Modeling and Analysis of Hybrid Converters

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***Abstract*—A generic behavioral average-circuit model that analyzes hybrid converters that include a switched inductor and switched capacitors is developed. The model can be used to calculate or to simulate the average static, dynamic and small signal responses of hybrid converters. The model is valid for all operation modes of the Switched Capacitor Converters operating in CCM and DCM modes of the Switched Inductor Converter and is compatible with circuit simulators that include dependent sources. The model was verified by simulation and experimentally. The experimental converter included a Boost converter followed by x3 switched capacitor converter. Good agreement was found between the behavior of the proposed average model, full circuit simulation and experimental results.**

I. INTRODUCTION

Hybrid converters include a switched inductor PWM converter that is cascaded with a Switched Capacitor Converter (SCC) [1-3]. The hybrid converters exhibit most of the advantages of the basic PWM switched inductor converters, while being able to achieve much higher conversion ratios, both bucking and boosting. The beneficial attributes of the hybrid converter are lower stress on the switches and a high total conversion gain in the CCM mode. If isolation is not required, capacitor multipliers can replace a transformer with a large turns ratio, which exhibits significant leakage inductance and winding capacitance. These parasitic elements cause voltage and current spikes and increase losses and noise [3]. Furthermore capacitor multipliers preclude the need for HV diodes, which normally have poorer reverse recovery characteristics. Hybrid converters are therefore good candidates for applications such as laser drivers, X-ray systems, ion pumps, electrostatic systems and many others [3]. Other applications such as grid-connected renewable energy systems can also benefit from the characteristics of hybrid converters. Renewable energy sources such as a single PV panel have, in general, a low output voltage, which needs to be boosted so that it will be capable of injecting energy into the grid. This can be conveniently done by hybrid converters with multiple capacitors, such as the one shown in Fig. 1, which is a "Boost SCCx3" that has a voltage gain of six when operating at 50% duty cycle. Other examples are the converters proposed in [4, 5].

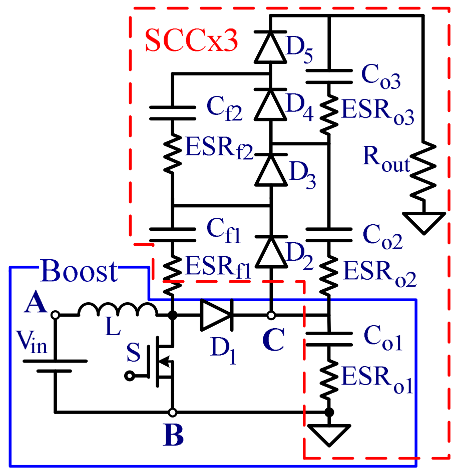

Figure 1. Hybrid Boost SCCx3 converter.

Dynamic and static analyses of hybrid converters have been carried out in a number of earlier studies [1, 3, 6]. Some of the analyses concentrated on a particular converter [1, 3], others concentrated on the large signal response of this family of converters [6]. The objective of this study was to develop a generic average model of hybrid converters based on PWM switched inductor and SCC systems that are compatible with available simulation packages and that will be capable of reproducing the static and dynamic behavior of the converter, including the small signal responses. The proposed average modeling approach applies the concept of equivalent circuit models of switched inductor converters and SCC [7-9]. The latter is based on a loss model described in [7-11]. The major contribution of the present study as compared to previously published work is the fusion of two distinctive equivalent circuit models into a single generic equivalent circuit model, which provides a tool to evaluate hybrid converter structures in a generic way. The resulting generic equivalent circuit model is a linear circuit that, on the one hand, preserves the average behavior of the original non-linear switch-mode converter, while, on the other hand, is

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transparent to the switching action. Being a continuous analog circuit, the model can be solved symbolically using any network analysis method or employing numerical tools, which include mathematical software tools and simulation software packages.

