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H^∞ control of boost converters

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Abstract **- The tasks of the controller in a PWM power converter are to stabilize the system and to guarantee an almost constant output voltage in spite of the perturbations in the input voltage and in the output load. We propose to design the controller using H^∞ control theory, via the solution of two algebraic Riccati equations. The almost optimal H^∞ controller is of the same order as the converter and has a relatively low DC gain (as opposed to the classical approach of maximizing the gain at low frequencies). In a case study, for a typical low power boost converter, we obtain significantly improved frequency responses of the closed loop system, as compared to classical voltage mode, current mode or feedforward control.**

Pulse width modulation (PWM) DC-DC converters are nonlinear and time variant systems. After linearization of the average model (see, e.g. Sum [5]) we obtain a linear time invariant model that describes approximately the behavior of the system for frequencies up to half of the switching frequency. Boost and buck-boost converters (in continuous conduction mode) have a zero in the right half-plane (RHP) in their transfer function from the control input (the duty cycle) to the output voltage.

The tasks of the controller are to stabilize the system and to minimize the fluctuations of the output voltage, in spite of the perturbations in the input voltage and the output load. The classical design method, based on Bode and Nyquist plots, is to impose a high loop gain in a frequency band as wide as possible, without distroying stability, see for example the Unitrode manual [4] or Sum [5]. The existence of the RHP zero limits the achievable loop bandwidth and thus the frequency response of the controlled system.

We show that the objectives of the controller match very well with the standard problem dealt with in H^∞ control theory (see, e.g., Francis [3] or Doyle *et al* [2]). The suboptimal solution of the standard problem can be found via the description of the system in state space and the solution of two algebraic Riccati equations, for which reliable programs are available in MATLAB (in the Robust Control Toolbox). The nearly optimal H^∞ controller is of the same order as the converter and has a relatively low DC gain (in contrast with the classical idea of using high gain at low frequencies), and so it is easy to implement.

As a design example we took the typical low power boost converter shown in Fig. 1, which transforms a nominal 12V input voltage into 24V at its output, with a nominal output current of 0.5A. The switching frequency was 240KHz. The role of the current source in the output is to simulate changes in the load (changes in the output current). The series resistance of the inductor, of the switch, of the diode and of the capacitor are all shown.

The controller imposes the duty cycle D, which is the ratio between the time T_{on} when the switch is closed and the period T. After linearization of the average model (see Sum [5]), we can translate the problem of designing a controller to the standard problem of H^∞ control, as illustrated in Fig. 2.

In this diagram, the plant P is the converter, C is the controller to be designed and W is a weight function. The vector w contains the perturbations, the output z is the weighted error signal, the vector y contains the measurement outputs of the plant and the control input u is the duty cycle.

In H^∞ control the aim is to find a controller C wich minimizes the H^∞ norm of the transfer function T_{zw} from w to z. The H^∞ norm is the maximal gain, so that in fact

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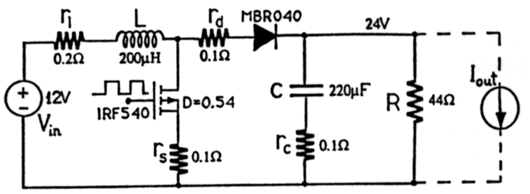

Fig. 1. The boost converter in the design example

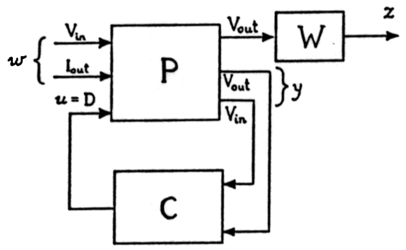

Fig. 2. The control of the converter as a standard H^∞ control problem

we are minimizing z for the worst possible perturbation w. The weight function expresses the relative importance of different frequencies (it is bigger for frequencies whose presence is more disturbing). By adjusting the weight function we can obtain, for example, better performance at lower frequencies at the expense of worse attenuation at higher frequencies, and the other way round. Note that the component of T_{zw} are: the input to output voltage disturbance attenuation and the output impedance, both multiplied by W. Thus our design objective includes minimization of the weighted otput impedance.

