A UNIFIED BEHAVIORAL AVERAGE MODEL OF SEPIC CONVERTERS WITH COUPLED INDUCTORS

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Abstract - An average model of SEPIC converters with coupled or uncoupled inductors was developed and verified against cycle-by-cycle simulations. The model can be used as-is by any modern circuit simulator to run steady state (DC), large signal (transient) and small signal (AC) analyses. The leakage inductances were taken into account and treated separately from the mutual inductance. The primary and secondary coupling coefficients were incorporated as parameters in the model. The coupling coefficients can be set to a value from zero to almost one, representing the complete range of possible SEPIC topologies.

I. INTRODUCTION

Practical SEPIC converters are usually built with coupled inductors (L_s and L_p, Fig. 1) to lower production costs and to steer away the input current ripple [1]. Yet, the SEPIC dynamic behavior is still poorly understood and the single simulation model proposed hitherto for this topology is valid only for the uncoupled inductors situation [2]. The SEPIC with coupled inductors case appears to represent a still open simulation issue. It involves two coupled inductors acting as a transformer, in the sense that current flows on both sides at the same time, while both sides are loaded by capacitors (C_{fi}, C_s and C_p, Fig. 1). The same modeling problem appears in other topologies with coupled inductors (e.g. C'uk converter). A general modeling solution, suitable for coupled inductors converters, is proposed in this paper.

The average modeling methodology [3, 4] used here hinges on the observation that by continuously applying the running average voltage (for a window of one switching period) across an inductor, the inductor will produce a running averaged current. The same reasoning applies to switched capacitors. Namely, by injecting the running average current into a capacitor, the capacitor will be charged to the running averaged voltage. This observation is based on the assumption that the switching frequency is much higher than the power stage bandwidth. Therefore, the average voltages across the capacitors and average currents through the inductors do not change appreciably during one switching period. Namely, the voltages and currents that will be produced by the average model of the proposed methodology are the ripple-less values, as if we have operated the switch at infinite frequency but with the same duty cycle values.

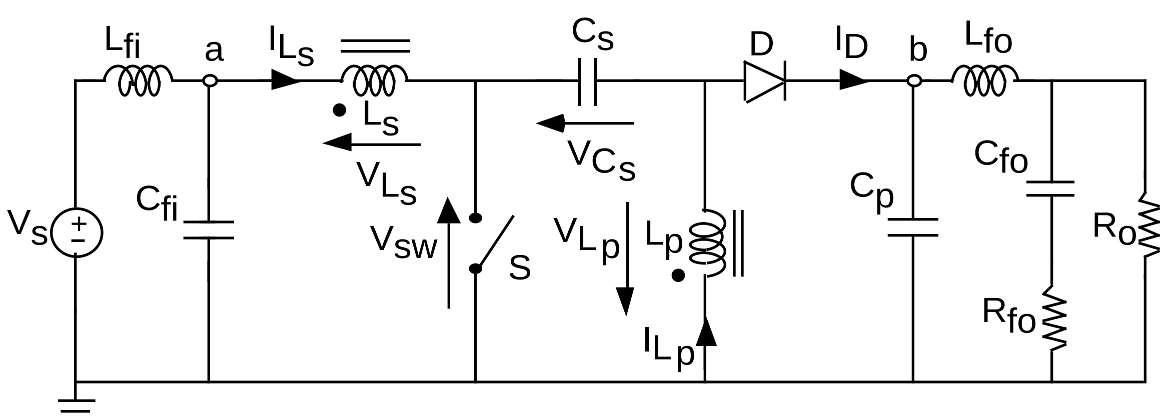

Fig. 1 DC to DC SEPIC converter with coupled inductors (L_s and L_p), including input and output filters.

