A CHAOS MODEL OF SUBHARMONIC OSCILLATIONS IN CURRENT MODE PWM BOOST CONVERTERS

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ABSTRACT

Chaos concepts formulated in a discrete form were applied to examine instability conditions in a current mode PWM Boost converter under open and closed outer-loop conditions. A simple expression for the maximum duty cycle for subharmonic-free operation was developed and applied to asses the effects of the outer loop on subharmonic oscillations in the converter under study.

I. Introduction

Switch Mode system are notorious for their potential to develop instability (by which we mean the onset of parasitic oscillations). A coarse examination of the nature of these instabilities suggests that they might have two distinct and possibly unrelated origins. One is associated with 'analog' instability that can be explained in terms of linear feedback theory. The second, a more devious one, is apparently associated with the sampling or discrete nature of switch mode systems. An example of the latter is the onset of sub-harmonic oscillations in current mode (CM) converters. These unstable conditions were recently explained in terms of a Chaos model [1-2] which seems to fit the nature of switch mode systems. Indeed, it has been shown [3] that subharmonic oscillations in CM is a manifestation of a chaotic behavior. This phenomena was originally explained [4] by considering the propagation of a disturbance in a CM controlled system. This fundamental explanation and its extension [5-6] are insufficient, though, to quantize subharmonic phenomena encountered in complex, closed loop systems such as reported in [7].

The objective of this study was to describe and explain by a Chaos model the behavior of a CM Boost converter under open and closed outer loop situations. The study was motivated by the feeling that a quantitative model can help to examine the effect of the outer loop components on the onset of sub-harmonic oscillations and to quantize the nature of the oscillation in terms of harmonic content under open and closed outer loop conditions. Once developed the model can be used to examine other situations in which instability can be expected.

II. CHAOS MODEL OF A CURRENT MODE BOOST CONVERTER

The CM Boost converter considered in this study (Fig. 1) is based on the generic topology as described in [5]. It is assumed that the converter is operating in the continuous conduction mode. The circuit diagram of Fig. 1 serves as a reference for the three cases discussed below:

1. An open outer loop configuration with a constant control voltage (V_C) (solid line part) and no slope compensation ($m_C = 0$).
2. An open outer loop with slope compensation.
3 A closed outer loop with no slope compensation.

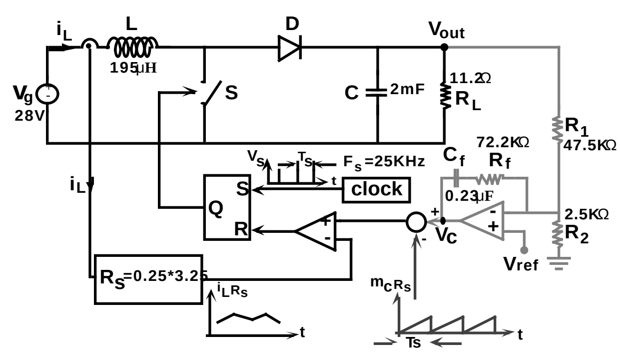

Fig. 1. Circuit diagram of the generic Current Mode controlled boost converter considered in this study.

The basic waveforms related to the current controlled programming (Fig. 2) include a current control signal V_C/R_S (referred to the inductor current), slopes m_1 and m_2

of the inductor current and a compensation slope m_c as normally added to ensure stability over the complete duty cycle range. The turn off instant occurs when the peak inductor current reaches the value of the control current in Fig 2(a) or the combined signal of V_c plus m_c (Fig. 2b) .

