

A Simple and Intuitive Graphical Approach to the Design of Thermoelectric Cooling Systems

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Abstract - In various applications, thermoelectric active cooling systems can help maintain electronic devices at a desired temperature condition better than passive coolers. Thermoelectric Coolers (TEC) are especially useful when the temperature of a device needs to be precisely controlled. This study proposes a user-friendly graphical method for calculating the steady-state operational point of a TEC based active cooling system, including the heatsink role. The method is simple and intuitive and provides comprehensive information about the cooling system such as its feasibility, required heatsink, the TEC current, temperatures of the cold side and others. The method could help designers to examine and choose a thermoelectric module from catalogues to meet a specific cooling problem. To start using the method, designers need only the experimental TEC data provided by practically all manufacturers of such devices. The experimental results of this study verify the high accuracy of the proposed model and graphical approach.

I. INTRODUCTION

The demands for small-size active cooling equipment has increased in recent years, since the traditional passive cooling systems (heatsink and fan) are not powerful enough to cope with the task of cooling a variety of modern electronic devices. One potential alternative solution is active cooling [1] - [3]. The difference between passive and active cooling is depicted in Fig. 1. The passive cooling system (Fig. 1a) includes a heatsink, possibly with a fan, with thermal resistance Θ_k , (K/W). The active cooling system (Fig. 1b) uses an energy conversion process to absorb the thermal energy from the surface needed to be cooled and to pump this energy out. Active cooling can be realized by applying a thermoelectric (Peltier) cooler (TEC) and a heatsink of thermal resistance Θ_k .

The main problem of designing thermoelectric active cooling systems is the fact that the system depends on a large number of parameters. The parameters involved are thermal resistance of the heatsink, temperature of the ambient air, the many parameters of the TEC, and the electrical current through it.

The objective of this work was to simplify the treatment of TEC based cooling systems by applying a unified model. The paper proposes a universal graphical method for the design of TEC-based active cooling systems.

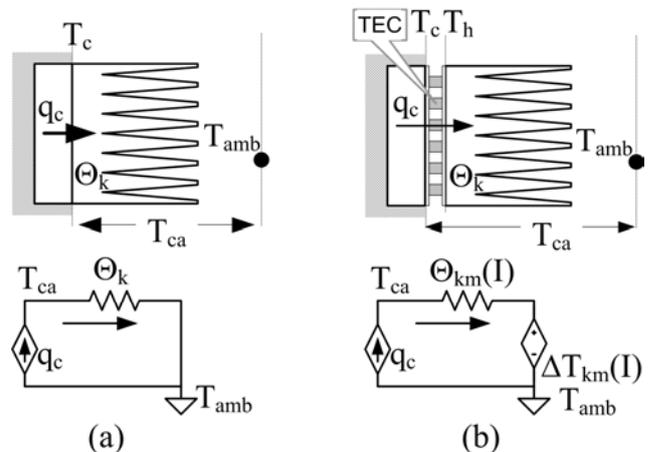


Fig. 1 Schematic representation of a passive (a) and an active (b) cooling systems, shown under similar system terms. T_c - temperature of the surface of interest, q_c - heat dissipated by the thermal load, T_{amb} - ambient temperature, T_{ca} - temperature difference between the surface and the ambient.

II. THE UNIFIED TEC MODEL

The behavior of a thermoelectric couple is determined by three fundamental parameters: Θ - the thermal resistance of the couple in the direction of the heat flow, R - the electrical resistance of the couple, and α - the Seebeck coefficient. Commercial TECs include N couples. Assuming that all couples are identical, and that the heat flow is unidirectional, the lumped parameters of the TEC α_m , Θ_m , and R_m will be:

$$\alpha_m = \alpha N \quad (1)$$

$$R_m = RN \quad (2)$$

$$\Theta_m = \Theta / N \quad (3)$$

Practically all TEC manufacturers ([4] - [6] and others) use the following parameters to specify their products: ΔT_{max} - is the largest temperature difference (K) that can be obtained between the hot and cold ceramic plates of a TEC for the given level of hot-side temperature T_h , I_{max} is the

input current (A) which will produce the maximum possible temperature drop ΔT_{\max} across a TEC, and U_{\max} is the DC voltage (V) that will deliver the maximum possible temperature drop ΔT_{\max} at the supplied I_{\max} .

