Statics and Dynamics of Fluorescent Lamps Operating at High Frequency: Modeling and Simulation

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Abstract- **The static and dynamic response of a fluorescent lamp operating at high frequency was studied and modeled by a SPICE compatible behavioral equivalent circuit. Good agreement was found between model simulations and experimental results. It was also found analytically that the proposed model predicts a zero at the half right side of the complex plane as observed experimentally.**

I. INTRODUCTION

It is well documented that the electrical V-I characteristic of a fluorescent lamp under high frequency excitation is, to a first approximation, resistive [1]. Indeed, for many practical purposes the lamp can be replaced by an equivalent linear resistor [2,3]. However, a closer examination of fluorescent lamps characteristics reveals that this approximation is very crude and does not take into account many intricate phenomena that may be significant in the analysis and design of electronic ballasts. The deviations from the linear resistor model can be divided into three categories: the dependence of the V-I curve on the lamp's power level [4-6], the nonlinearity of the V-I curve at a given operating condition [7, 8] and the time domain response of the lamp to a change in excitation [9, 10]. The latter aspect has many theoretical and practical implications since the dynamics of the lamp is of prime importance when considering stability and response of lamp/ballast systems in open and closed loop configurations [10]. Furthermore, the nature of the dynamic response must be seriously considered in applications that call for modulation of the lamp's 'carrier' signal [11]. It is thus clear that a SPICE compatible model that will emulate the electrical static and dynamic characteristics of fluorescent lamps operating at High Frequency (HF) could be highly useful from the practical and theoretical points of view.

The objective of this study was to develop a SPICE compatible model that will exhibit two major electrical features of the fluorescent lamp operated at HF: the dependence lamp's resistance on power level and its dynamic response to changes in electrical excitation. It is still assumed that at any given operating point the lamp is resistive. Based on experimental observations as well as some physics based reasoning we developed a behavioral model to emulate the electrical response of a fluorescent lamp. Once developed, the model was calibrated against experimental data and then verified by independent measurements.

II. OBSERVATIONS

As already reported earlier, the impedance of fluorescent lamps operation at high frequency is approximately resistive. Following [4], the equivalent resistance (R_{eq}) at high frequency can be expressed as :

$$R_{eq} = \frac{V_s}{I_{rms}} - R_s \qquad (1)$$

where:
V_s , R_s - lamp's constants
I_{rms} - rms current through lamp

Relationship (1) is valide for a limited power range from about 30% to nominal power. For a wider power range the $V_{lamp}=f(I_{lamp})$ is non-linear and hence a higher order polynomials is required to fit the lamp's characteristic. This issue is further discussed in Section VII below.

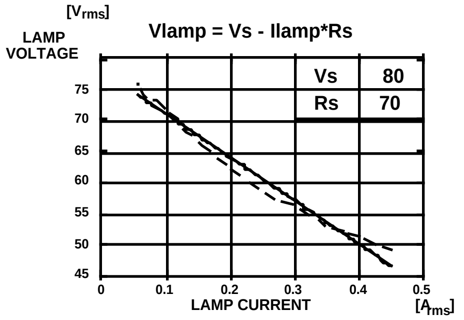

Fig. 1. Lamp voltage versus lamp current (OSRAM L 18W/10). Broken line: experimental. Solid line: linear fit.

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The parameters of equation (1) (V_S, R_S) can be obtained from experimental data [4]. The degree of matching is depicted in Fig. 1. The linear curve fitting for this lamp (OSRAM L 18W/10) yields the constants : $V_S = 80V$; $R_S = 70\Omega$.

The dependence (1) is for static conditions (constant HF drive). That is, at steady state conditions the high frequency V-I curve of the lamp depends on the power (or rms current) of the lamp as shown in Fig. 2. For a low power (P1) the lamp represents a relatively high resistance (R_{eq1}) while at a higher power (P2) the resistance is smaller (R_{eq2}). For a fast current change the response is approximately linear (arrows A & B, Fig. 2) while for a slow change in rms current the V-I curve will slide along arrow C (Fig. 2). The physical interpretation of this observation is linked to the lamps time constant associated with the generation/recombination of charge carriers in the lamp's plasma. A fast current change does not alter appreciably the density of the charge carriers in the plasma. Consequently the lamp behaves (to a first approximation) as a linear resistor. For slow changes the density of the carriers in the plasma will change and hence the resistance will shift. This behavior is clearly observed when exposing the lamp to a step change in the power level (Fig. 3). In this experiment the lamp was driven by a square wave voltage source in series with an inductor. Power level change was forced by changing the excitation frequency from 50kHz to 24kHz. For the high frequency carrier the lamp behaves as a linear resistor. The response to a change of power level (Fig. 3) is associated with a low frequency time constant that controls the rate at which the resistance of the lamp is varying. It is postulated here that for relatively small power variations the response is associated with a single low-pass time constant (pole).