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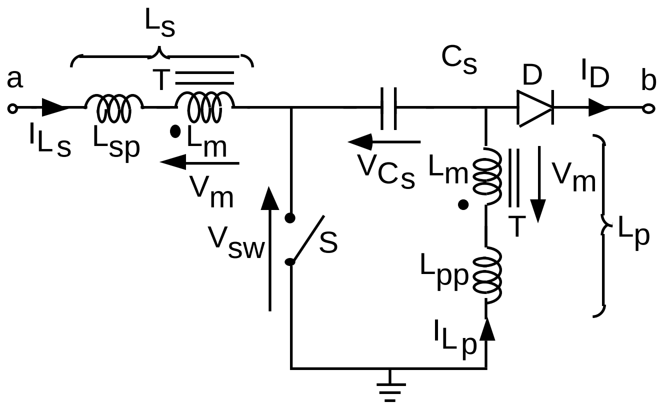


Fig. 2 Simplified diagram of the SEPIC topology with coupled inductors.

II. MODEL DERIVATION

Fig. 2 shows a simplified diagram of the coupled inductors SEPIC separated from the input and output filters. The voltages at terminals (a) and (b) are assumed to be constants within the switching period. The coupled inductors (L_s and L_p, Fig. 1) were split each into leakage (L_{sp} and L_{pp}) and mutual (L_m) inductances with ideal coupling. Assuming that the capacitors (C_{fi}, C_s and C_p Fig. 1) are large enough so that their voltages do not change appreciably during one cycle, they can be replaced by voltage sources. Therefore, we have in fact a transformer connected to two voltage sources at the input and output at the same time (Fig. 3). Obviously, the leakage inductances (L_{sp} and L_{pp}) cannot be neglected in this case. The voltage sources of the two terminals, V_{L_s} and V_{L_p} of Fig. 3, represent the voltages across the two inductors L_s and L_p respectively, including the leakage inductances. These voltage sources have different values during the 'on' and 'off' time intervals, but can be considered almost constant within each interval. The magnitude of the voltages, including the diode and switch conduction voltage drops (V_{dion} and V_{swon} respectively), are as follows:

$$V_{L_s} = \begin{cases} V_a - V_{swon} & ;\ t_{on} \\ V_a - V_{cs} - V_{dion} - V_b & ;\ t_{off} \end{cases} \tag{1}$$

$$V_{L_p} = \begin{cases} V_{cs} - V_{swon} & ;\ t_{on} \\ -V_b - V_{dion} & ;\ t_{off} \end{cases} \tag{2}$$

In the case of the SEPIC converter with coupled inductors, the leakage inductances (L_{sp} and L_{pp}, Figs. 2,3) act as the switched inductors [5]. In order to derive the average voltages across them, one has to evaluate the internal voltage (V_m) across the mutual inductance (L_m, Figs. 2,3).

A simple way to derive the expression for V_m is to start with the currents equation at the primary of the coupled inductors model of Fig. 3 :

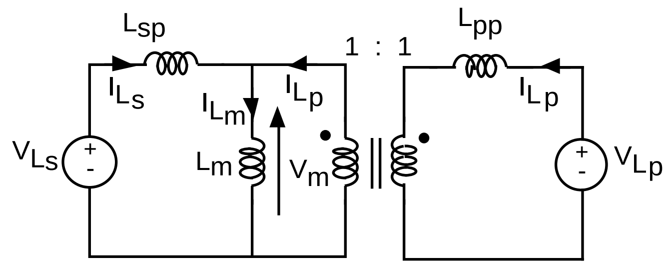


Fig. 3 The coupled inductors model with voltage sources representing the capacitors voltages.

$$I_{L_m} = I_{L_s} + I_{L_p} \tag{3}$$

where I_{L_m}, I_{L_s} and I_{L_p} are per the notations of Fig. 3.

