Adiabatic Charging of Capacitors by Switched Capacitor Converters with Multiple Target Voltages

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***Abstract*—A method for adiabatic charging of a capacitor by Switched Capacitor Converters (SCC) with multiple target voltages is presented, analyzed and verified experimentally. The SCC automatically changes the target voltages from low to high and back, thereby emulating voltage steps. This reduces the energy losses due to small voltage deviations between the SCC output and the charged capacitor and by enabling the stored energy to be recovered back into the power supply. In comparison with the previous method a considerable economy in hardware is achieved by replacing a set of voltage sources by a single SCC. It is shown that the voltage source and the switch resistance in the regular charging circuit can be emulated by the SCC equivalent circuit. Furthermore, such emulation provides the same step response if the SCC is operated close to the no-charge (NC) mode. This theoretical prediction was approved experimentally using the binary SCC.**

I. Introduction

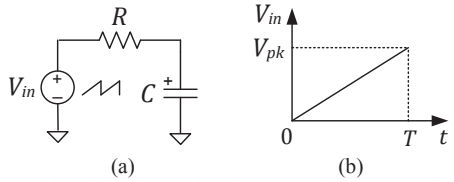
Some applications, such as LED laser drivers, call for repetitive capacitor charging, while others call for repetitive charge and discharge cycles. It is well known that if a capacitor, C, with zero initial voltage is charged fully by a constant voltage, ΔV, through a resistor, R, half of the delivered energy is lost regardless of how low R is:

$$E_R = \frac{C\Delta V^2}{2} \tag{1}$$

This loss can be reduced if a capacitor is charged by a ramp voltage source [1-3] as depicted in Fig. 1, where $V_{in}(t) = V_{pk}(t/T)$, and V_{pk} is the voltage across C at the moment T. The energy loss in linear charging is [3]:

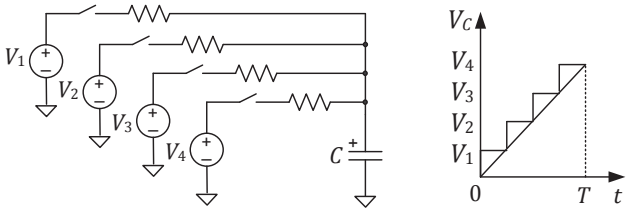
$$E_R(\beta) = \frac{CV_{pk}^2}{\beta}\left(1 - \frac{1 - e^{-\beta}}{2\beta}\right) \tag{2}$$

where $\beta = T/RC$. The asymptotic limit of (2) in the case of $\beta \to 0$ ($T \ll RC$) is exactly the energy loss given by (1).



(a) (b)

Fig 1: Circuit for linear charging (a) and the input voltage (b).



(a) (b)

Fig. 2: Stepwise charging of a capacitor by multiple voltage sources (a) and the resulting stepped voltage (b).

For $\beta \to \infty$ ($T \gg RC$) the asymptotic limit of (2) is

$$\lim_{\beta \to \infty} E_R(\beta) = \frac{CV_{pk}^2}{\beta} \tag{3}$$

This is exactly the energy loss in the case of charging by a constant current source [3]. The main disadvantage of the above method is that the realization of a ramp voltage or current source in practice is lossy in itself. An alternative approach is to approximate a ramp voltage by a set of constant voltage sources [4], [5] as shown in Fig. 2. The energy loss in the stepwise charging depends on the number N of voltage steps and their heights. If all the voltage steps are equal to $V_i = \Delta V/N$ then [4], [5]:

$$E_N = \frac{C\Delta V^2}{2N} \tag{4}$$

The disadvantage of this approach is the large number of voltage sources that need to be generated. The objective of this paper is to investigate the option of replacing the set of constant voltage sources by a single switched capacitor converter with multiple target voltages.

