Average Modeling and Simulation of Series-Parallel Resonant Converters by SPICE Compatible Behavioral Dependent Sources

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Abstract **- A new methodology for developing average models of resonant converters is presented and verified against cycle by cycle simulation. The proposed modeling approach applies the concept of $R_{ac}(t)$ which represent the instantaneous effective load of the resonant network. Unlike the treatment given in previous studies, the value of R_{ac} is evaluated here dynamically as a function of the temporal low-frequency-average of other relevant variables. Once defined, the model can be used as is to run steady state (DC), large signal (transient) and small signal (AC) simulations on any modern circuit simulators. The proposed methodology was used to develop a behavioral model of a series-parallel resonant converter. Excellent agreement was found between simulation by the proposed model and cycle by cycle simulation.**

I. INTRODUCTION

Resonant converters have many favorable advantages. They can be designed for Zero Voltage Switching (ZVS), Zero Current Switching (ZCS) in either current fed or voltage fed topologies. Indeed, they were shown to be useful in a multitude of applications ranging from basic DC-DC converters [1], active power factor correction circuits [2] to capacitor chargers [3].

A prerequisite for a solid engineering design of resonant converters is a good model that describes their operation in the time as well as in the frequency domain. Two basic approaches have been used hitherto to develop such models. One approach applies analytical relationships to derive the expressions that describe the behavior of a given converter in the various domains [4]. A second approach developed by Steigerwald [5] uses the first harmonic approximation and the R_{ac} concept. By this, the converter is described as a simple resonant network with a load dependent damping (or quality) factor which can then be examined by basic (steady state) network equations. The limitation of the second approach is the difficultly of applying it to more than just the steady state (DC) voltage ratio relationships.

In this study we overcome this deficiency of the R_{ac} approach by extending the behavioral modeling methodology [6] to resonant converters. The advantage of the average models derived by the proposed high level presentation, is their ability to emulate the DC, large signal and small signal responses of the corresponding switch mode or resonant system. Once derived, the models can be run as-is on practically any modern circuit simulation package to obtain open or closed loop responses in the time and/or frequency domain. The fundamental ideas of the proposed approach are exemplified by developing the behavioral model of a series-parallel resonant converter and verifying the validity of the model against cycle by cycle simulation. Following the same line of reasoning, other resonant topologies can be captured by analogous behavioral representation.

II. MODEL DERIVATION

Following Steigerwald [5], the basic operation of a resonant converter, such as a series-parallel converter (Fig. 1), can be represented by a damped resonant network (Fig. 2). In this representation the virtual AC resistor (R_{ac}) expresses the effect of the dissipative nature of the load (R_{out}, Figs. 1, 2) on the resonant circuit. The value of (R_{ac}), under steady state conditions, can be obtained by equating the AC power dissipated by it to the DC power delivered to the load (R_{out}). This yields:

$$R_{ac}=\frac{\pi^2}{8}R_{out} \qquad (1)$$

where R_{ac} and R_{out} are per the notations of Figs. 1, 2. It is assumed that the switching frequency is above the resonant frequency. For switching frequency below the resonant frequency, the accuracy of the model may by poor [5].

R_{ac} is in general time dependent. For example, in the transient state, or in steady state when the switching frequency is modulated by a low frequency perturbation, the 'load' seen by the resonant network is not constant. Under these conditions, some reactive energy circulates in the output filter components. Yet, the average 'load' seen by the resonant network at any given moment is resistive. This stems from the fact that the current through L_{out} (Figs. 1, 2) can be considered constant over one switching cycle and the fact that the current drawn by the output section is always in phase with the voltage across C_p (Figs. 1, 2) [5]. Consequently, R_{ac} can be considered as a time dependent resistor. The value of $R_{ac}(t)$ at any given time can be derived dynamically by

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dividing the average of the absolute value of the voltage across C_p by the average current of L_{out} (Fig. 2). Namely:

$$R_{ac}(t) = \frac{\pi^2}{8}\frac{\overline{|V_{cp}(t)|}}{\overline{I_{L_{out}}(t)}} \qquad (2)$$

where:

($\overline{|V_{cp}(t)|}$) is the average of the absolute value of the voltage across C_p and ($\overline{I_{L_{out}}(t)}$) is the average current of L_{out}.