Taking the derivative of both sides implies:

$$\frac{dI_{L_m}}{dt} = \frac{dI_{L_s}}{dt} + \frac{dI_{L_p}}{dt} \tag{4}$$

Assuming constant voltages over one switching cycle and substituting the current derivatives by the voltage to inductance ratio for each inductor, we obtain (see notations in Fig. 3):

$$\frac{V_m}{L_m} = \frac{V_{L_s} - V_m}{L_{sp}} + \frac{V_{L_p} - V_m}{L_{pp}} \tag{5}$$

which yields an explicit expression for V_m:

$$V_m = \frac{V_{L_s} L_m L_{pp} + V_{L_p} L_m L_{sp}}{L_m L_{sp} + L_m L_{pp} + L_{sp} L_{pp}} \tag{6}$$

Notice that the voltage V_m is an algebraic function of the voltages V_{L_s} and V_{L_p}. This implies that it also does not change significantly within the 'on' or the 'off' time intervals.

Based on equation (6), all the average voltages of the SEPIC switched inductors can be evaluated and used to generate the inductors average currents (I_{ls}, I_{lp}). The inductors currents will be used to derive the averaged current of the capacitor C_s. Thus, a complete behavioral average model for the SEPIC converter can be developed, similar to the method described earlier [5].

The coupling coefficients k_1 and k_2 are defined as the ratios between the mutual inductance L_m to the primary and secondary inductances L_s and L_p respectively, namely:

$$k_1 = \frac{L_m}{L_s} \tag{7}$$

$$k_2 = \frac{L_m}{L_p} \tag{8}$$

The leakage inductances can be defined as:

$$L_{sp} = L_s - L_m = (1 - k_1) L_s \tag{9}$$

$$L_{pp} = L_p - L_m = (1 - k_2)L_p \qquad (10)$$

were the mutual inductance L_m is:

$$L_m = k_1 L_s = k_2 L_p = k\sqrt{L_s L_p} \qquad (11)$$

and k is defined as:

$$k = \sqrt{k_1 k_2} \qquad (12)$$

In the coupled inductors SEPIC we identify two switched inductors (L_{sp} and L_{pp}, Figs. 2,3) and one switched capacitor (C_s, Figs. 2,3). Note that, the mutual inductance is functioning only as a parameter in the average model (eq. 6). Following the procedure developed earlier [5], the switching elements are replaced by sub models, containing a dependent voltage source across each of the inductors (L_{sp}, L_{pp}) and a dependent current source that inject an averaged current into the capacitor (C_s).

The complete average model is shown in Fig. 4. The upper circuit is the main average model while the lower one contains dependent voltage sources representing time dependent variables used in the upper part. The dependent voltage source E_{lsp} produces the average voltage at one terminal of the inductor L_{sp} while the other terminal (a) is assumed to be at constant voltage (approximately, within one cycle). E_{lpp} imposes the average voltage across the inductor L_{pp} while G_{cs} injects the average current into the capacitor C_s. Finally, the dependent current source G_b generates the average current flowing out of terminal b.

The expressions for the dependent sources are as follows:

$$E_{lsp} = [V(mon) + V(swon)]*V(don) + + [V(moff) + V(cs) + V(dion) + V(b)]*V(doff) \qquad (13)$$

$$E_{lpp} = [-V(swon) + V(cs) - V(mon)]*V(don) + - [V(b) + V(dion) + V(moff)]*V(doff) \qquad (14)$$

$$Gcs = -I_{lpp}*V(don) + I_{lsp}*V(doff) \qquad (15)$$

$$G_b = (I_{lsp} + I_{lpp})*V(doff) \qquad (16)$$

$$E_{mon} = [V(a) - V(swon)]*k_{l1} + + [V(cs) - V(swon)]*k_{l2} \qquad (17)$$

$$E_{moff} = [V(a) - V(cs) - V(dion) - V(b)]*k_{l1} + - [(V(b) + V(dion)]*k_{l2} \qquad (18)$$

where:

$$k_{l1} = \frac{L_m L_{pp}}{L_m L_{sp} + L_m L_{pp} + L_{sp} L_{pp}} \qquad (19)$$

$$k_{l2} = \frac{L_m L_{sp}}{L_m L_{sp} + L_m L_{pp} + L_{sp} L_{pp}} \qquad (20)$$

$$E_{doff} = 1 - V(don) \qquad (21)$$

All voltages are node voltages reefer to 'ground' (Fig. 4).