II. BASIC THEORETICAL CONSIDERATIONS

A. *The switched inductor average model basics:*

Switched inductor PWM converters share a common topology-independent module (Fig. 2a). There are several works dealing with the modeling of PWM switched inductor converters [7, 8, 12-16]. The model selected here, is the Generic Switched Inductor Model (GSIM) (Fig. 2b) [8].This model has the capability of modeling the CCM and DCM operation modes in a continuous way [12]. For example, during simulation of the transient response the model will move seamlessly from CCM to DCM and back without the need to interrupt the simulation run and readjust the model. The GSIM includes four dependent sources that are used to interface the module with the rest of the converter. The dependent sources represent the average currents flowing into or out of the module and the dependent voltage source represents the average voltage drop applied to the inductor. According to [7, 8] equation (1) describes these average dependent sources:

$$G_a = I_{Lav}; \quad G_b = \frac{I_{Lav} D_{on}}{D_{on} + D_{off}}; \quad G_c = \frac{I_{Lav} D_{off}}{D_{on} + D_{off}};$$
$$E_L = V_{(a,b)} D_{on} + V_{(a,c)} D_{off} \qquad (1)$$

where I_{Lav} is the average inductor current, $V_{(a,b)}$ and $V_{(a,c)}$ are the average voltages between the points a and b and a and c, respectively, of the GSIM (Fig. 2b), $D_{on}= T_{on}/T_s$ is the duty cycle, T_s is the switching period, $D_{off} = 1-D_{on}$ for the case of CCM and is given by (2) for the case of DCM.

$$D_{off} = \frac{2 I_{Lav} L f_s}{V_{(a,b)} D_{on}} - D_{on} \qquad (2)$$

where L is the inductance of the choke, $f_s = 1 / T_s$ is the switching frequency. Selection of the DCM-CCM mode can be done on the fly by calculating the two D_{off} values and selecting the minimum between the DCM case (2), and the CCM case $(1- D_{on})$ [8].

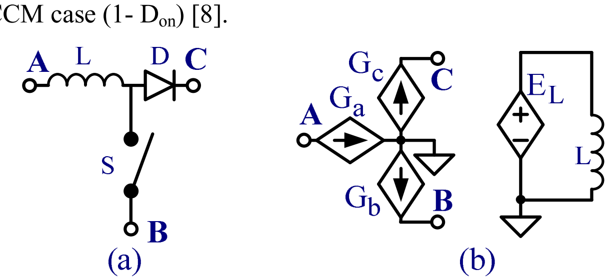

Figure 2. PWM Converters (a) Common block, (b) Generic switched inductor model - GSIM.

B. *SCC average model basics:*

The basic subcircuit of an SCC includes a flying capacitor, C_f, connected to its charging/discharging source, V_s, which could be a voltage source or another capacitor. The generic capacitor's charge/discharge process, which takes place in this SCC subcircuit, can be represented by the equivalent circuit of Fig. 3a, in which ΔV_i is the initial voltage difference between the capacitor and the charging/discharging voltage source of a subcircuit notated "i", C_i is the total capacitance of the subcircuit i and R_i is the total resistance of the loop (any switch resistances and capacitors' ESRs). It is further assumed that the capacitor is periodically charged and discharged at a frequency of f_s. The charge/discharge duration is T_i, and the charge and discharge processes are modeled as two independent subcircuits, which are analyzed separately.

The average power, P_i, dissipated by the subcircuit during a switching duration, T_i, can be expressed as a function of the average current, I_{avi}, in the subcircuit (averaged over the switching period $T_s=1/f_s$) [10]:

$$P_i = \left(I_{av_i}\right)^2 \cdot \left\{ \frac{1}{2 f_s C_i} \cdot \frac{(1+e^{-\beta_i})}{(1-e^{-\beta_i})} \right\}; \quad \beta_i = \frac{T_i}{R_i C_i} \qquad (3)$$

The term in curly brackets can be defined as the equivalent resistance, R_{ei}, of subcircuit i:

$$R_{e_i} = \left\{ \frac{1}{2 f_s C_i} \cdot \frac{(1+e^{-\beta_i})}{(1-e^{-\beta_i})} \right\} = \frac{1}{2 f_s C_i} \cdot \coth\left(\frac{\beta_i}{2}\right) \qquad (4)$$

These results are valid for all operational modes of the SCC i.e. when T_i is larger, about equal to, or smaller than R_iC_i. These three situations are denoted as Complete Charge (CC), Partial Charge (PC) and no Charge (NC), respectively [11]. Based on these formulations, the average behavior of the instantaneous equivalent circuit for duration T_i (Fig. 3a) can be represented by a generic **average** equivalent subcircuit (Fig. 3b), in which all the variables are average values (averaged over the switching period): V_{Cavi} is the value of the capacitor voltage during the time frame T_i, I_{avi} is the average current in the subcircuit, calculated by integrating the charge transferred in the i^{th} subcircuit during

