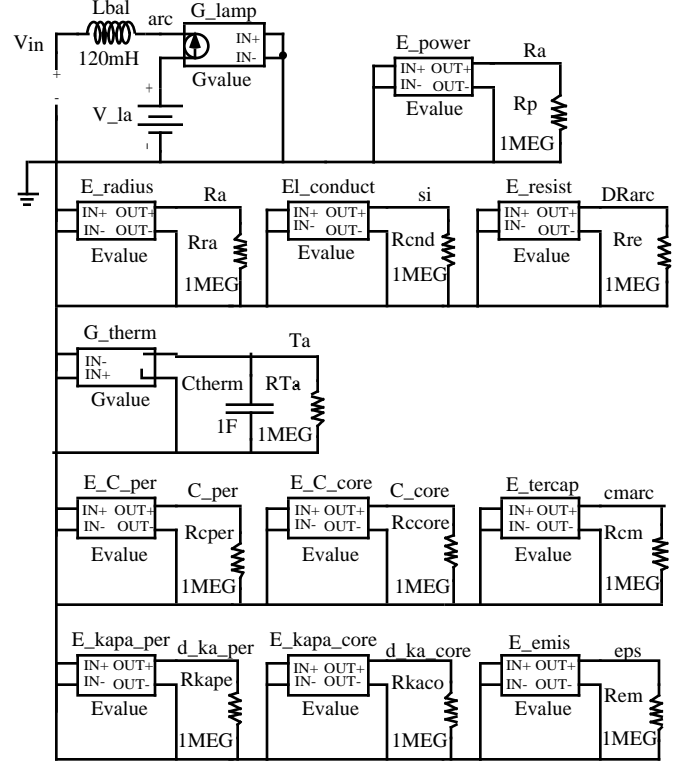


Fig. 4.SPICE model. See text for definitions of dependent voltage sources (EVALUE) and current source (GVALUE)

iterations.  $R_{arco}$  - arbitrary number,  $R_{arco} \gg R_{nom}$  - lamp nominal resistance.

$V_{la}$  - zero-voltage Vdc-type IVS; used to measure the instantaneous lamp current  $I(V_{la})$ .

$E_{power} = V(lamp) * I(V_{la})$  -

Value-type dependent voltage source that creates voltage  $V(Power)$  numerical equal instantaneous lamp power.

$El_{conduct} = sia_{Na} * PWR(V(Ta), 0.25) * EXP(-$

$Ek_{Na}/V(Ta))/5.244$  - Value-type according to (3) creates voltage  $V(si)$  that is equal the lamps conductivity. This expression takes into consideration the relationship:

$$p = \frac{p_i}{T_{core}} T; \quad (12)$$

where  $p_i$  is the initial partial pressure magnitude,  $T_{core}$  - the initial average core temperature (Appendix B).

$E_{resist} = \frac{R_{arco} - Lg/(sio + V(si))/Pi/((Rao + V(Ra))*(Rao + V(Ra)))}{R_{arco}}$  - Value-type source that produces a voltage  $V(DRarc)$  equal to the lamp's resistance  $R_{arco}$  is added to offset the factor added at the denominator of  $G_{value}$  (see above).

$E_{radius} = Rat$ . Value-type source that produces a voltage  $V(Ra)$  equal the arc's radius.

It was found that emulating the radius as  $V(Ra)$ , rather than applying it as a parameter help convergence.

$G_{therm} = (V(power)/Lg -$

$(V(eps) + (kapa_o + V(d_ka_core)) * (V(Ta) -$

$Tamb)) * 2 * Pi * (Rao + V(Ra)) / V(cmarc) -$  Gvalue-type source,

that is the analog of (9). It produces the voltage  $V(Ta)$  that is equal to the arc's core average temperature.

$E_{C\_core} = Co + V(d_{ka\_core}) * dD/ro\_core * 1.732/Lmfp\_core$   
 $SQRT(mHg/k/V(Ta))$ - Evaluate-type source that produces a voltage  $V(C\_core)$  equal to the heat capacity of core.

$E_{C\_per} = Co + V(d_{ka\_per}) * dD/ro\_per * 1.732/Lmfp\_per$   
 $*SQRT(mHg/k/(k\_tred * V(Ta)))$ ;- Evaluate-type source that produces a voltage  $V(C\_per)$  equal to the heat capacity of arc's periphery as above.

$E_{tercap} = V(C\_core) * Pi * (Rao + V(Ra)) * (Rao * V(Ra)) * ro\_core +$   
 $V(C\_per) * Pi * (Rtu - V(Ra)) * (Rtu - V(Ra)) * ro\_per$  - Evaluate-type source that produces a voltage  $V(cmarc)$  equal to the arc's heat capacitance (per unit length).