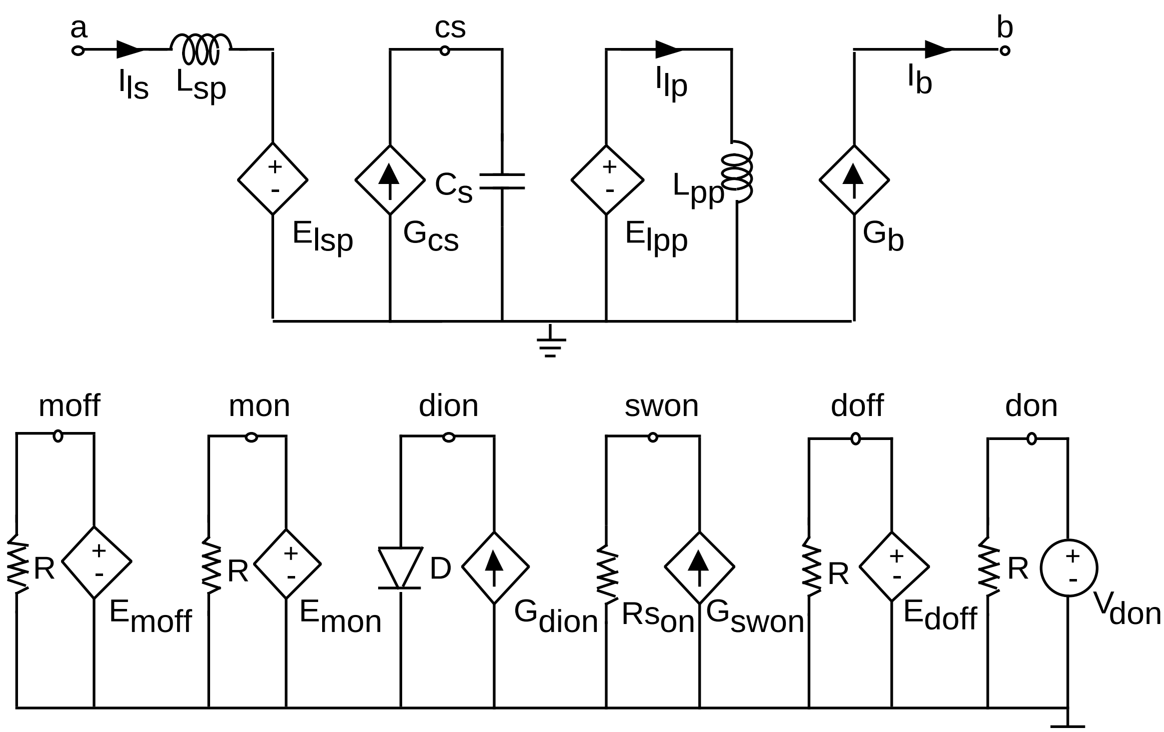


Fig. 4 Proposed behavioral average model for SEPIC converters with coupled inductors. Nodes names are marked by -o-.