We have written a program based on the Robust Control Toolbox of MATLAB, which computes the optimal H^∞ controller for any boost converter. The program is based on the state space discription of the plant and uses the MATLAB function hinf. The averaged and linearized plant is described by the equations:

$$\dot{x} = Ax + B_1 w + B_2 u$$
$$V_{out} = C_1 x + D_{11} w + D_{12} u$$
$$y = C_2 x + D_{21} w + D_{22} u$$

where in our design example:

$$A = \begin{bmatrix} -4208 & -2283 \\ 2086 & -103.1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 4975 & 228.3 \\ 0 & -4535 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 119540 \\ -5370 \end{bmatrix}$$

$$C_1 = [0.046 \quad 1 \qquad C_2 = \begin{bmatrix} \quad \end{bmatrix}$$

$$D_{11} = [0 \quad 0.1], \quad D_{12} = [-0.118]$$

$$D_{21} = \begin{bmatrix} 0 & 0.1 \\ 1 & 0 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} -0.118 \\ 0 \end{bmatrix}$$

In particular, the transfer function from D to V_{out} is:

$$P_{12}(s) = \frac{-0.118(s + 45460)(s - 42240)}{s^2 + 4311s + 5.2 \cdot 10^6}.$$

Note the RHP zero in $P_{12}(s)$. For our design example, we chose the weight function

$$W(s) = \frac{s + 2\pi \cdot 3500}{s + 2\pi \cdot 500}.$$

After incorporating a realization of the weight function into the extended plant (which is of order three). we obtained the almost optimal H^∞ controller $C_0(s)$ as:

$$\begin{bmatrix} \frac{-2.43 \cdot 10^7 (s - 1.02 \cdot 10^8)(s + 12140)(s + 4120)}{(s - 4.47 \cdot 10^{14})(s + 3140)(s + 45460)} & \frac{1.87 \cdot 10^{13}}{s - 4.47 \cdot 10^{14}} \end{bmatrix}$$

This raw controller $C_0(s)$ has an unstable pole. actually an extremly big positive pole which can not be realized by any means. (The closed loop system would be stable even if we could realize $C_0(s)$ and would use it). To overcome this problem. the unstable pole was approximated by a constant:

$$\frac{K}{s - a} \sim \frac{-K}{a} \qquad \text{for } a >> 1.$$

Further investigation has shown that this approximation does not affect the performance of the controller over the frequency range of interest (up to about 100KHz). The approximate H^∞ controller is:

$$C(s) = \begin{bmatrix} \frac{-5.56(s + 4120)(s + 12140)}{(s + 3140)(s + 45460)} & -0.0417 \end{bmatrix}$$

This controller improves the frequency response of the closed loop system, as compared to classical voltage mode control, current mode control or feedforward control. We