The dynamic response of the fluorescent lamp, discussed above, can be observed from a different angle: exposing the lamp to a modulated HF signal [9, 10]. In this case, the nature of incremental (modulation) resistance should be a function of the modulating frequency. This is depicted in the experimental results of Fig. 4. At low modulating frequency (Fig. 4a), the current and voltage variations are in opposite direction, implying that the incremental resistance is negative. This can be explained by the fact that the V-I curves wobbles along the path 'C' of Fig. 2 which represents a negative resistance. For a high modulating frequency (Fig. 4c), the charge carrier density in the plasma is about constant and the incremental resistance is positive. In this case the V-I trace is moving along the HF resistance (paths A, B, Fig. 2).

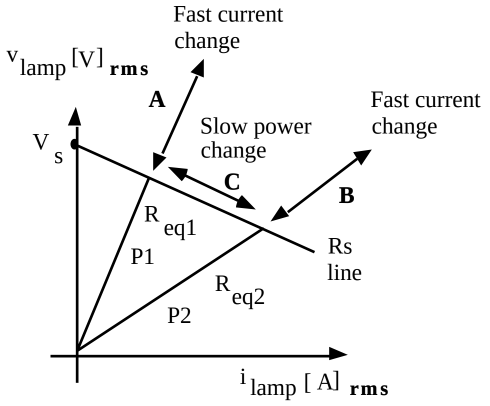


Fig. 2. Dynamic changes of V-I characteristics of a fluorescent lamp operating at HF.

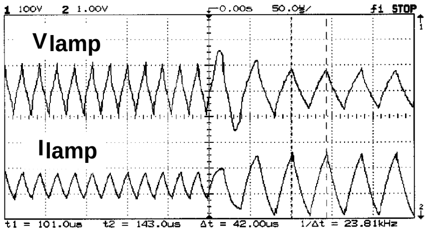


Fig. 3. Response of a fluorescent lamp voltage (upper trace, 100V/div) and current (lower trace, 1A/div) to a change in power level. Scale: 50mS/div (horizontal). Lamp: OSRAM L 18W/10.

III. MODEL DEVELOPMENT

Considering the above observations, it is evident that the electrical response of the lamp can be emulated by a behavioral dependent source whose resistance is defined by (1) [4, 5]. Equation (1) can be modified to a dynamic model if the rms current is first derived by measuring the lamp's rms current and then passing it through a low pass filter that matches the slow response of the lamp. This slow varying rms value can then be plugged back into (1) so as to change the lamp's resistance according to its static and dynamic behavior. The proposed SPICE compatible model of Fig. 5 follows these ideas. In this model, the lamp is represented as a dependent current source (G_1) that emulates a variable resistance according to (1). The output of the dependent voltage source E_1 is proportional to the square of the lamp current (i(lamp)). Namely, the voltage at node (isq) is

$$v(isq) \equiv i[(lamp)]^2 \qquad (2)$$

where the notation v(x) means: the voltage at node "x". This {v(isq)} signal is then passed through a low pass filter (R_1C_1, Fig. 5) to obtain its low frequency component. For frequencies $f > 1/(2\pi R_1C_1)$ and for times $t > R_1C_1$ the average voltage on C_1 (node (p)) will be:

$$v(p) = \frac{1}{T}\int_0^T v(isq)dt \equiv \frac{1}{T}\int_0^T \left[i(lamp)\right]^2 dt \qquad (3)$$

where T is the time constant R_1C_1.