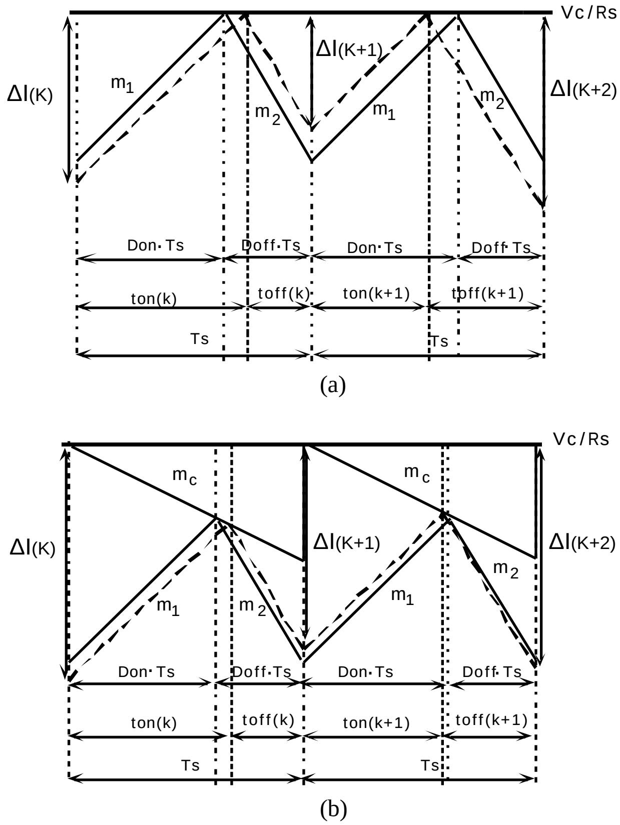

Fig. 2. Propagation of perturbation in inductor current, when V_c is const (opened outer loop). (a) without m_c. (b) with m_c

Examination of the waveform associated with the propagation of a perturbation over two cycles (Fig. 2, dashed-line) reveals that the deviations of inductor current (ΔI) (at the beginning of each cycle) from the control signal V_C/R_S are related to other basic parameters (marked in Fig. 2b) by the following relationships:

$$\Delta I(k) = (m_1 + m_c)\, ton(k) \qquad (1)$$

$$\Delta I(k+1) = m_2\, toff(k) + m_c\, ton(k) \qquad (2)$$

where k is the (discrete) cycle index and ton(k) and toff(k) represent the 'on' and 'off' time in a perturbed cycle k.
The discrete time difference equation of the system is thus:

$$\Delta I(k+1) = -\frac{m_2 - m_c}{m_1 + m_c}\Delta I(k) + m_2\, Ts \qquad (3)$$

For stability we require:

$$\left|\frac{d\Delta I(k+1)}{d\Delta I(k)}\right| < 1 \qquad (4)$$

And hence the stability criterion for the case under study can be expressed as:

$$\frac{m_2 - m_c}{m_1 + m_c} < 1 \qquad (5)$$

In the absence of slope compensation, $m_c = 0$ and under steady-state conditions (Fig. 2(a) solid line):

$$\Delta I = m_1\, Don\, Ts = m_2\, Doff\, Ts \qquad (6)$$

where all parameters refer to their steady-state values.
In this case (no slope compensation, open outer loop), (5) compresses to:

$$\frac{m_2}{m_1} = \frac{Don}{Doff} < 1 \qquad (7)$$

which implies that stability is assured for Don < 0.5, as is well known.

For nonzero m_c, we can apply (5) to determine the minimum value of m_c required to ensure stability:

$$m_c > m_1\left(\frac{1}{2Doff} - 1\right) \qquad (8a)$$

or

$$m_c > m_2\left(1 - \frac{1}{2Don}\right) \qquad (8b)$$

The relationship (3) can be used to develop the discrete map of $\Delta I(k+1) = f(\Delta I(k))$. This was accomplished by a MATLAB (MathWorks Inc.) subroutine that was run for a hundred cycles . As evident from Fig. 3(a), with $m_c = 0$ and Don = 0.6 the Boost converter Fig. 1 under open outer loop is unstable. In contrast, the single point of Fig. 3(b) implies stability for the same converter with a slope compensation .