A basic assumption of the present modeling approach is that quasi steady state conditions prevail during any given switching cycle. That is, we assume that rate of change of the disturbances is sufficiently low such that steady state solutions of the resonant network equations are a good approximation of the instant input to output relationships. Under this assumption, the average voltage across C_p can be obtained by a simple steady state transfer function e.g.:

$$\overline{|V_{cp}(t)|} = V_{dc}\frac{8}{\pi^2}|H(j\omega)| \qquad (3)$$

where:

$$|H(j\omega)| = \frac{\omega C_s R_{ac}(t)}{\left[\left(1-\omega^2 L_r C_s\right)^2 + \left(\omega R_{ac}(t)\left(C_s+C_p-\omega^2 C_s C_p L_r\right)\right)^2\right]^{\frac{1}{2}}} \qquad (4)$$

and V_{dc} is the DC input voltage.

Equations (2) and (3) can now be solved for $\overline{|V_{cp}(t)|}$ and $R_{ac}(t)$ assuming that all other variables are known. In the present approach, the chores of deriving the solution are left to circuit simulators such as PSPICE [7] that have a build-in capabilities to handle behavioral dependent sources. To accomplish this, we first transform the problem into an equivalent circuit representation which is compatible with practically all modern analog circuit simulators.In this portrayal, all time dependent variables are coded into voltages or currents (Table 1). Next, we present the relevant equations by dependent sources that are a function of the coded variables and constants. Finally, we add the excitation and the output section to complete the picture. The final result for the series-parallel converter is the equivalent circuit of Fig. 3. The definitions of the independent and dependent sources are as follows:

$$\mathrm{Vin} = \frac{4}{\pi}V_{dc} \qquad (5)$$

Vf = f [Hz] (Switching frequency) (6)

Ew = 2πv(f) [1/Sec] (Angular switching frequency) (7)

$$\mathrm{ERac} = \frac{\pi^2}{8}\left|\frac{\mathrm{v(cp)}}{\mathrm{-i(Ecp)}}\right| \quad \text{(Temporal value of } R_{ac}\text{)} \qquad (9)$$

Variable	Reference Figure	Coded	Model Notation (Fig. 3)
$R_{ac}(t)$ [Ω]	2	Yes	v(Rac) [Volt]
$\overline{\lvert V_{cp}(t)\rvert}$ [volt]	2,3	No	v(cp) [Volt]
$\overline{I_{L_{out}}(t)}$ [Amp]	2	No	-i(Ecp) [Amp]
V_{dc} [Volt]	1	No	v(in) [Volt]
f [Hz]		Yes	v(f) [Volt]
ω [1/Sec]		Yes	v(w) [Volt]

Table 1. Time dependent variables representation.

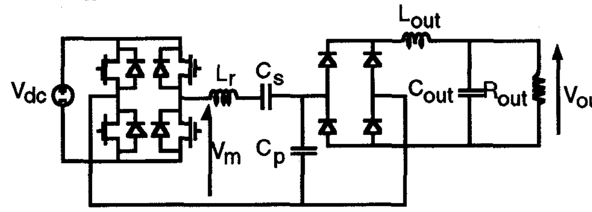

Fig. 1. Basis configuration of the series-parallel resonant converter topology.

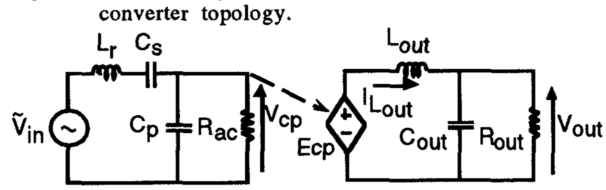

Fig. 2. First-harmonic approximation of the series-parallel resonant converter.

It should be pointed out that the average model of Fig. 3 is transparent to the switching frequency. Namely, at steady state, all the voltages and currents in the model (Fig. 3) are DC. During a transient state, the voltages and currents are time dependent.

$$\mathrm{Ecp} = \frac{\mathrm{v(in)(2/\pi)v(w)}C_s\mathrm{v(Rac)}}{\left[\left(1-(\mathrm{v(w)})^2\mathrm{LrCs}\right)^2 + \left(\mathrm{v(w)v(Rac)}\left(\mathrm{Cs+Cp-(v(w))^2CsCpLr}\right)\right)^2\right]^{\frac{1}{2}}} \quad \text{(Average voltage across } C_p\text{)} \qquad (8)$$

For a constant switching frequency, the excitation Vf (Fig. 3) is a DC voltage source. For FM modulated switching frequency, Vf will comprise a DC component plus an AC component that represents the frequency deviation. In transient analysis Vf is time dependent.