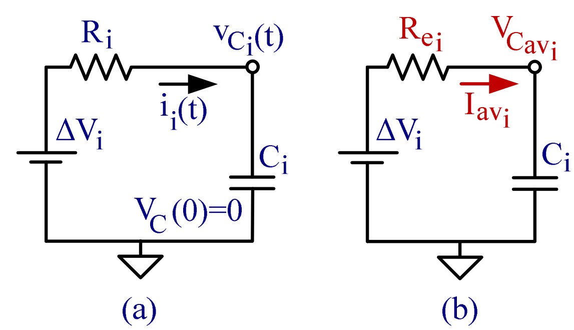

Figure 3. SCC subcircuit (a) Generic instantaneous charge/discharge circuit, (b) Generic average model.

T_i and divided by the full switching period, T_s, R_{ei} is the equivalent resistance of subcircuit i, as summarized in (4) [10, 11]. It should be noted that each flying capacitor will appear in at least two subcircuits, for an SCC with at least two phases (when charging and discharging), or in a number of subcircuits for the multi-phase SCC.

The interconnection of the subcircuits to form the complete model can be carried out directly if the flying capacitor shares a common potential in all subcircuits. In other converters, such as in the case of the Boost SCCx3 circuit discussed here (Fig. 1), the capacitors are floating at different potentials and direct connection is impossible. This is resolved by an isolation element - a "DC transformer" - which is realized by dependent sources E_T and G_T (Fig. 4). The relationships between currents and voltages of the "DC transformer" are given in (5), consistent with the notation of Fig. 4.

$$\begin{bmatrix} V_{T2} \\ I_{T_2} \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix} \begin{bmatrix} V_{T1} \\ -I_{T1} \end{bmatrix} \tag{5}$$

where n is the transformation ratio between the primary and the secondary of the transformer.

III. The Hybrid Modeling Approach

The fundamentals of the proposed modeling approach are exemplified by an average equivalent circuit of a hybrid converter, PWM Boost with SCC x3 multiplier (Fig. 1).

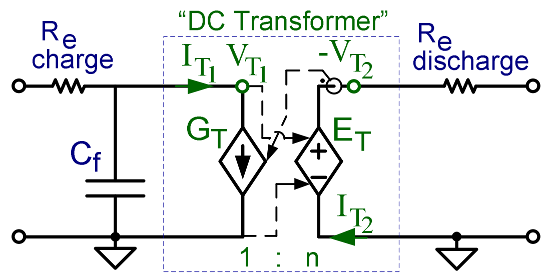

Figure 4. Average dynamic model of flying capacitor, including isolated charge transfer mechanism "DC Transformer".

The multiplication value of three was selected to emphasize the gain difference between the bare Boost converter and the hybrid converter, and to highlight the capabilities of the model to predict dynamic behavior of complicated multi-capacitor systems rather than a simple doubler. The model development is demonstrated by a PSIM simulation package implementation (Powersim Inc. v. 9.0.3), as shown in Fig. 5. It should be noted that the proposed approach is compatible with any circuit simulator that includes dependent sources, such as PSpice, NL5 etc.. The PSIM simulator was selected for demonstrating the proposed modeling approach since it is capable of running simultaneously a full circuit (cycle-by-cycle) simulation along with the small signal (AC) analysis of the average model simulation. This facilitates displaying the two results side by side for comparison. Some aspects of PSpice implementation of SCC and GSIM average models are discussed later on in this section and more details can be found in [7-9].