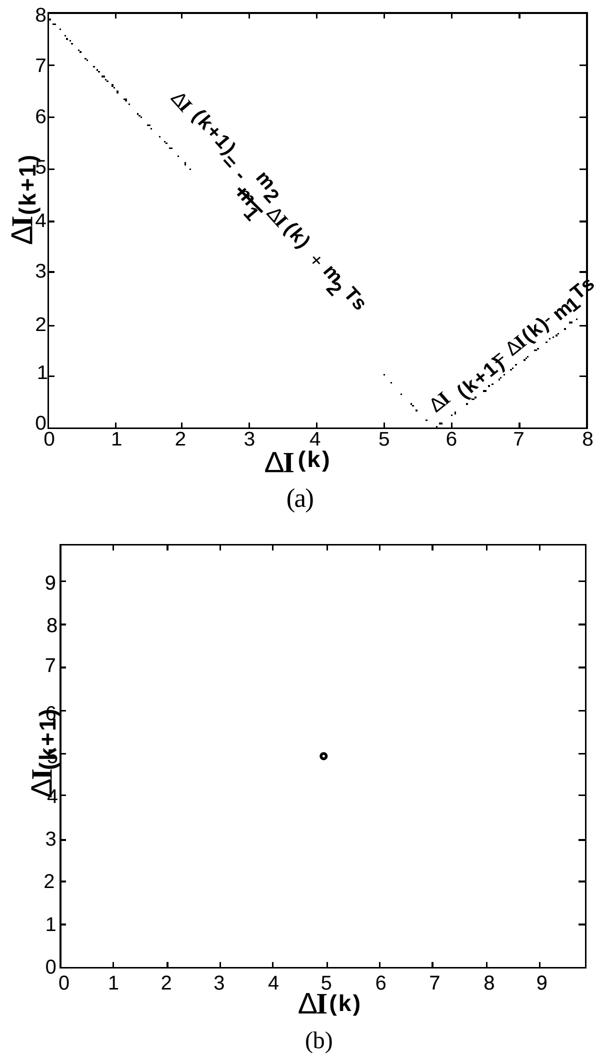

Fig. 3. A Discrete map of Δ I **Error!**=Constant) conditions with (a) no slope compensation (m_c =0) (b) with slope compensation (m_c =0.54 m_1). Produced by MATLAB (MathWorks Inc.) for Don=0.6. Plots represent a sequence of one hundred cycles from k=900 to k=1000.

Another important instrument for examining and explaining the stability properties of a chaotic system is the bifurcation diagram [3]. Fig. 4(a) illustrates the creation of sub-harmonic oscillation (Don > 0.5) as function of duty cycle in open loop converter for a zero m_c . For nonzero m_c (Fig. 4(b)) the borderline between the stable and chaotic region moves to a higher duty cycle according of (8).

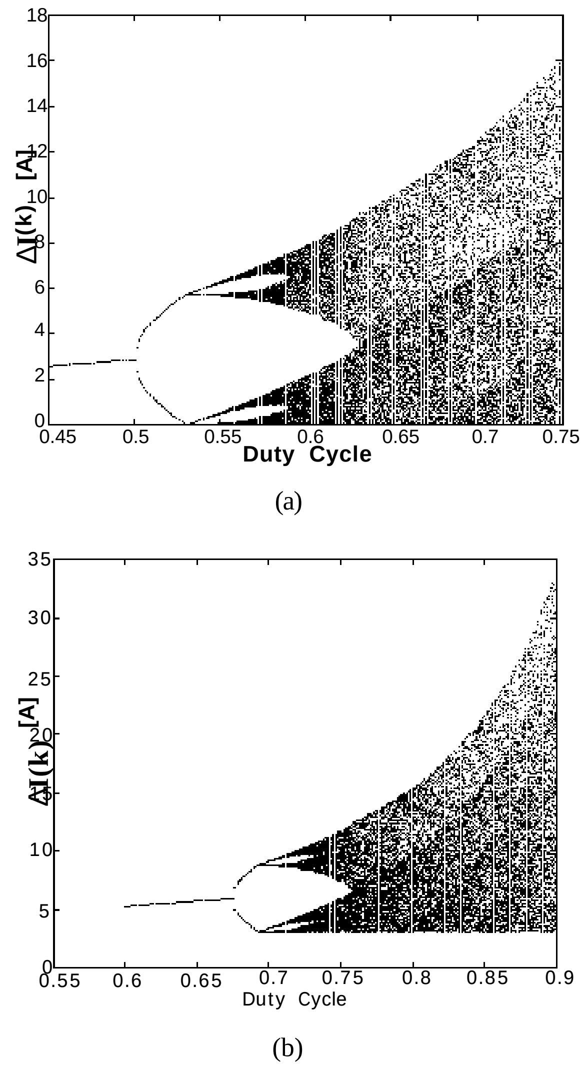

Fig. 4. Bifurcation diagram produced by sweeping the duty cycle parameter (a) over the range 0.45 to 0.75 and m_c =0. (b) over the range 0.6 to 0.9 and m_c = 0.54 m_1 .

III. THE EFFECT OF THE OUTER VOLTAGE FEEDBACK LOOP

The inductor current waveform under closed outer voltage loop conditions is shown in Fig. 5 (refer to Fig. 1 for notations). The solid lines represent the steady-state condition whereas the dashed line shows a perturbed waveforms of the inductor current .