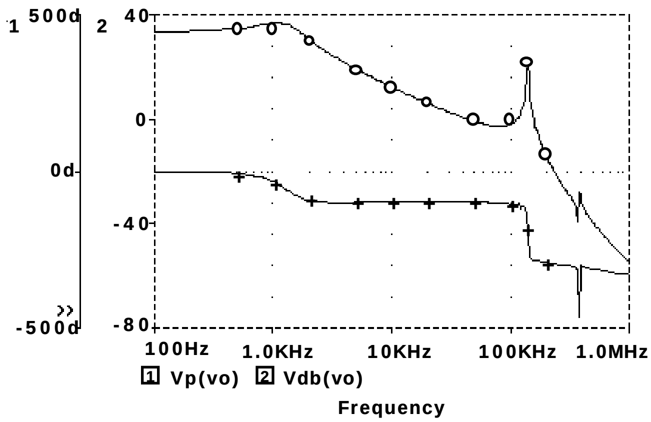


Fig. 5 AC small signal control-to-output $\left[\frac{V_o}{V_{don}}(f)\right]$ response of SEPIC converter with coupled inductors. Cycle-by-cycle simulations (**o** for Magnitude[db], + for Phase[deg]). compared to average model simulations (continuous lines). Operating point: D_{on}=0.14, R_o=5Ω, k_1=k_2=0.9.

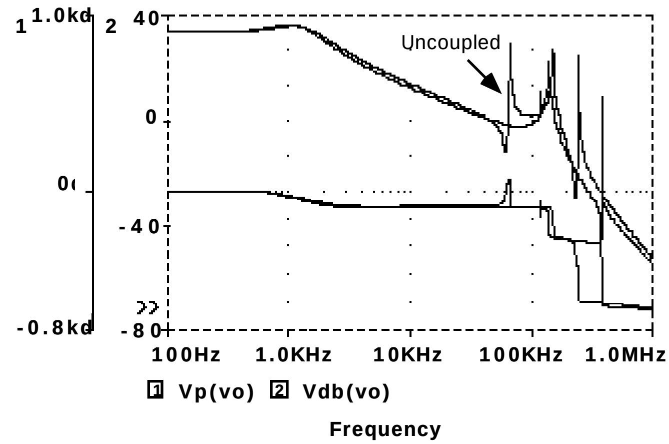


Fig. 6 AC small signal characteristics of SEPIC topology with coupled (k_1=k_2=0.9) and uncoupled inductors. Operating point: D_{on}=0.14, R_o=5Ω.

The voltage dependent sources E_{mon} and E_{moff} (eq. 17, 18) generate the voltage V_m (eq. 6) of the 'on' and 'off' time intervals respectively.

The resulting voltages (V(mon) and V(moff)) are incorporated in the expressions for E_{lsp} and E_{lpp} (eq. 13,14). The average currents produced by the inductors (I_{lsp}, I_{lpp}) are used in the dependent current sources G_{cs} and G_b (eq. 15,16).

The dependent voltage source E_{doff} generates a time dependent voltage which is an analog to the 'off' time ratio (D_{off}, eq. 21), assuming Continuous Conduction Mode (CCM). The independent voltage source V_{don} (Fig. 4) emulates the duty cycle (D_{on}) for open loop simulations. This source can be replaced by a dependent voltage source along with the corresponding control circuitry for closed loop simulations.

III. MODEL VERIFICATION

The average model was verified against a complete time domain simulation of the switched SEPIC converter. The parameters of the converter were as follows (see Fig. 1 for notations):

V_s = 36V, L_{fi} = 2.75μH, C_{fi} = 0.2μF,

L_s = L_p = 9.75μH.

C_s = 0.3μF, C_p = 0.44μF, L_{fo} = 3.8μH,

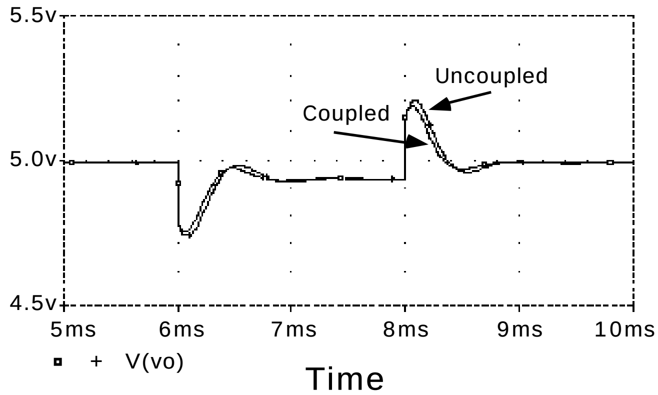
C_{fo} = 940μF, R_{Cfo} = 90mΩ, $R_{o(nominal)}$ = 5Ω,