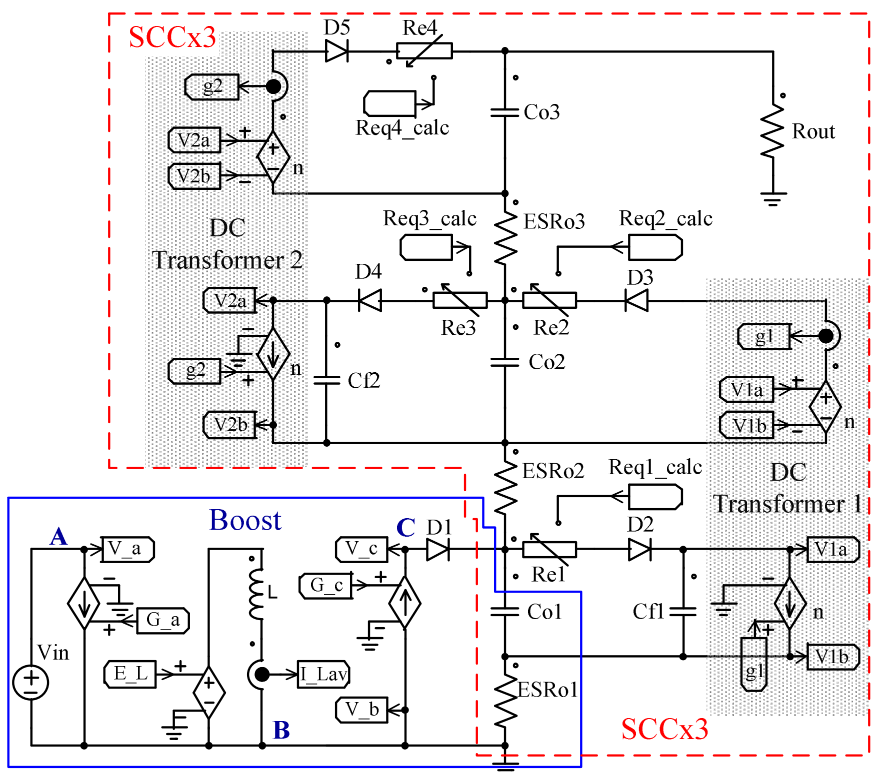

Figure 5. Average dynamic model of Boost SCCx3 system, including isolated charge transfer mechanism "DC Transformer", Implemented in PSIM.

The module encircled in Fig. 5 by a solid line is the average model of the Boost section. It follows the structure of Fig. 2b, as configured for the Boost stage (the shorted current source G_b of Fig. 2b is absent from Fig. 5. For the sake of brevity, choke, switch, and capacitors' parasitic resistances are neglected at this point.

The SCC module (encircled by a dashed line) comprises the three serially connected capacitors (C_{o1}-C_{o3}) and the flying capacitors, C_{f1} and C_{f2}, which are connected to the complimentary subcircuits by two "DC Transformers". The equivalent resistances, which represent the losses of each subcircuit, are implemented by controlled resistors R_{e1}-R_{e4}. Also included in the average model are the diodes of the SCC, which were assumed to be ideal in the present case. The values of the equivalent resistors are calculated on the fly by (4) in which all the parameters are known except T_i, which is a function of the duty cycle. The duty cycle is also the control variable of the dependent sources of the Boost section, as per (1) and (2).

In PSIM, to run a "small signal" simulation a small sinusoidal signal is used as a control signal to excite the system and the response is evaluated as the ratio of the resulting output perturbation to this signal for different excitation frequencies. In the demonstrated system, the excitation includes a DC component to set the operating point (DC duty cycle) plus a sinusoidal source (D_{AC}, D_{DC}) to generate the combined signal V_{inj} (Fig. 6). In the proposed model, the excitation signal (duty cycle perturbations) affects all dependent sources of the Boost module and all the variable resistors in the SCC module. That is, the control signals of these dependent elements are a function of the duty cycle via eqs. (1), (2) and (4). It is thus required that these expressions be evaluated on the fly and that the results are fed to the control port of the dependent elements. In the model presented here, the calculations are performed by mathematical blocks of PSIM, which are represented by blocks 'm' in Fig. 6. All the constants are made accessible to the math blocks by defining them in a "Parameter File" block, while variables that are modified on the run are connected to the math blocks by wires. These include the current duty cycle (V_{inj}), the average current of the inductor (I_{Lav}), and the voltages V_a, V_b, V_c. These are then used to calculate the current control signals of the dependent elements, G_a, G_b, G_c, Req_1_cal, Req_2_cal, Req_3_cal (Fig. 5). By this, the duty cycle excitation will affect all dependent elements in order to emulate the behavior of the physical system. Clearly, the model shows only the average values of the variables and is transparent to the switching duty cycle. Nonetheless, in the PSIM environment, "small signal" analysis is carried out in the time domain and the frequency dependent data is gathered by carrying out the analysis for specified signal frequencies and then displaying the result in a conventional small signal response plot.