In Fig. 5 the reference current is denoted Iref corresponding to V_c /R_s in the open loop case (Fig. 2(a)). The slopes m_{c_1} and m_{c_2} are an approximation of the instantaneous rising and falling portions of the control voltage (V_c), scaled by the current feedback network (R_s).

Unlikely the case of the open outer loop situation, the voltage control (V_c) in the closed outer loop system is affected by the output voltage ripple and therefore is not a constant even at steady state. Furthermore, under sub-harmonic oscillation conditions V_c could be highly variable.

The interception point $i_{top}(k)$ (Fig. 5) of the slopes m_1 and m_{c_1} can be obtain from simple geometrical relationships:

$$\Delta I(k) = (m_1 - m_{c_1})\, Don\, Ts =$$
$$=(m_2 - m_{c_2})\, Doff\, Ts = \Delta I(k+1) \qquad (9)$$

Consequently,

$$\frac{m_2 - m_{c_2}}{m_1 - m_{c_1}} = \frac{Don}{Doff} = \frac{m_2}{m_1} \qquad (10)$$

and

$$\Delta I_L = \Delta I + \Delta I_{mc} \qquad (11)$$

Under steady state conditions, ΔI_L and ΔI_{mc} are the ripple of inductor current and control voltage(V_c) respectively, scaled by R_s.

Under perturbed conditions (dashed-line):

$$\Delta I(k)=(m_1 - m_{c_1})\, ton(k) \qquad (12)$$
$$\Delta I(k+1) = (m_2 - m_{c_2}(k))\, toff(k) \qquad (13)$$

Which can be transformed into the difference equation:

$$\Delta I(k+1)=-\frac{m_2 - m_{c_2}(k)}{m_1 - m_{c_1}}\Delta I(k)+(m_2 - m_{c_2}(k))Ts \qquad (14)$$

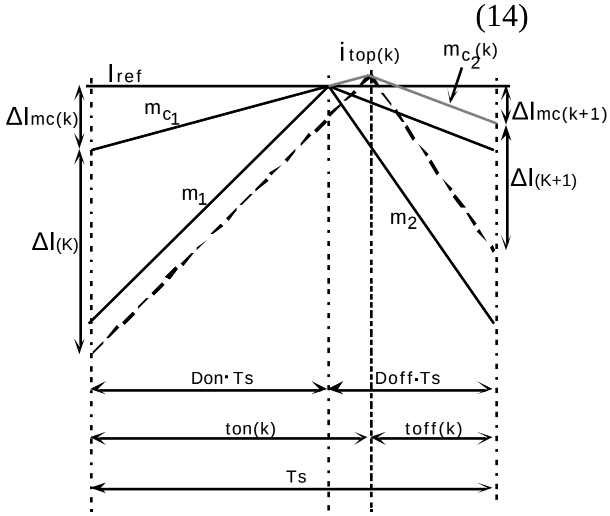

Fig. 5. Propagation of a perturbation in inductor currentwhen the outer loop is closed and m_c =0.

A first order approximation of m_{c_1} and $m_{c_2}(k)$ was obtained by deriving the analytical expression for Vout (Fig. 1) and applying Taylor series expansion. It was found that the scaled (by R_s) slopes can be expressed as:

$$m_{c_1} \quad \frac{V_{out} R_f}{R_1 R_s \tau} \qquad (15)$$

$$m_{c_2}(k) \quad \frac{V_g R_f}{R_1 R_s}\left\{\frac{\omega_d^2 toff(k)}{2} + \frac{toff(k)}{(2\tau)^2} - \frac{\omega_d^2 toff(k)}{4\tau}\right\} + \frac{V_g R_f}{Doff R_1 R_s}\left\{-\frac{1}{\tau} - \frac{\omega_d^2 toff(k)}{2} + \frac{toff(k)}{(2\tau)^2} + \frac{\omega_d^2 toff(k)}{4\tau}\right\} + \frac{itop(k) R_f}{C R_1 R_s}\left\{1 - \frac{toff(k)}{2\tau}\right\} \qquad (16)$$