II. Basics of Switched Capacitor Converters

Switched Capacitor Converters (SCC), also referred to as charge pumps, are used as stand-alone power converters for low-power applications and are often embedded in VLSI circuits. It is well known that the SCC exhibits high efficiency only when its output voltage, V_o, is very close to the target voltage, $V_{TRG}=M \cdot V_{in}$, where M is the no-load conversion ratio. The SCC efficiency can be approximated by $\eta=V_o/V_{TRG}$ and decreases when the SCC is loaded. This efficiency drop is due to the inherent power losses, which can be modeled by the equivalent circuit shown in Fig. 3.

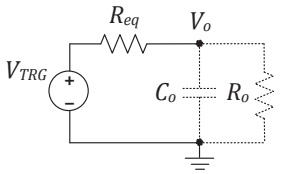


Fig. 3: The equivalent circuit of SCC.

This circuit includes the target voltage source, V_{TRG}, and an equivalent resistor, R_{eq}, which represents the power losses in the switch resistances and the capacitors' ESR [6-8]. As follows from Fig. 3, the highest efficiency will be obtained if R_{eq} is kept as small as possible. Thus, to provide different output voltages one needs to adjust V_{TRG} by changing M. This, however, is a difficult problem since M depends on the SCC topologies and can take only discrete values. An effective way to realize multiple values of M is to use a binary SCC [8], [9], where for n flying capacitors we have 2^n-1 different values of M spaced with a small gap of $1/2^n$, as shown in Fig. 4 for n=3.

The topologies of the binary SCC are synthesized by the extended binary (EXB) representation [8], [9], which, in the case of M_3=3/8, yields a set of the coefficients $a_{i,j}$ that are given in Table I. These coefficients represent the connection polarity of the flying capacitors C_j (-1 implies that C_j is charged, 1 implies that it is discharged, and 0 implies that C_j is not connected). The values of $a_{i,0}$ are restricted to be 1 or 0 depending on whether V_{in} is connected or not, while $a_{i,4}$ is equal to -1 for all i, since C_o is always charged.

TABLE I: Set of the coefficients $a_{i,j}$ for M_3=3/8.

j / i	$a_{i,0}$	$a_{i,1}$	$a_{i,2}$	$a_{i,3}$	$a_{i,4}$
$a_{1,j}$	1	-1	-1	1	-1
$a_{2,j}$	0	0	1	1	-1
$a_{3,j}$	0	1	0	-1	-1
$a_{4,j}$	1	-1	0	-1	-1

In normal operation, the switches commutate the flying capacitors C_j according to the above rules, which leads to the topologies shown in Fig. 5. Once topology i is configured, a total capacitor $C_i=1/\sum(1/C_j)$ starts to charge (or discharge) through a total resistor R_i, which represents all the parasitic resistances in the actual topology, namely $R_{ds(on)}$ of the switches and the ESR of the capacitors. The time allotted for each topology is denoted by t_i, and the corresponding time constant by $\tau_i = R_iC_i$. Depending on the ratio t_i/τ_i, the SCC can operate in three different modes. The charging of C_i is almost completed if $t_i \geq 5\tau_i$ and this case is denoted by CC. In the case of $t_i \approx \tau_i$ the charging is partial (PC) and if $t_i \ll \tau_i$ there is no effective charging (NC).

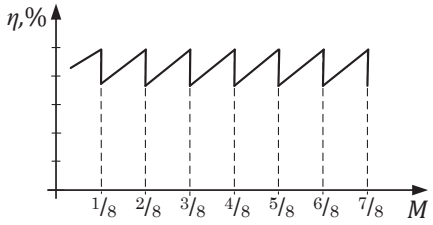


Fig. 4: Theoretical efficiency of the binary SCC.