$E_{kapa} = k * (Eef/k/V(Ta)) * (Eef/k/V(Ta)) * EXP(-Eef/k/V(Ta))$ - Evaluate-type source that produces a voltage  $V(d_{kapa\_core})$  equal to the heat transfer coefficient increment.

$E_{eps} = eps\_0 * (Eef/k/V(Ta)) * (Eef/k/V(Ta)) * EXP(-Eef/k/V(Ta))$ - Evaluate-type that produces a voltage  $V(eps)$  equal to the emissivity increment.

Each source is loaded by a resistor to avoid floating node situations. All global constants, internal and input data are assignment in PSPICE "PARAMETERS" (APPENDIX B).

## V. EXPERIMENTAL VERIFICATION

The proposed model was verified against experimental data for low frequency (50Hz), high frequency (30kHz) and small signal modulation (carrier frequency 30kHz at modulation frequency range 50Hz - 5000Hz). Experiments were carried out on a 400W high pressure Sodium lamp (HPS) (OSRAM VIALOX NAV-T-400). The experimental and simulated dynamic responses (Figs. 5 and 6) were found to be in good agreement

## VI. CONCLUSIONS

A HID lamp model, based on the arc's thermodynamics, was shown to emulate the electrical static and dynamics responses of a HPS lamp operating at high and low excitation frequencies. The model was implemented by an equivalent circuit that includes behavior dependent sources. It is thus compatible with general purpose electronic circuit simulators such as PSPICE (MicroSim Inc.). The main difficulty with the model is the difficulty of getting the many parameters of the lamp. In particular those dependent on the composition and pressure of inner gases. Consequently, future use of this approach for any given lamp will be conditioned on the availability of this data. The suggested solution to the problem is to follow the practice already adopted by vendors of electronics components. Namely, having the SPICE related parameters published by lamp manufacturers.

## APPENDIX A

TABLE 1

400-W Sodium-lamps data [2]

No	Parameter	Magnitude
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1	Partial gas pressure $P_i$ in Na..... $1 * 10^4$ Pa operating lamp,	Hg..... $8 * 10^4$ Pa Noble gas (Ar, Kr, Ne, Xe) $2 * 10^4$ Pa
2	Gas temperature at axis, $T_{ax}$	4000 °K
3	Arctube wall temperature, $T_w$	1500 °K
4	Radiant power efficiency, 56 % rad	
5	Lamp current, $I_{rms}$	4.4 A
6	Lamp voltage, $V_{rms}$	105 V
7	Arctube inner diameter, $D_{tu}$	8 mm
8	Arctube inner long, $L_{tu}$	102 mm
9	Electrode diameter, $D_{el}$	1.35 mm
10	Electrode spacing, $L_g$	82 mm

\*The values in lines 7,8,9 come by from real lamp measurement.

The electro and thermo-physicals parameters of the arc tube's medium were derived detailed under the auxiliary assumptions :

9. The mixture in the arc tube is treated as a perfect monatomic [2] gas.
10. The noble gases recipe default is forced to replace it by hypothetical noble gas with averaged atom mass.
11. The radial temperature distribution in the arc tube has a parabolic profile:

$$T_r = T_w + (T_{ax} - T_w) \left( 1 - \frac{r^2}{D_{tu}^2/4} \right) \quad (A1)$$

where r is a radial coordinate.

13. Since the arc's steady-state ionization is governed by the species with lowest ionization potential, the arc's conductivity was been calculated by considering (in the Na- lamp) only Na.

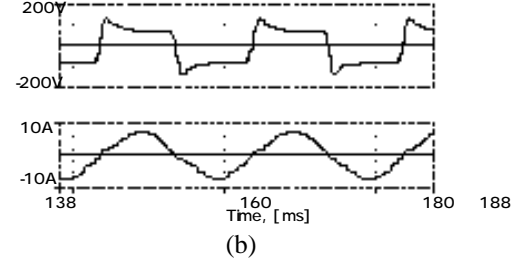
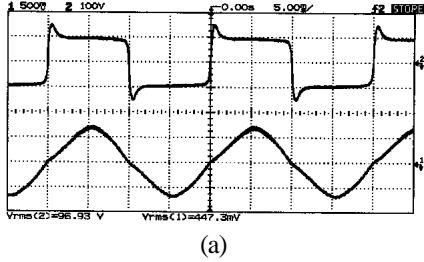


Fig.5. Dynamic characteristics of the Na-lamp (NAV-T-400, OSRAM) at 50 Hz: (a) experimental, (b) model simulation. Upper trace - voltage, lower trace - current. In (a): voltage - (100V/div), current - (5A/div).