The average voltage across the capacitor C_1 (node 'p' in Fig. 5) is thus a smoothed value of the squared rms current. The filtered I_{rms} is then obtained by E_2 (node 'rms' in Fig. 1) as the square root of the average voltage across the capacitor C_1 (node 'p'). Consequently:

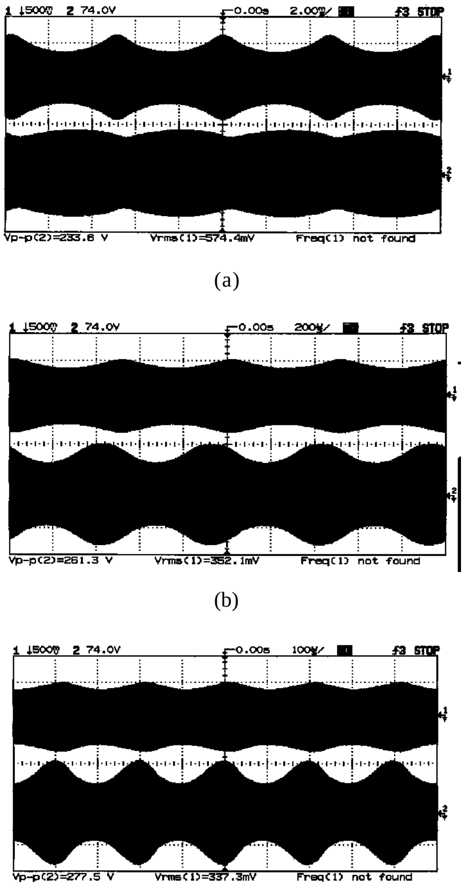
$$v(rms) \equiv \sqrt{\frac{1}{T}\int_0^T \left[i(lamp)\right]^2 dt} \qquad (4)$$

The definitions of the dependent sources are thus (per the notation of Fig. 5):

$$G_1 \equiv \frac{v(lamp)}{\frac{V_S}{v(rms)} - R_S} \qquad (5)$$

$$E_1 \equiv \{i(lamp)\}^2 \qquad (6)$$

$$E_2 \equiv \sqrt{v(p)} \qquad (7)$$



(a)

(b)

(c)

Fig. 4. Response of a fluorescent lamp (OSRAM L 18W/10) to a modulation on the high frequency excitation. Modulation frequency 200Hz (a), 2kHz (b), 5kHz (c). Upper traces: Lamp current (0.5A/div). Lower traces: Lamp voltage (74V/div). Carrier frequency: 50kHz.

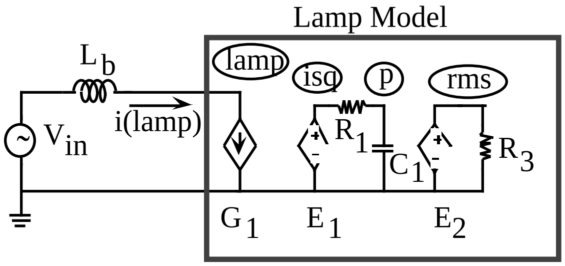


Fig. 5. Proposed fluorescent lamp model connected to a high frequency ballast.

The dynamic response of this model to changes in power level clearly depends on the time constant R_1C_1. It is postulated and subsequently proven that this single time constant can produce the observed dynamic responses of the physical lamp.

IV. SMALL SIGNAL RESPONSE

The response of the proposed model (Fig. 5) to a modulated signal on the carrier can be derived from (1). The lamp's incremental impedance ($Z_{inc}(f_L)$) is found by first representing (1) in a way that will distinguish between the AC voltage and current of the lamp and the filtered component of the lamp's current :

$$\tilde{V}_{lamp} = \tilde{I}_{lamp}\left(\frac{V_s}{I_{lamp0}} - R_s\right) \qquad (8)$$

where:

$\tilde{V}_{lamp}$ - lamp voltage (phasor)

$\tilde{I}_{lamp}$ - lamp current (phasor)

I_{lamp_0} - lamp RMS current

The small signal relationship is obtained by taking the derivative of (8):

$$v = i\left(\frac{V_s}{I_{lamp_0}} - R_s\right) - I_{lamp0}\frac{V_s i_0}{\left(I_{lamp0}\right)^2} \qquad (9)$$

where:

v - small signal perturbation of $\tilde{V}_{lamp}$

i - small signal perturbation of $\tilde{I}_{lamp}$

i_o - small signal perturbation of I_{lamp_0}

The small signal component i_o is obtained by passing the lamp current through the transfer function $H(j\omega)$ of the low pass filter (Fig. 5):

$$i_0 = iH(j\omega) \qquad (10)$$

From (9) and (10):

$$v = i\left(\frac{V_s}{I_{lamp0}} - R_s\right) - \frac{V_s i}{I_{lamp0}} H(j\omega) \qquad (11)$$

or:

$$v = i\left(\frac{V_s - R_s I_{lamp0} - V_s H(j\omega)}{I_{lamp0}}\right) \qquad (12)$$