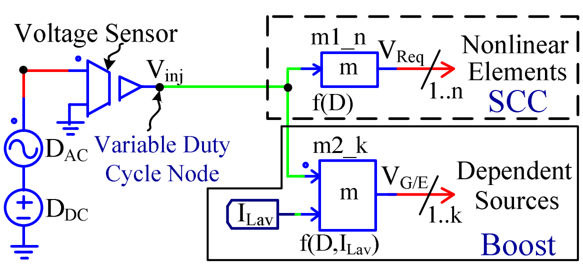

Figure 6. PSIM implementation of the average control model.

The PSIM example given above represents an implementation of the proposed modeling approach in the discrete event simulators environment, which lacks the option of small signal, (AC) analysis. SPICE-based simulators such as OrCAD PSpice that include AC analysis have a built-in linearization procedure that converts a nonlinear system, as the one discussed here, to its linearly approximated circuit around the operating point [7]. The implementations of the proposed average model in SPICE-based simulators will follow the procedure shown above, apart from two points; 1. the excitation will be a (phasor) AC source and 2. the analysis will be AC analysis that includes the linearization preprocessing by the simulator, which is a procedure that is transparent to the user.

The implementation of the proposed model in the OrCAD PSpice environment is given below. The equivalent circuit model of Boost SCCx3 will be the same as in Fig. 5, with some minor changes. First, the equivalent resistances are emulated by dependent resistors. This can be conveniently realized by dependent current sources that are a function of the voltage across them, V_R, and a control variable, I_G, (Fig. 7) such that $I_G = V_R/r$. In this case, the emulated resistance, R, will be equal to r. An example of variable resistance implementation in PSpice is depicted in Fig. 8. In this case, the switching frequency is emulated by a voltage ($V \equiv f_s$, node V_inj Fig. 8) which is the variable in the expression of GVALUE (6). A convenient emulation factor would be 1Hz ≡ 1V.

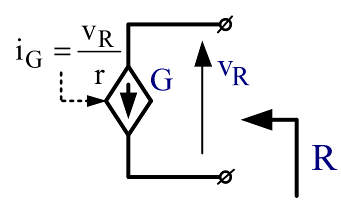

Figure 7. Emulation of a dependent resistor by a dependent current source, R = r.

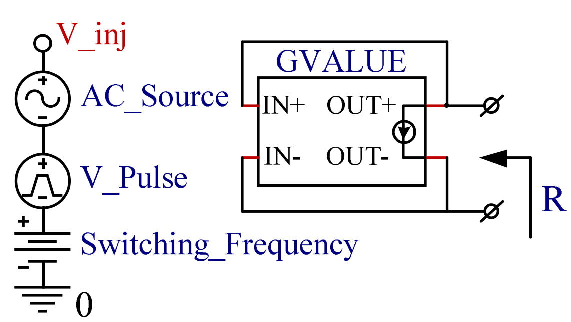

Figure 8. Implementation of a dependent equivalent resistance in PSpice.

The expression (EXPR) of the GVALUE (Fig. 8) will thus be:

$$\mathrm{EXPR} = \frac{\{V(\%IN+) - V(\%IN-)\}}{\left\{ \frac{1}{2\cdot V(V\_inj)\cdot C\_1} \cdot \frac{1+e^{-\frac{1}{2\cdot V(V\_inj)\cdot R\_1\cdot C\_1}}}{1-e^{-\frac{1}{2\cdot V(V\_inj)\cdot R\_1\cdot C\_1}}} \right\}} \quad (6)$$

where R_1, C_1 are the total resistance and capacitance of the subcircuit '1'; V(%IN+)-V(%IN-) is the input voltage to the GVALUE, and V(V_inj) is the voltage of node V_inj (Fig. 8).

The (static) dependence of V_o on the switching frequency f_s can be obtained by running a DC analysis on the circuit, in which the DC voltage source (Switching_Frequency, Fig. 8) is swept over the desired range. This same schematic can also be used to simulate the output voltage response to a step change in switching frequency. The relevant analysis in this case is TRANSIENT analysis, and the excitation source that mimics the frequency change will be V_Pulse (Fig. 8).