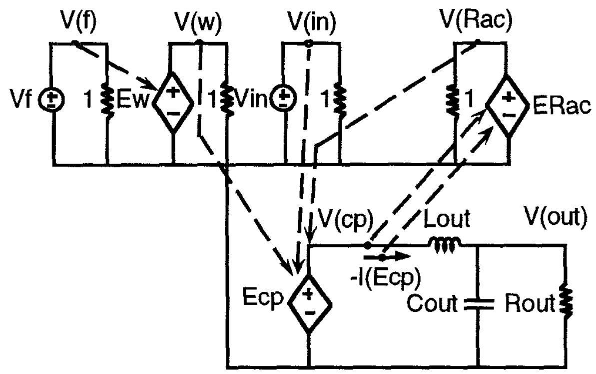

Fig. 3. Average model of the series-parallel resonant converter by applying PSPICE (Micro Sim Inc) behavioral dependent sources.

III. RESULTS AND DISCUSSION

The proposed model methodology was verified by comparing the model behavior against a full, cycle by cycle, PSPICE simulation.

The parameters of the resonant converter studied were as follows (Fig. 1):

$V_{dc} = 100V$

$L_r = 78\mu H$ $C_s = 43nF$ $C_p = 43nF$

$L_{out} = 1mH$ $C_{out} = 1\mu F$ $R_{out} = 15\Omega - 120\Omega$

The comparison was made for steady state (DC), large signal (transient) and small signal analyses (AC). The procedures and results for each case are given below:

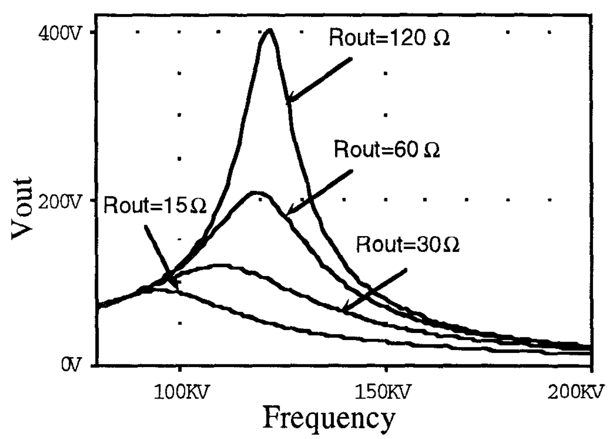

Fig. 4. DC gain for different load resistors obtained by proposed model. Horizontal axis: 1V=1Hz

A. *DC Analysis*

The 'DC' transfer function (output voltage as a function of switching frequency) was obtained by running a DC analysis on the model. This was done by sweeping the voltage source (Vf, Fig. 3), which represents the switching frequency, over the desired voltage-coded frequency range. Typical responses for different load resistors are depicted in Fig. 4. The original response of the actual switching circuit was obtained by running a cycle by cycle PSPICE simulation, of the original circuit, in the transient (TRAN) mode. Each simulation run was for one switching frequency and the asymptotic value of say, the output voltage, was used as the steady state solution. Comparisons between the results of the model simulation and the cycle by cycle simulation (Fig. 5) reveal that the two are practically identical. It is important to point out that the time required to obtain one point by the cycle by cycle simulation was two orders of magnitude longer than the time required for the complete DC sweep simulation by the model. Typical running time (with a 33MHz 486 machine) were as follows. Simulation time for one point in cycle by cycle simulation : 475 seconds; total running time for a complete DC sweep by model: 3.7 seconds.

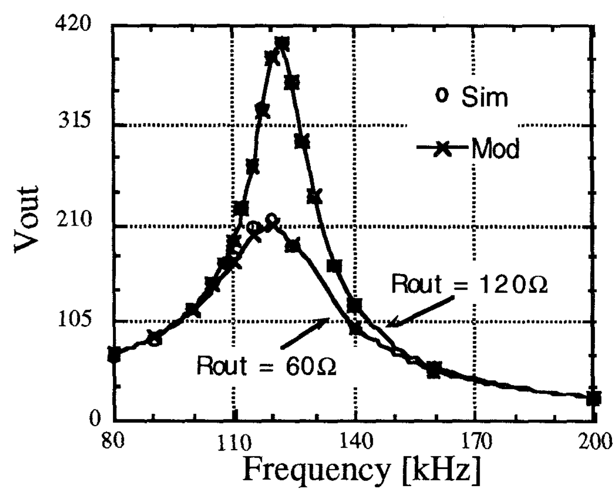

Fig. 5. Comparisons between the DC gain results of model simulation (Mod) and the cycle by cycle simulation (Sim) for $R_{out} = 120\Omega$ and $R_{out} = 60\Omega$.