The transfer function of the low pass filter is:

$$H(j\omega) = \frac{1}{1 + j\omega T}$$

where $T = R_1 C_1$.
From which:

$$v = i\left(\frac{(V_s - R_s I_{lamp0})(1 + j\omega T) - V_s}{I_{lamp0}(1 + j\omega T)}\right) \qquad (13)$$

The lamp's incremental impedance (Z_{inc}) is thus:

$$Z_{inc} = \frac{v}{i} = \left(\frac{(V_s - R_s I_{lamp0})\, j\omega T - R_s I_{lamp0}}{I_{lamp0}(1 + j\omega T)}\right) \qquad (14)$$

or:

$$Z_{inc} = R_s\left(\frac{\frac{R_{eq}}{R_s}\, j\omega T - 1}{j\omega T + 1}\right) \qquad (15)$$

Hence, the response of the proposed lamp model (Fig. 5) when driven by high carrier frequency which is modulated by a low frequency (f_L) is found to be:

$$Z_{inc}(f_L) = R_S \frac{\frac{jf_L}{f_0}\frac{R_{eq}}{R_s} - 1}{\frac{jf_L}{f_0} + 1} \qquad (16)$$

where $f_o = \frac{1}{2\pi R_1 C_1}$. It is of great interest to observe that response of the proposed model (16) includes a pole and a zero at the half right side of the complex plane (RHPZ) as discussed in [9,10].

The mathematical representation of (16) is in excellent agreement with the physical reasoning given above in connection with Fig. 2. For low modulation frequency (f_L -> 0) $Z_{inc}(f_L)$ -> $-R_S$ and for high modulating frequency (f_L -> ∞), $Z_{inc}(f_L)$ -> R_{eq} - as expected.

V. MODEL CALIBRATION

The proposed model of Fig. 5 can be calibrated to the response of a given lamp by evaluating V_S, R_S and the time constant R_1C_1. The first two can be obtained by measuring the rms voltage and current of the lamp for various power levels as shown in Fig. 1. R_1 C_1 can be obtained by matching the incremental response of the model to experimental results similar to those of Fig. 4. This was done experimentally by driven the experimental lamp by a high frequency carrier that was PM modulated. The PM modulation causes an AM modulation due to the series inductor of the ballast [9, 10]. The lamp current and voltage traces are then used to identify the location of the low frequency pole ($T = R_1$ C_1 of Fig. 5). Since only the time constant is of importance, one of the component (say, C_1) is chosen arbitrarily while the other is calculated from the experimentally measured T.

VI. EXPERIMENTAL RESULTS

Experiments conducted on an 18W lamp (OSRAM L 18W/10) for which the model parameters were extracted as discussed above. The degree of matching is shown by considering the step (Fig. 6) and modulated (Fig. 7) responses. These should be compared to Figs. 3 and 4 respectively. As can be seen the behavioral matching is very good. A more rigorous comparison was made by examining a complex plan plot the measured and simulated increment impedance (Fig. 8). This plot revels that the model predicts