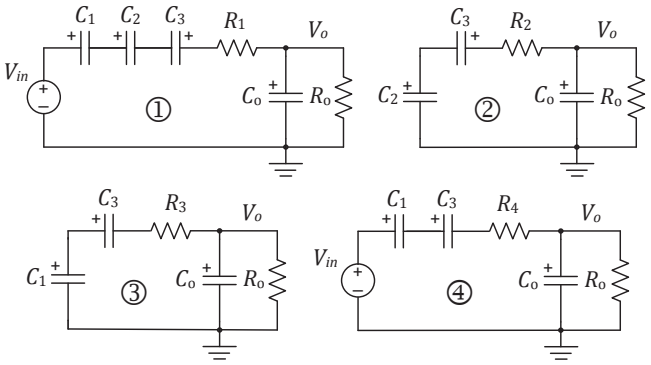


Fig. 5: Topologies of the binary SCC with the conversion ratio M_3=3/8.

III. Proposed Approach

In the proposed approach, the set of constant voltage sources shown in Fig. 2 is replaced by the binary SCC. To emulate the voltage steps, the SCC automatically changes the conversion ratios from low (1/8) to high (7/8) and back as shown in Fig. 6. This introduces a ramp of small steps and allows the stored energy to be returned to the power supply when the capacitor is ramped back.

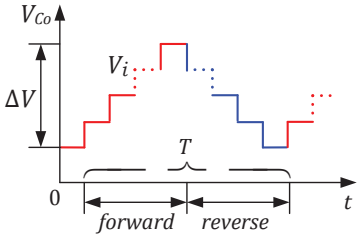


Fig. 6: Voltage across C_o in the proposed approach.

The average (DC) current I_{av} flowing in and out of C_o in different charging circuits is shown in Fig. 7. In comparison with the regular charging, where the spike of momentary current is high (Fig. 7a), the stepwise charging offers a sequence of lower current spikes (Fig. 7b), while in the charging by current source (Fig. 7c) the momentary current is simply constant. Considering I_{av} as a common reference, one can conclude that the RMS of i_{Co} is minimal in the case of Fig. 7(c) and therefore, the energy loss in this case will be minimal. Comparing to the worst case of Fig. 7(a), the sequence of lower spikes in Fig. 7(b) has lower RMS, which decreases with increasing the number of voltage steps.

Since C_o should be charged (discharged) fully, there is a need to define the minimal duration of each voltage step V_i. Furthermore, it is highly advantageous to know the law according to which the voltage across C_o is changing. These issues are solved in the proposed approach by considering the SCC equivalent circuit.

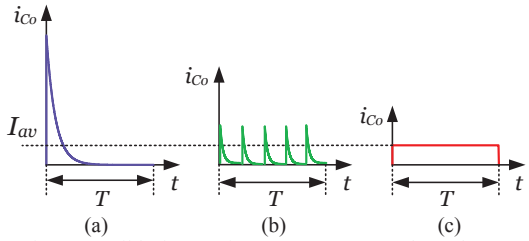


Fig. 7: Possible forms of momentary current through C_o.

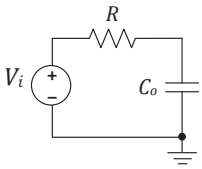


Fig. 8: Elementary charging circuit.

In the elementary charging circuit depicted in Fig. 8, the voltage source V_i=V_{TRG} and serial resistor R_i=R_{eq}. Under the condition that the SCC switching frequency, $f \gg 1/T_i$, the minimal step duration needs to be $T_i \geq 5R_{eq}C_o$.

As was shown in [6-8] the equivalent resistor in the class of SCC where the flying capacitors are always connected in series is given as:

$$R_{eq} = \frac{1}{2f}\sum_{i=1}^{n+1}\frac{k_i^2}{C_i}\coth\left(\frac{\beta_i}{2}\right) \tag{5}$$

where i is the topology number, k_i=I_i/I_o is the ratio of the average topology current, I_i , to the average output current I_o and β_i=t_i/τ_i. Under the condition that the time slots of all the topologies are equal, t_i=t, the coefficients k_i can be found as a solution of a system of linear equations [6-8].