B. *Transient Analysis*

The transient response obtained by the model was also validated against a cycle by cycle simulation. We tested the effect of a step in frequency. In the model, the frequency step was represented by a step in Vf. In the cycle by cycle simulation, the switching frequency was instantaneous switched from one frequency (155kHz) to another (165kHz). Typical results are given in Fig. 6.

C. Small Signal Analysis

An important advantage of the proposed model is the swiftness by which small signal analysis can be carried out. To do that, one defines Vf or Vin (Fig. 3) as a DC plus an AC voltage source and runs an AC analysis on the circuit. That is, the derivation of the linearized response is left to the simulator. All that is needed is to properly scale the frequency perturbations, i. e. 1V=1Hz. To verify the model behavior we compare it also to cycle by cycle simulation. The latter is extremely tedious and time consuming. For each frequency point the switching frequency was modulated and after steady state was reached, the modulation component at the output was examined for amplitude and phase.

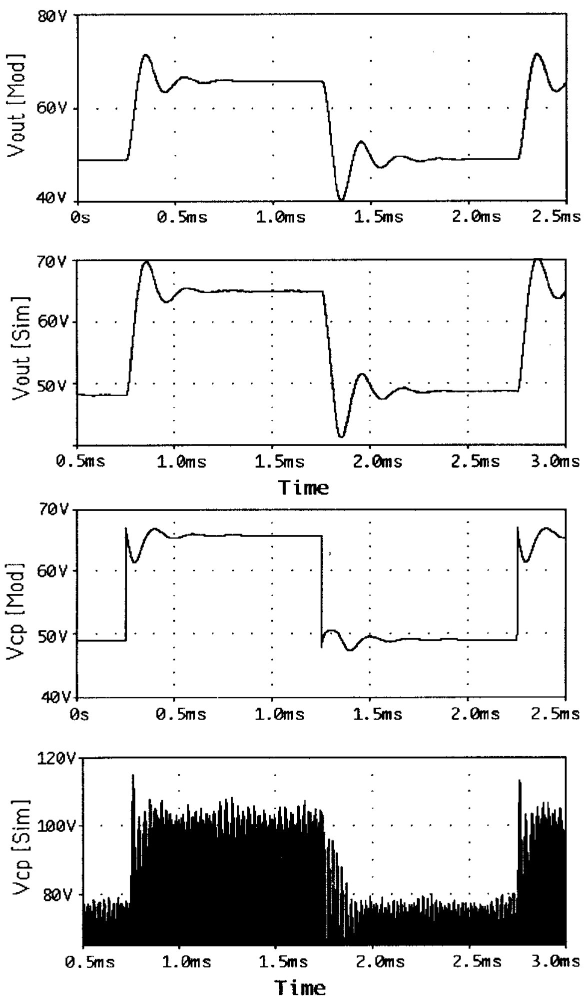

Fig. 6. Comparisons between Vout (Upper two traces) and Vcp (Lower two traces) response to a step in frequency. Upper traces: model simulation (Mod); Lower traces: cycle by cycle simulation (Sim).

The relevant relationships for the cycle by cycle simulation are as follows [8]:

1. Frequency-perturbations to output-voltage.

The modulated signal (V_m, Fig. 1):

$$V_m = V_{dc}\,\mathrm{sgn}\left[\sin\left\{2\pi f_s t+\beta\sin\left(2\pi f_m t\right)\right\}\right] \quad (10)$$

where:

$$V_{ref} = \cos\left(2\pi f_m t\right) \quad (11)$$

The amplitude and phase responses were then calculated by:

$$v_{ac}[\mathrm{dB}] = 20\mathrm{Log}\left(\frac{\Delta V_{out}(p\text{-}p)}{2\beta f_m}\right) \quad (12)$$

$$\varphi v_{ac}[\mathrm{deg}] = \varphi V_{out} - \varphi V_{ref} \quad (13)$$

2. Line to output transfer function.

The modulated signal (V_m, Fig. 1):

$$V_m = \left[V_{dc}+\beta\sin\left(2\pi f_m t\right)\right]\mathrm{sgn}\left\{\sin\left(2\pi f_s t\right)\right\} \quad (14)$$

where:

$$V_{ref} = \sin\left(2\pi f_m t\right) \quad (15)$$

The amplitude and phase responses were then calculated by:

$$v_{ac}[\mathrm{dB}] = 20\mathrm{Log}\left(\frac{\Delta V_{out}(p\text{-}p)}{2\beta}\right) \quad (16)$$

$$\varphi v_{ac}[\mathrm{deg}] = \varphi V_{out} - \varphi V_{ref} \quad (17)$$

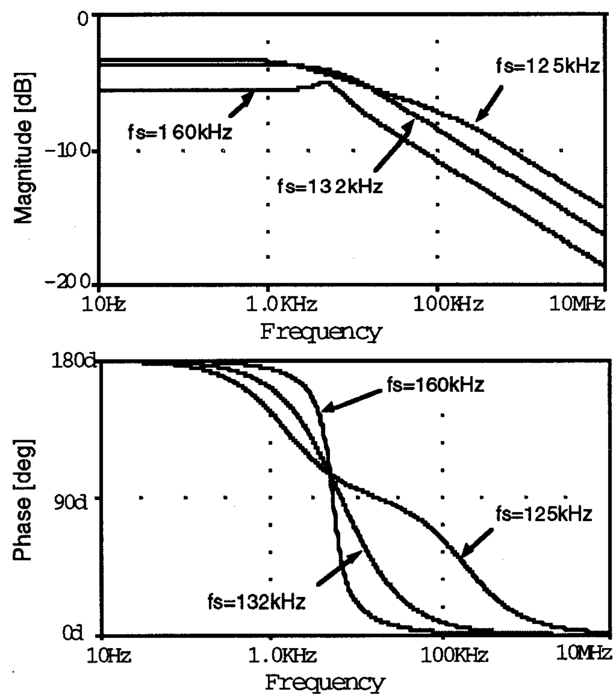

Fig. 7. Small signal, switching frequency to output voltage response for different center frequencies as obtained by the proposed model.

Results of typical simulation runs are given in Figs. 7, 8.

The agreement between the model behavior and the cycle by cycle simulation (Fig. 8) was found to be excellent.

IV. CONCLUSIONS

The behavioral modeling methodology for resonant converters, developed in this study, was found to yield an accurate model that checks well against cycle by cycle simulations. The main advantages of the model are the ease of its derivation and the fact that the basic average and high level model is directly applicable to DC, transient and small signal analysis. The derivation of the model is carried out for large signal, leaving the task of linearization to the simulator. Following the same reasoning, similar models can be developed for other resonant topologies.

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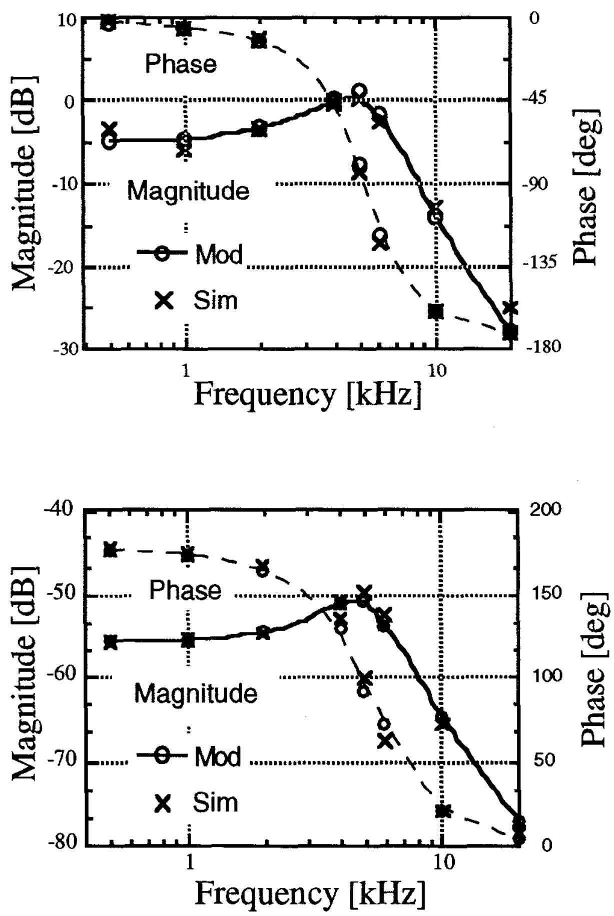

Fig. 8. Comparisons between small signal responses. Upper traces: switching frequency to output voltage response. Lower traces: Line to output voltage response (constant switching frequency: f_s = 160kHz). Model simulation (Mod): circles. Cycle by cycle simulation (Sim): crosses.