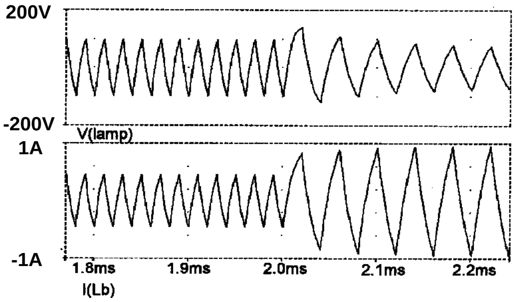
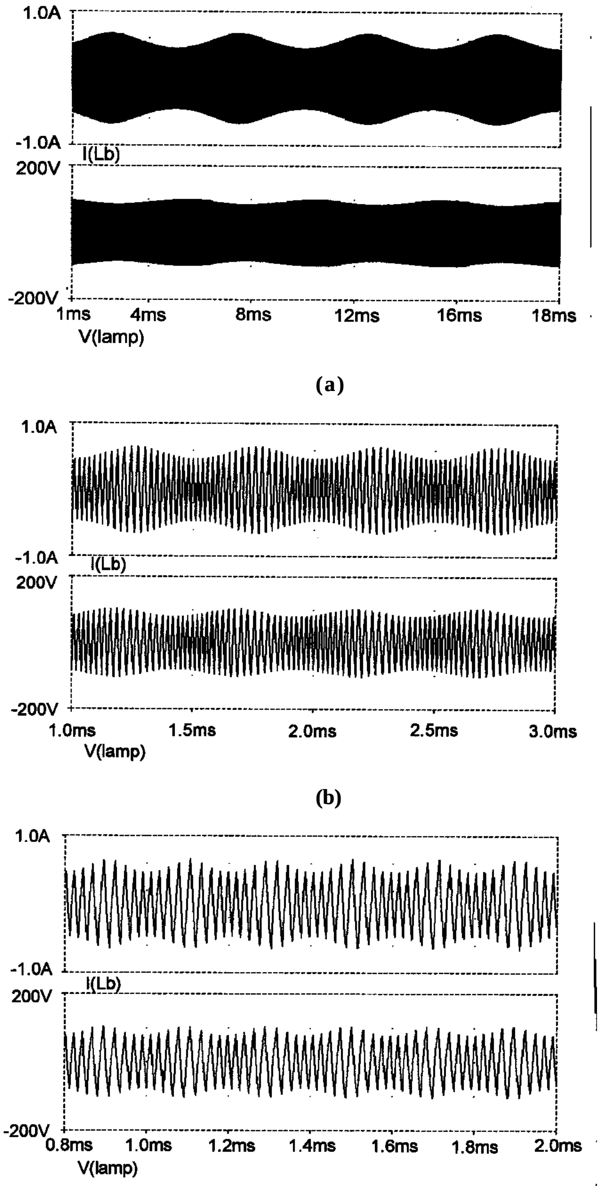


Fig. 6. Simulated response of a fluorescent lamp voltage (upper trace, 100V/div) and current (lower trace, 1A/div) to a change in power level. Simulation was carried by the model of Fig. 5 for the experimental conditions of Fig. 3.



(a)

(b)

(c)

Fig. 7. Simulated response of a fluorescent lamp to a modulation on the high frequency excitation. Simulation was carried by the model of Fig. 5 for the experimental conditions of Fig. 3.

faithfully the phase but has a considerable error in the amplitude. The reason for this discrepancy is traced down to the nonlinearity of the V-I curve of the experimental lamp (Fig. 9) . The simulation model assumes a linear response at any power level (Fig. 10 - Simulation). However, since the experimental measurements of the modulated component are made around peak values they measure a higher incremental resistance (Fig. 10 - Experimental).

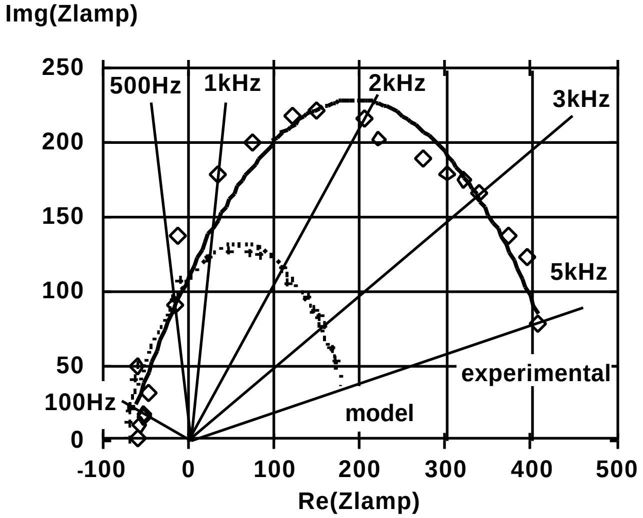


Fig. 8. Complex plan plot of measured and simulated incremental impedance of a fluorescent lamp (in Ohms) . Carrier frequency 50kHz, modulation frequency as marked.

VII. DISCUSSION AND CONCLUSIONS

The results of this study clearly suggest that the model of Fig. 5 is a good first order approximation of a fluorescent lamp behavior under static and dynamic conditions. The main reason for the deviation between the experimental and simulation results is the assumption that the lamp behaves as a linear resistor.

Implementation of the nonlinearity of the V-I curve (Figs. 9, 10) [7, 8] may render an even more accurate model. However the basic features of the present model are already very useful. It emulates the static and dynamic response of a fluorescent lamp operating at HF and exhibits the predicted RHPZ response [9, 10]. The model can thus be useful in simulating open and closed loop responses of lamp/ballast systems. When applying to model for a wide range of dimming one has to take into account that (1) is valid over a small power range.