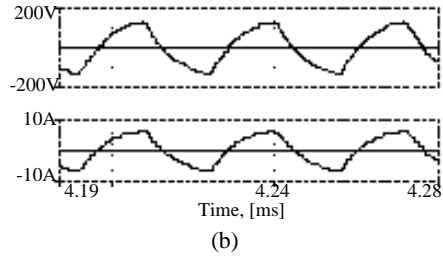
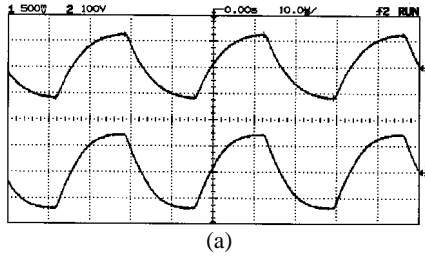


Fig.6 Dynamic characteristics of the Na-lamp (NAV-T-400, OSRAM) at 30 kHz: (a) experimental, (b) model simulation. Upper trace - voltage, lower trace - current. In (a): voltage - (100V/div), current - (5A/div).

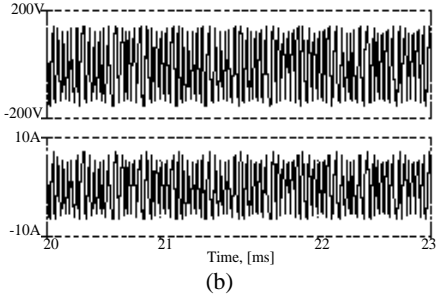
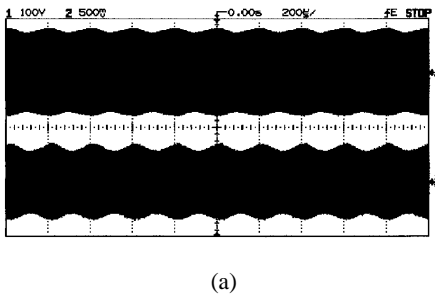


Fig 7. Response of the HPS lamp to low frequency modulation. Carrier frequency 30kHz, modulation frequency 5kHz. (a) experimental (b) model simulation. Upper trace - voltage, lower trace - current. In (a): voltage - (100V/div), current - (5A/div).

APPENDIX B  
PSPICE Model "PARAMETERS" data  
INPUT DATA

<u>PARAMETERS:</u>	<u>PARAMETERS:</u>	<u>PARAMETERS:</u>	<u>PARAMETERS:</u>
F_in 50Hz	Ltu 102m	Dtu 8m	ro_core 0.55
Vin 320V	Lg 82m	Del 1.33m	ro_per 0.7
Tamb 310			
<u>PARAMETERS:</u>	<u>PARAMETERS:</u>	<u>PARAMETERS:</u>	<u>PARAMETERS:</u>
N_core 22E+16	sia_Na 3.1E+6	k0 7286	k_tred 0.756
Co 80	sio 300	Eef 43.83E-20	eps 0.045
		kapa-o 3.6	
<u>INTERNAL PARAMETERS</u>			
<u>PARAMETERS:</u>	<u>PARAMETERS:</u>	<u>PARAMETERS:</u>	<u>PARAMETERS:</u>
eVi_Na 5.139	m_aNa 45	Q_Na 37E-20	Pi 3.14159
eVi_Hg 10.437	m_aHg 200	Q_Hg 31E-20	ga 56.703n
eVi_Sc 6.561	m_aSc 45	Q_Na_i 43E-20	k 1.381E_-23
<u>PARAMETERS:</u>	<u>PARAMETERS:</u>	<u>PARAMETERS:</u>	<u>PARAMETERS:</u>
mNa {m_aNa*1.67495E-27}	No_core {N_core/(Pi*Del*Del/4*Lg)}		
mHg {m_aHg*1.67495E-27}	No_per {No_core*ro_per/ro_core}		
mSc {m_aSc*1.67495E-27}			
<u>PARAMETERS:</u>	<u>PARAMETERS:</u>	<u>PARAMETERS:</u>	<u>PARAMETERS:</u>
Ei_Na {eVi_Na*1.6E-19}	Lmfp_core {0.707/(Q_Hg*No_core)}	Rat {Del/2}	
Ek_Na {Ei_Na/2/k}	Lmfp_per {0.707/(Q_Hg*No_core)}	Rao 0.1 m	
	dD {(Dtu-Del)/2}	Rtu {Dtu/2}	

## PSPICE FILE

To make possible the application of proposed model by other workers, the PSPICE file is available for down load . Please check the WEB site: <http://www.ee.bgu.ac.il/~pel> - Downloads Section.

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