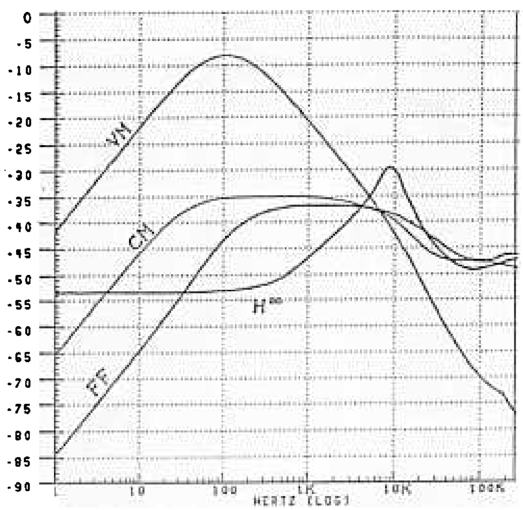

Fig. 3. The input to output voltage disturbance attenuation, in dB

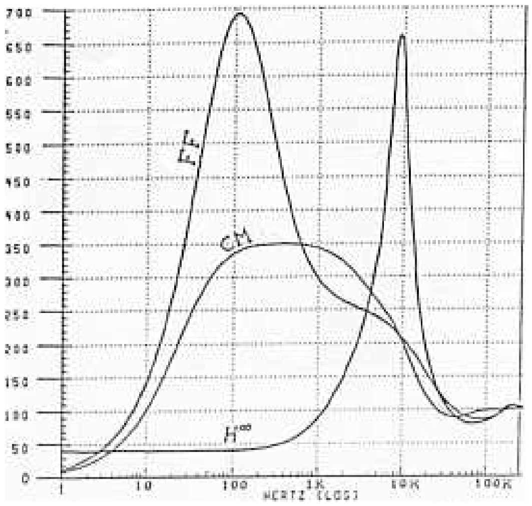

Fig. 4. The output impedance, in mΩ

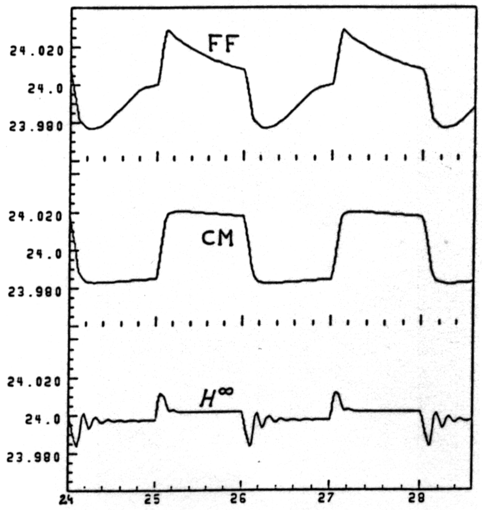

Fig. 5. The output voltage with rectangular perturbation in the input voltage, ±1V

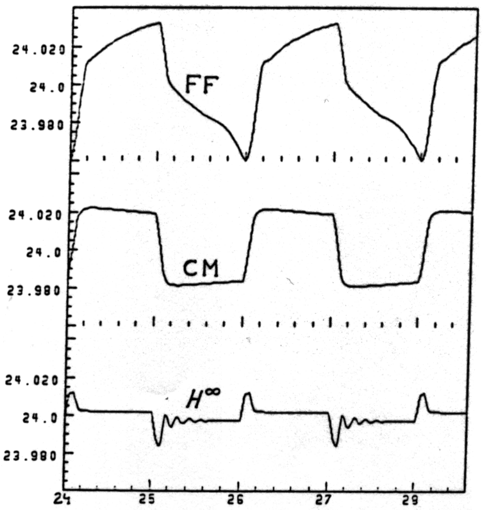

Fig. 6. The output voltage with rectangular perturbation in the load, ±50mA

have followed the method in [4] to design these controllers, and have obtained the following:

Voltage mode controller:

$$C_{VM}(s) = \frac{-3(s+730)^2}{s(s+45460)}.$$

Current mode, the outer controller:

$$C_{CM}(s) = \frac{10^5(s+210)}{s(s+45460)}.$$

Feedforward controller:

$$C_{FF}(s) = \left[\frac{-3(s+730)^2}{s(s+45460)} \quad -0.046 \right].$$

HSPICE simulation in time domain and in frequency domain (AC analysis, based on the switched inductor model of [1]) has been performed to confirm the theoretical results. In Fig. 3 and 4, the voltage disturbance attenuation and the output impedance of the closed loop systems is shown, for the four different controllers mentioned above, according to the simulation. The plots are marked VM (voltage mode control), CM (current mode control), FF (feedforward control) and H^∞ (H^∞ control). We see that at very low frequencies, the H^∞ controller is not as good as the others, but it maintains practically constant performance levels up to about 1KHz, and it outperforms the other controllers between 10Hz and 3KHz, especially with regard to output impedance.

Fig. 5 shows the steady state output voltage of the closed loop system, with the different controllers, when a square wave input voltage disturbance of ±1V is added to the nominal input voltage of 12V, at 500Hz. The curves are marked CM, FF and H^∞, as before. The voltage mode control is not shown, because it performs much worse than the others. The time runs from 14ms to 28.6ms, measured from the moment when the circuit was turned on.

Fig. 6 is similar, but now the disturbance is added to the output current, and it is ±0.05A (over the nominal 0.5A). Again the voltage mode control is not shown, this time the reason being that its curve overlaps with that of the feedforward control.

We have implemented the H^∞ controller and the experimental results are close to those predicted by theory or simulation.

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