Finally, the small signal frequency-to-output voltage response of an SCC can be emulated. The relevant excitation will be the VAC source (AC_Source, Fig. 8), simulation type will be "AC Analysis" and it will be carried out around the operating point set by the DC source (Switching_Frequency, Fig. 8) that emulates the switching frequency at the operating point.

IV. Experimental and Simulation Model Validation

The dynamic average modeling approach of hybrid PWM SCC systems was verified by cycle-by-cycle (full, switched circuit) simulation, and experimental results. Boost SCC x3 converter of Fig. 1 was selected as a representative hybrid PWM SCC system. Simulations were carried out on the PSIM package, and experimental trials on a Boost SCCx3 laboratory prototype. The experimental prototype parameters, which were also used in the simulations, are as follows: input voltage, V_{in}= 9.61V; switching frequency, f_s = 105.6kHz; large signal (operation point) duty cycle, D = 48.2%; small signal (injection) duty cycle, d_{AC} = 0.17%; main inductor, L = 2mH; MOSFETs (R_{ds_on} assumed to be zero in simulation trials), IRF540; all the capacitors (electrolytic), C = 100μF; ESR of all the capacitors, 225mΩ; load resistance, R_{out} = 500Ω; all the diodes (assumed to be ideal for the simulation trials), MUR420.

Steady state output voltage values obtained by simulation and experimentally were as follows: average model simulation 54.3V; full simulation 54.3V; experimental results 50.9V. AC response of the laboratory prototype Boost SCCx3 converter was measured by a network analyzer (Core Technology Group, Model: SA-10) and the results are presented in Fig. 9. Experimental data was compared to the average model calculation and cycle-by-cycle simulation results (Fig. 10), which represents the control to output response, (v_o/d(f)), of the Boost SCCx3 system. Some mismatches, such as lower steady-state voltage, more heavily damped response in the experimental results and phase lag of the experimental system at lower frequencies, as observed in Fig. 10, are due to the omission of the loss components in the present discussion.

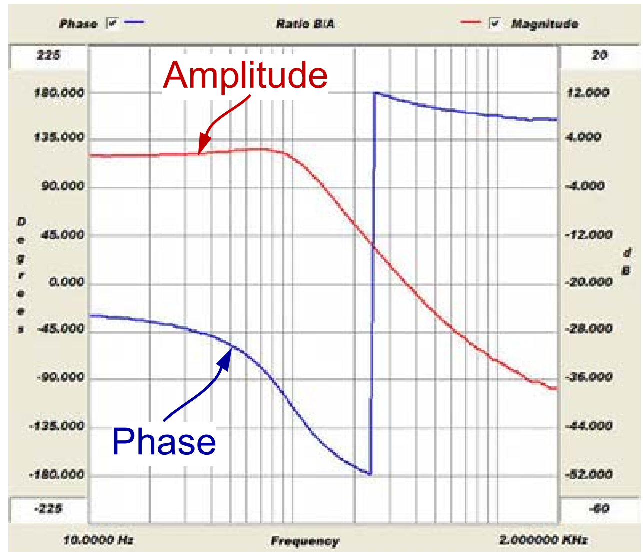

Figure 9. Experimental results of the control to output response of a Boost SCCx3 hybrid converter.

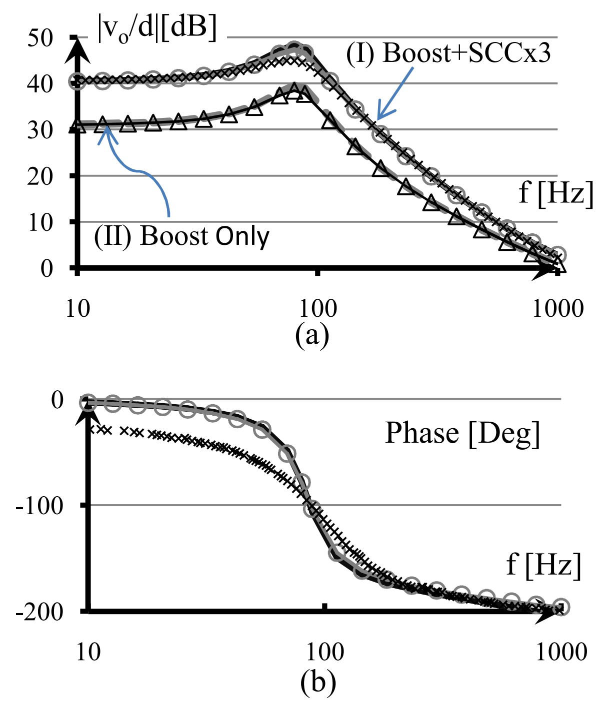

Figure 10. Control to output response of Boost only and hybrid converter: (a) Amplitude response; (I) Hybrid: experimental, cycle-by-cycle simulation, average model; (II) Boost only: cycle-by-cycle simulation, average model; (b) Phase response of hybrid converter: experimental, cycle-by-cycle simulation, average model.