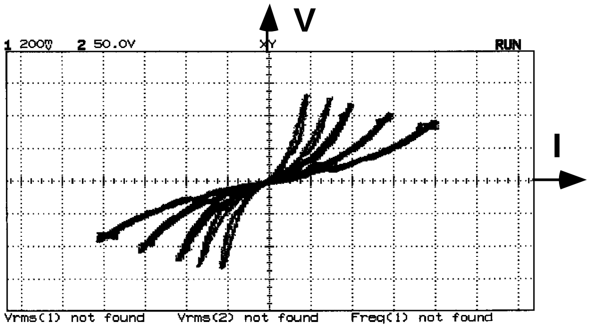


Fig. 9. V-I curve of experimental lamp when driven at 50kHz for various power level. Scales: 50V/div (vertical); 200mA/div (horizontal).

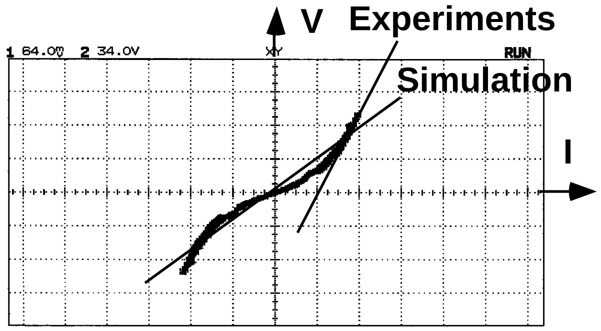


Fig. 10. Average equivalent resistance (Simulation) and peak equivalent resistance (Experimental). Scales: 34V/div (vertical); 54mA/div (horizontal).

If required, the fitting range can be extended by applying a second (or higher order) polynomial. For a second order polynomial:

$$R_{eq} = \frac{K_1}{I_{rms}} + K_2\ I_{rms} + K_3 \qquad (17)$$

where: K_1-K_3 are constants of the lamp. This extension was found to emulate the behavior of the lamp over a 1:10 dimming range. For such a range a second order polynomial fitting seems a better choice (Fig. 11). Notice that over this range the lamp's equivalent resistance increases 15 fold (Fig. 12).

The proposed model (Fig. 5) can be used with any polynomial fitting. The only adjustment required is to replace the denominator of (5) by the expression of the equivalent resistance derived from the fitting by evaluating:

$$\frac{V_{lamp_{rms}}}{I_{lamp_{rms}}} = f(I_{lamp_{rms}}) \qquad (18)$$

For the linear fit the expression is that of (1). For a second order polynomial expression is that of (17).

In the case of a high order polynomial, the negative resistance R_S (1)varied with the operating point. It represents, in fact, the local slope of the $V_{lamp(rms)}=f(I_{lamp(rms)})$ curve. This local slope is derived automatically by the simulator when runnig the proposed model (Fig. 5).

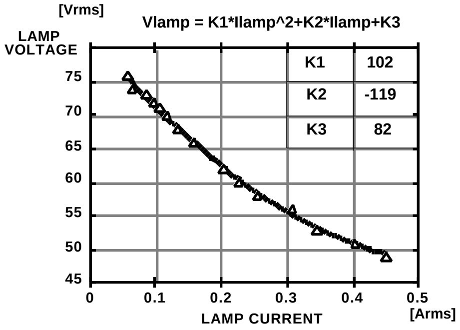


Fig. 11. Second order polynomial fitting the lamp voltage versuses lamp currentt. Same data as is Fig. 1.

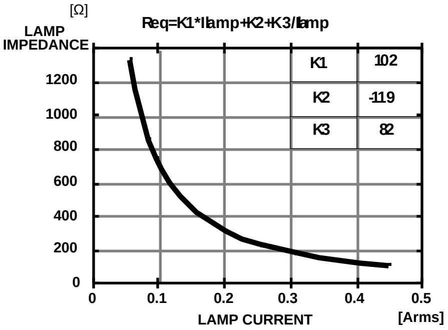


Fig. 12. Equivalent resistance of the experimental lamp derived from the data of Figs. 1 and 11.

Although developed through behavioral considerations, the model seems to suggest that the small signal dynamics of the physical lamp is governed by a single time constant. The overall small signal response is more complex having both a pole and RHPZ due to the nonlinearity of (1).

ACKNOWLEGMENT

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