As shown earlier [7], [8], the following expressions can be used to calculate the fundamental TEC's parameters from the set of data given by manufacturers (T_h , ΔT_{\max} , U_{\max} , I_{\max}):

$$\alpha_m = \frac{U_{\max}}{T_h} \quad (4)$$

$$R_m = \frac{U_{\max}}{I_{\max}} \frac{(T_h - \Delta T_{\max})}{T_h} \quad (5)$$

$$\Theta_m = \frac{\Delta T_{\max}}{I_{\max} U_{\max}} \frac{2T_h}{(T_h - \Delta T_{\max})} \quad (6)$$

Following the first law of thermodynamics, one can express the energy equilibrium at each side of the thermoelectric module, defined as the cold (c) and hot (h) junctions. Note that all parameters taken, in first order approximation are assumed to be time invariable and temperature independent, and that the contribution of the Thomson effect is neglected, as suggested in [1], [3].

For the absorbing (cold) side one can write:

$$q_c = \alpha_m T_c I - \frac{\Delta T}{\Theta_m} - \frac{I^2 R_m}{2} \quad (7)$$

and for the emitting (hot) side:

$$q_h = \alpha_m T_h I - \frac{\Delta T}{\Theta_m} + \frac{I^2 R_m}{2} \quad (8)$$

where q_c is the heat absorbed at the cold side of the TEC, q_h - heat dissipated at the hot side, T_c and T_h - temperatures of cold and hot sides respectively in K, and $\Delta T = T_h - T_c$.

The electrical section of the module is described as an electrical resistance R_m in series with an emf-source:

$$V = \alpha_m \Delta T + I R_m \quad (9)$$

Finally, the temperature of the hot side of the TEC can be expressed as a function of the heat transferred from that side to the passive heat removal heatsink.

$$T_h = T_{\text{amb}} + q_h \Theta_k \quad (10)$$

Applying the set of equation (7) - (10) one can eliminate the variables q_h , T_h , and V and get an expression for T_c , the temperature of the absorbing side of the TEC. The solution can then be used to develop an equivalent circuit type model of the TEC system (Fig. 1b) for which the temperature difference between the cooled side (T_c) and the ambient (T_{amb}), (T_{ca}), is expressed as:

$$T_{\text{ca}} = (T_c - T_{\text{amb}}) = q_c \cdot \Theta_{\text{km}}(I) + \Delta T_{\text{km}}(I) \quad (11)$$

where

$$\Theta_{\text{km}}(I) = \left(\frac{\Theta_k + \Theta_m - I \alpha_m \Theta_m \Theta_k}{1 + I \alpha_m \Theta_m - I^2 \alpha_m^2 \Theta_m \Theta_k} \right) \quad (12)$$

is the apparent thermal resistance of the TEC (Fig. 1b), and

$$\Delta T_{\text{km}}(I) = \left(\frac{\Theta_m + \Theta_k (2 - I \alpha_m \Theta_m)}{1 + I \alpha_m \Theta_m - I^2 \alpha_m^2 \Theta_m \Theta_k} \right) \frac{I^2 R_m}{2} + \left(\frac{I \alpha_m \Theta_m (I \alpha_m \Theta_k - 1)}{1 + I \alpha_m \Theta_m - I^2 \alpha_m^2 \Theta_m \Theta_k} \right) T_{\text{amb}} \quad (13)$$

is the apparent temperature pump (Fig. 1b). This representation provides an intuitive understanding of the cooling process of a TEC. It shows that the TEC introduces a heat pump of temperature difference ΔT_{km} and an apparent thermal resistance Θ_{km} both of which are dependent on the TEC parameters and the current I that drives it.

Although the above expressions seem complex, they include only the fundamental parameters of the TEC that can be calculated from the manufacturers' data by (4)-(6). All expressions can of course be easily evaluated by using commonly available software.

III. GRAPHICAL METHOD

Since a direct application of the analytical solution of (11) for the design of TEC based systems could be rather involved, we propose here a simple and intuitive graphical approach. It is based on a parametric representation of current dependent values of (11) denoted here as the $S(I)$ curve. The x and y values of the $S(I)$ (current dependent) curve are defined as:

$$S(I) = \begin{cases} x = \Theta_{\text{km}}(I) \\ y = \Delta T_{\text{km}}(I) \end{cases} \quad (14)$$

It should be noted that in this presentation, the x-axis variable is thermal resistance, whereas the y-axis variable is temperature. A typical $S(I)$ plot is shown in Fig. 2. Notice that each point of the $S(I)$ curve corresponds to a specific TEC current. The graphical solution for a given active cooling system is facilitated by drawing on same plot a line, denoted as the d line that represents the objective thermal problem on hand, defined as:

$$y = T_{\text{car}} - q_{\text{cr}} \cdot x \quad (15)$$

where y- and x- axis are as above. The slope of this line q_{cr} , corresponds to the power needed to be dissipated by the cooled unit whereas T_{car} is the required surface temperature of the cold side of the TEC (assumed to be equal to the surface of the heat source), referred to the ambient temperature. The intersection(s) between the $S(I)$ curve and the d line (if existing) satisfies both equations and is thus the solution of the given TEC cooling system (11).