V. DISCUSSION

The average circuit model proposed in this study is an extension of previously published average models of switch mode systems. As such, the model is compatible with CCM and DCM operation modes and capable of seamless sweeping between them as is the GSIM model it is based on [8]. It is compatible with all operational modes of SCC - CC, PC, and NC [9, 11, 17]. The model provides an insight into the operation of the hybrid converters and can be used to analyze the effects of the circuit parameters such as values of the components on the static and dynamic responses. The proposed modeling methodology could be helpful to researchers and design engineers as well as for educational purposes. The average model can help in the optimization of hybrid converters and, particularly, in their control in order to achieve desired regulations. Furthermore, the model can be used to delineate the contribution of each section to the total response. For example, the plots of Fig. 10 can be used conveniently to investigate the difference between the bare Boost, the SCC and the total hybrid system dynamic responses. Using the proposed model, the impact of capacitors' ESR on the output voltage can be evaluated (Fig. 11) in a fast and easy way.

A Boost SCCx3 system was selected for experimental and simulation tests to maintain a fair model evaluation under different conditions, in terms of gain achieved by the hybrid converter and the stresses that the components of the system are exposed to, when compared to the bare Boost converter. The stresses in a hybrid converter (in comparison to the bare Boost converter) are somewhat released by the SCC multiplying stage and allow selection of much more moderate components, such as switches or output capacitors that are required to withstand only one third of the output voltage, while carrying the same current. The diode in a bare Boost converter operated at a high boost ratio will be exposed to an extreme reverse recovery current due to the large input-output voltage difference. Moreover, operation at duty cycles close to unity will result in very high currents through the diode and hence a momentary high forward voltage. These facts will significantly reduce the overall boost efficiency in comparison with the hybrid converter.

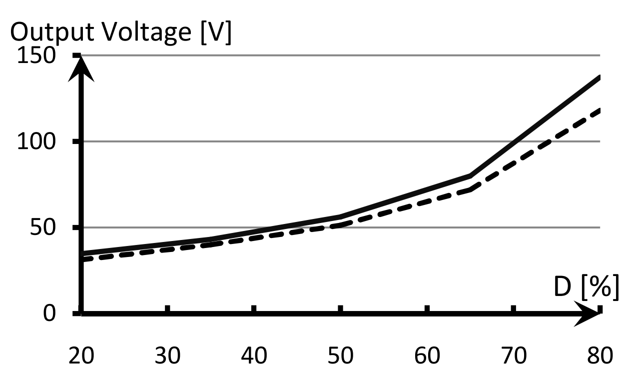

Figure 11. Boost SCCx3 model predicted output voltage as a function of duty cycle for V_{in}=10V: solid trace–capacitors' ESR=50mΩ; dashed trace–capacitors' ESR=1Ω.

VI. CONCLUSIONS

This paper presents a unified behavioral average circuit model of hybrid converters. These converters include PWM-controlled switched inductors paired with SCC systems and are useful in high gain applications. An implementation of the model on modern simulation software packages is proposed and demonstrated on PSIM and OrCAD PSpice simulators for a Boost SCCx3 multiplier converter.

The model was found to be useful in calculating and simulating average static, dynamic and small signal responses of hybrid converters. The model is valid for all operation modes of the SCC and operation in CCM and DCM modes of the Switched Inductor Converters.

The proposed average circuit model was verified by full circuit simulations and experimentally, and good agreement was found between the results. The minor discrepancies are probably due to the fact that this model did not take into account diode voltage drops and switch resistances. This resulted in a less damped response of the full simulation and of the model calculated results. It should be noted that a basic limitation of the model is the assumption that the capacitors' charge/discharge processes could be approximated by first order RC subcircuits [9-11].

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