For the binary SCC, analytic expressions for R_{eq} were derived in [8] and are given in Table II. In these expressions β=$1/[4(n+1)fRC]$, where R is a $R_{ds(on)}$ of a single switch and C is the capacitance of a single capacitor, C_j. The asymptotic limit of R_{eq} for $\beta\to0$ was found using f=$1/[4(n+1)\beta RC]$. This limit is called the no-charge (NC) operation mode (also known as FSL) and reached in practice if the SCC operates with a very high switching frequency so that $t_i \ll R_iC_i$. As shown in [6-8], the efficiency of SCC operating in the NC mode is maximal. The graph of R_{eq} as a function of f is built in Fig. 9 for the experimental SCC with M_3=3/8. As is evident from this figure, at the actual switching frequency, f=$50kHz$, the SCC operates very close to the NC mode.

Evaluation of the energy loss in the proposed approach is based on the average current concept introduced above and is realized in the experimental setup shown in Fig. 10. In the reverse operation, C_o is discharged to a lower target voltage. This implies that the flying capacitors recover the charge stored by C_o and return it to the power supply. Since the same sequence of spikes shown in Fig. 7(b) is reflected to the input and may cause high voltage ripple, a filter capacitor $C_{in} \gg C_o$ was placed before the ampermeter.

Table II: The analytical expressions of R_{eq}

M_n	Equivalent resistor expression	$\lim_{\beta\to0} R_{eq}$	R_{eq}, Ω
1/8, 7/8	$\frac{1}{64fC}\left[8\coth\left(\frac{\beta}{2}\right)+4\coth(\beta)+3\coth\left(\frac{3\beta}{2}\right)\right]$	$\frac{11}{2}R$	6.67
2/8, 6/8	$\frac{1}{8fC}\left[\coth\left(\frac{\beta}{2}\right)+\coth(\beta)\right]$	$\frac{9}{2}R$	5.44
3/8, 5/8	$\frac{1}{32fC}\left[7\coth(\beta)+3\coth\left(\frac{3\beta}{2}\right)\right]$	$\frac{9}{2}R$	5.51
4/8	$\frac{1}{4fC}\coth\left(\frac{\beta}{2}\right)$	$4R$	4.82

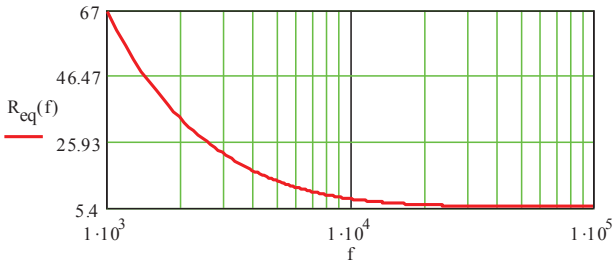


Fig. 9: R_{eq} as a function of f for the experimental SCC with M_3=3/8.

The energy losses in the charging and discharging will be practically equal, such that the total energy loss can be expressed as:

$$E_T = P_{in}T = V_{in}I_{DC}T \tag{6}$$

On the other hand, the average current through C_o during each voltage step is given by:

$$I_{av(i)} = \frac{C_oV_i}{T_i} \tag{7}$$

Since this current flows through the corresponding $R_{eq(i)}$, the power dissipation is:

$$P_i = \left(\frac{C_oV_i}{T_i}\right)^2 R_{eq(i)} \tag{8}$$

Substituting T_i= $5R_{eq(i)}C_o$ into (8), the energy loss over all the forward-reverse voltage steps can be found as:

$$E_A = \sum_{i=1}^{N} P_iT_i = \frac{2N}{5}C_oV_i^2 \tag{9}$$

IV. Experimental Results

The switch network used in the binary SCC is depicted schematically in Fig. 11. It was built around the CMOS bidirectional switches MAX4678 with $R_{ds(on)}$=1.2Ω, which were controlled by a microcontroller dsPIC33FJ12GP202. The flying capacitors C_1=C_2=C_3=$4.7\mu F$, while C_{in}=$4700\mu F$, C_o=$47\mu F$, and V_{in}=8V. The SCC is operated close to the NC mode with the time allotted for each topology being t_i=$5\mu s$. The duration of each voltage step is T_i = $5R_{eq(i)}C_o$ and lies in the range $1.13\div1.57ms$. The average (DC) input current measured in the experimental setup was I_{DC}=$2.5mA$, while the whole forward-reverse period was T=$17ms$. Substituting this value into (7) yields $E_T = 340\mu J$. On the other hand, the energy losses calculated by (4) and (9) are $E_N = 282\mu J$ and $E_A = 112.8\mu J$ respectively. In the worst case, when C_o is charged and discharged by a squarewave with ΔV=6V, the energy loss, given by (1), is equal to $E_R = 1692\mu J$.

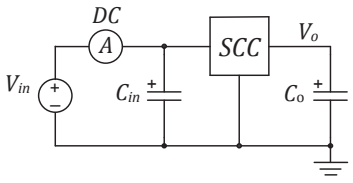


Fig. 10: Proposed experimental setup.

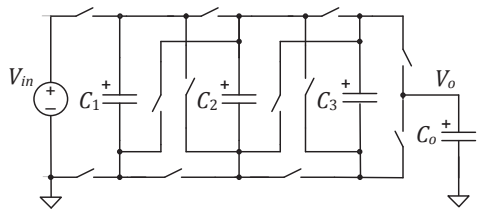


Fig 11: Switch network used in the binary SCC.

Comparing the above values, one can conclude that $E_N < E_T < E_R$. This fact could be explained by that the real SCC has some constant losses, which have a larger effect at the low average input current of I_{DC}=2.5mA. These constant losses include leakage currents and switching losses, as well as charge injections from the switch channels to ground. Since the SCC equivalent circuit of Fig. 3 takes into account only the conduction losses, the real values of R_{eq}, which include all the above losses, might deviate from the theoretical ones presented in Table II.

Fig. 12 shows the experimental waveforms of the voltage across the output capacitor and the charging/discharging current through it.

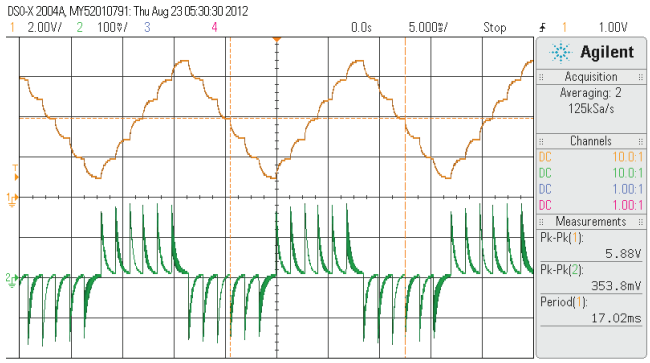


Fig 12: Experimental waveforms.
The upper trace (Ch.1) is the voltage across C_o, vertical scale: 2V/div.
The bottom trace (Ch.2) is the current through C_o, vertical scale: 0.1V/div.
Horizontal scale: 5ms/div.

V. Conclusions

A method for adiabatic charging and discharging of a capacitor by means of a multiple target SCC was analyzed and tested experimentally. The theoretical results predict an appreciable loss reduction as compared to the case of a voltage step charging method. The experimentally measured losses were higher than the theoretical estimate, which is explained by the parasitic effects of the SCC.

The advantage of the proposed approach is that the number of voltage steps is much greater than the number of flying capacitors (6 vs. 3). This is achieved due to the ability of the binary SCC to change the target voltages automatically, thereby emulating a complex multiple voltage source system. The energy loss measured in the experimental setup is considerably greater than the theoretical one for the stepwise charging, but about five times lower than that of squarewave charging.

Acknowledgement

This research was supported by the ISRAEL SCIENCE FOUNDATION (grant No. 517/11) and by the Kamin program of the Ministry of Industry Trade and Labor (grant No. 47189).

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