F_S = 1MHz (switching frequency)

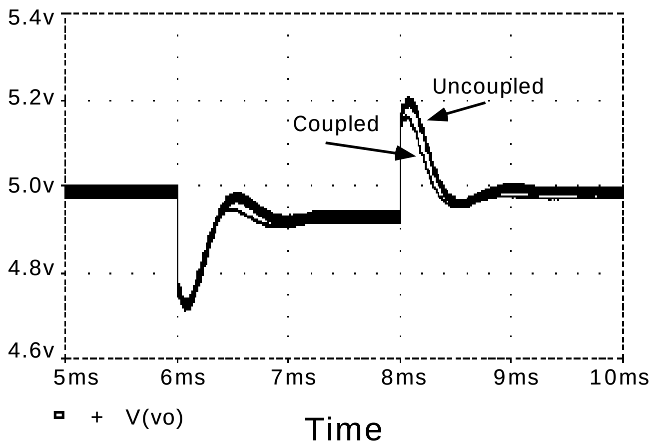
Excellent agreement was obtained for steady state (DC), large signal (transient) and small signal (AC) responses for the full range of coupling coefficient values. The DC traces of the coupled inductors case are identical to those of the uncoupled inductors case as reported earlier [5]. The small signal plots of Fig. 5 demonstrate the accuracy of the frequency response obtained by the proposed average model against time domain cycle-by-cycle simulations. The cycle-by-cycle values were collected tediously one by one. Each point was evaluated by running a transient simulation of the straightforward switched circuit while modulating the control voltage of the pulse generator (PWM) by a constant frequency sine wave. The average model results were obtained by one AC analysis sweep, in which an AC signal was superimposed on the duty cycle voltage source (V_{don}, Fig. 4).

The proposed average model can be used for both coupled and uncoupled cases without any modification. By setting the coupling coefficients parameters to zero, one gets an uncoupled SEPIC topology. In this case the inductances values of L_{sp} and L_{pp} (Fig. 4) would be equal to those of L_s and L_p respectively. Fig. 6 presents the differences in the frequency response of the coupled and uncoupled cases. It seems that the resonance effects in the coupled inductors dynamics were partly removed and shifted to higher frequencies. The uncoupled SEPIC frequency response was already verified [5].

The accuracy of the SEPIC large signal Dynamic response was tasted by a load step (Fig. 7). Both the coupled and uncoupled cases seem to have a very similar dynamic behavior. As can be seen, the proposed average model follows very well the running average of the cycle-by-cycle simulations. Notice the larger ripple in the uncoupled case. It might be due to the resonance's effect which seem to be stronger and at lower frequencies, in the uncoupled SEPIC, as indicated by the frequency response (Fig. 6). The CPU time of the average model time domain simulation was 285 times faster than the cycle-by-cycle simulation. A complete 'Schematic' [6] diagram of the experimental SEPIC converter is shown in Fig. 8.



(a)



(b)

Fig. 7 Transient response obtained by the Average Model with coupled and uncoupled inductors simulation (a), and by cycle-by-cycle simulation (b), for a load resistance step (from $R_O=5\Omega$ down to 1.43Ω and back).

IV. CONCLUSIONS

The behavioral average model presented here seems to be an excellent tool for the analysis and design of SEPIC converters. The model is compatible with any modern circuit simulator and can be used to run DC, AC and transient analysis. In AC analysis the task of linearization is left for the simulator.

The SEPIC with asymmetrical coupling as suggested by Dixon [1], is also supported by the proposed model. An extension to the Average Current Mode control [7] can be implemented by adding a 'Duty Cycle Generator' in a similar way to the Peak Current Mode control [5].

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See next page for Figure 8.