where

$$\tau = R_L\, C \qquad (17)$$

$$\omega_d = \sqrt{\frac{1}{LC} - \frac{1}{(2\tau)^2}} \qquad (18)$$

$$itop(k)=Iref - (\Delta I(k) + \Delta I_{mc}(k)) + m_1\, ton(k) \qquad (19)$$

$$Iref=\frac{V_g}{Doff^2\, R_L} + \frac{V_g\, Don\, Ts}{2\, L} \qquad (20)$$

Applying the above, the borderline Doff between the stable and unstable regions was derived to be

$$Doff^4\left\{\frac{\omega_d^2 Ts}{2\tau} - \frac{Ts}{4LC\tau}\right\} + Doff^3\left\{-\frac{\omega_d^2}{2} - \frac{1}{(2\tau)^2} - \frac{\omega_d^2 Ts}{2\tau} - \frac{R_f Ts}{2R_1R_sC\tau^2} + \frac{Ts}{4LC\tau}\right\} + Doff^2\left\{-\frac{1}{(2\tau)^2} + \frac{\omega_d^2}{2} + \frac{R_f}{R_1R_sC\tau} - \frac{2R_sR_1}{LTsR_f}\right\} + Doff\left\{\frac{1}{2\tau^2} + \frac{2}{\tau Ts} + \frac{R_sR_1}{LTsR_f}\right\} - \frac{1}{\tau Ts} = 0 \qquad (21)$$

The importance of this expression is its ability to predict the minimum Doff for sub-harmonic-free operation for the Boost converter. It should be noted that the polynomial equation is only a function of the converter's components' values and switching frequency. For example, for the nominal values of Fig. 1 [5] , the limit duty cycle before sub-harmonic develops is Doff=0.546 or Don=0.454. Changing C to 1mF will move the limit point significantly to Doff=0.686 or Don=0.314.

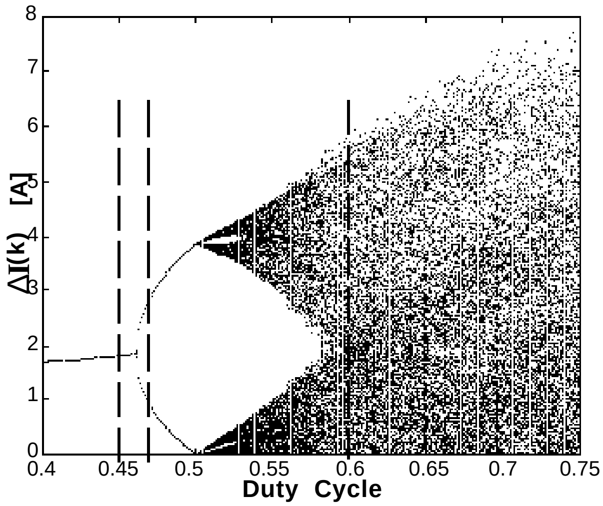

Fig. 11. Bifurcation diagram for the Boost converter of Fig. 1 under closed outer loop conditions. Fs=25KHz.

IV. DISCUSSION AND CONCLUSIONS

The results of this study clearly show that sub-harmonic oscillation in CM converters can readily be explained by the Chaos model developed in this investigation. The model was verified against exact circuit simulation and was found to predict faithfully the behavior of a CM Boost converter under various operating conditions. The Don of 0.5, which is often quoted as the borderline for sub-harmonic-free zone is correct for open outer loop conditions. When the outer loop is closed, the borderline might move significantly to rather low Don values. This implies that when slope compensation is not used a Don (max) of 0.5 is no guarantee for stability. When compensation slope is applied, stability is assured only if the slope is adjusted according to the criterion which takes into account the effect of the outer loop.

An examination of the expressions developed in this study reveals that the stability boundary is effected by the major power components (main inductor, output capacitor and load) and the voltage feedback network. The main conclusions are summarized as follows:

1. A decrease in the values of the switching frequency (F_s), output capacitor (C) and/or load resistor (R_L), will lower the Don limit for subharmonic-free operation in a current mode Boost converter.
2. An increase in the values of input inductor (L_{in}) and/or the high frequency gain of the outer loop (R_f/R_1), will lower the Don limit for subharmonic-free operation in a current mode Boost converter.
3. A preliminary analysis shows that the behavior of a current mode flyback converter is rather similar to that of a Boost converter.
4. A cursory exploration suggests that a current mode Buck converter is less sensitive to the to the outer loop as far as subharmonic oscillations ar concerned. However, under some operating conditions the limit Don is appreciably lower than 0.5.

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