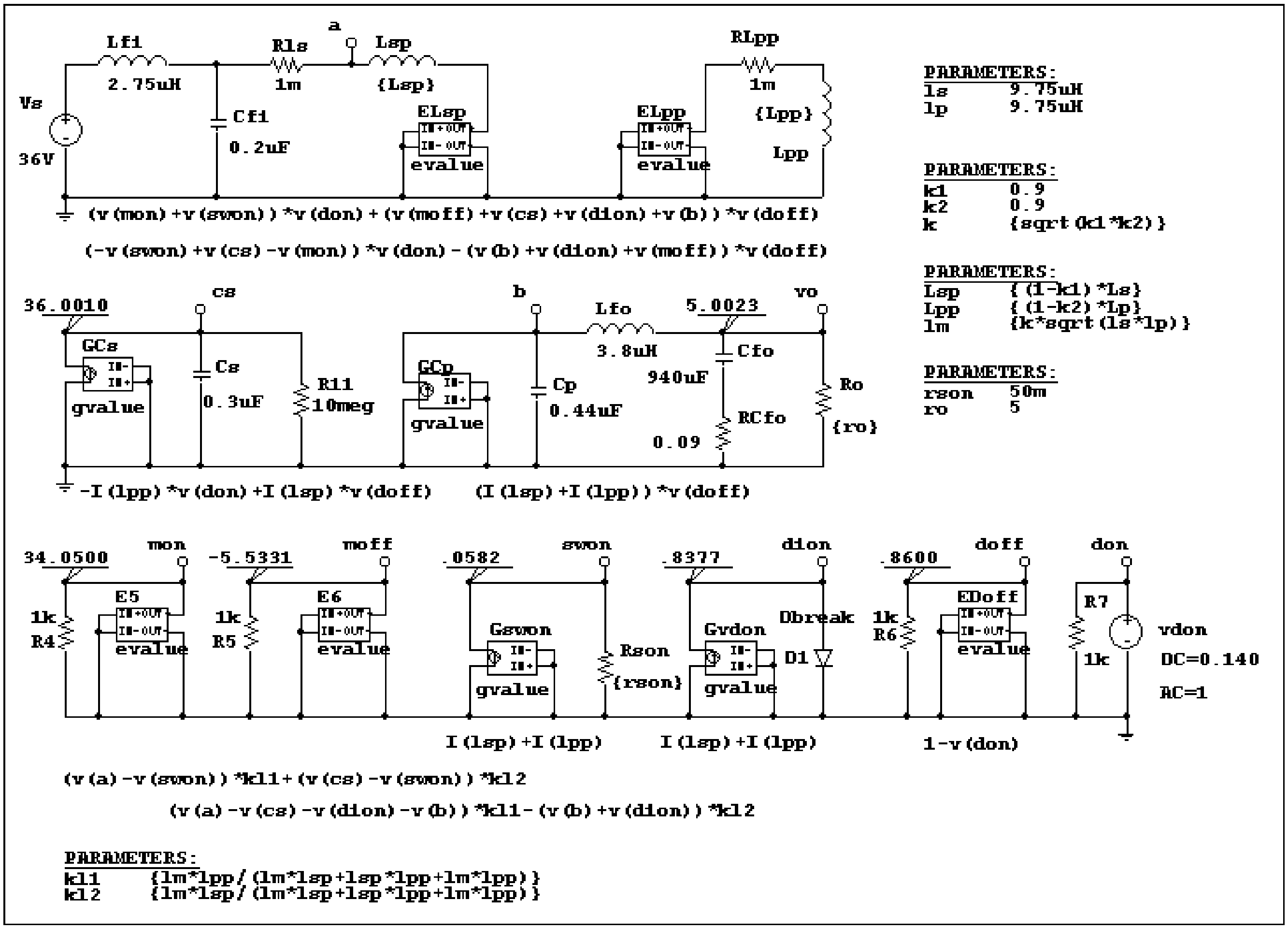

Fig. 8. 'Schematics' (MicroSim Inc.) diagram of the SEPIC average model for open loop simulations.