By way of illustration, consider an active heat removal case that applies a TB-127-1.2-1.4 module [4] and a

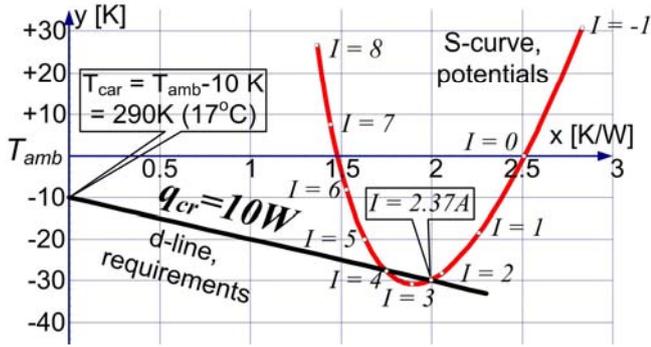


Fig. 2. Geometrical solution for the system: TEC (TB-127-1.4-1.2 [5]) with heatsink ($\Theta_k=1\text{K/W}$). $T_{\text{amb}}=300\text{K}$ and a specific thermal problem: $T_{\text{car}}=-10\text{K}$ (10K below the T_{amb}), $q_{\text{cr}}=10\text{W}$. The thermal requirement is met when the d line intersects the S-curve (at $I=2.37\text{A}$ and $I=4\text{A}$).

heatsink with $\Theta_k = 1\text{K/W}$, $T_{\text{amb}} = 300\text{K}$. The data supplied by manufacturer for the TEC are: $\Delta T_{\text{max}} = 70\text{K}$, $I_{\text{max}}=7.6\text{A}$, and $V_{\text{max}} = 15.9\text{V}$, under the condition that the temperature of the hot side $T_h = 300\text{K}$. The module has dimensions of 40mm by 40mm, and is enclosed between two ceramic plates. It is assumed that the power dissipated by the surface to be cooled is 10W and that the required surface temperature is ($T_{\text{amb}}-10\text{K} = 290\text{K}$).

Applying the transformation equations (4)-(6) we can first calculate the fundamental parameters of the TEC. They are found to be: $\Theta_m = 1.51\text{K/W}$, $\alpha_m = 53\text{mV/K}$, $R_m = 1.6\ \Omega$. Then, by using (12) and (13), one can plot the S(I)-curve. The d line is constructed according to the thermal data, as shown in Fig. 2. The intersection between the S(I) curve and the d line, are the solutions of the problem. The currents of the S(I) at the intersection points are the currents needed to drive the TEC in order to obtain the desired cooling effect. Between the two solutions that are normally obtained, one would obviously choose the lower current. The coordinates of the intersection points between the S and d curves correspond to the apparent thermal resistance and temperature pump of the equivalent circuit (Fig. 1b). In this example, the apparent thermal resistance (for $I = 2.37\text{A}$) is found to be $\Theta_{\text{km}} = 2\text{K/W}$ and the value of the temperature pump $\Delta T_{\text{km}} = -30\text{K}$. It should be noted that since the S(I) curve includes the information of the heatsink used, a different S(I) curve needs to be drawn for each thermal resistance of the alternative heatsinks as shown in Fig. 3.

IV. PROPOSED ACTIVE COOLING DESIGN PROCEDURE

The proposed graphical design procedure will be illustrated by way of an example in which a commercial TEC (TB-127-1.4-1.2 [4]) is considered. It is assumed that the surface of the unit to be cooled dissipates $q_{\text{cr}} = 40\text{W}$ and

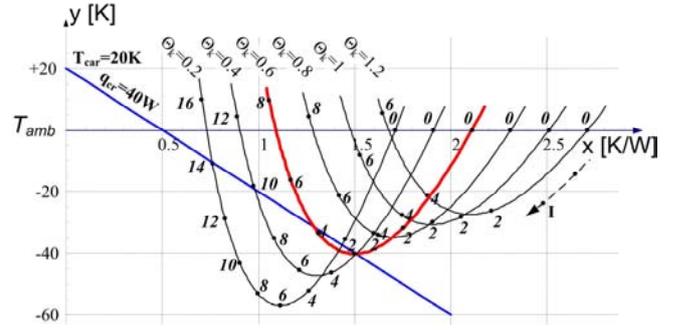


Fig. 3. The S-curve for TB-127-1,2-1,4 with different heatsinks (Θ_k from 0.2 to 2 K/W) for $T_{\text{amb}} = 300\text{K}$. Solid line specifies the requirements imposed on active cooling system ($T_{\text{car}} = 20\text{K}$, $q_{\text{cr}} = 40\text{W}$).

that it needs to be maintained at temperature of less than 50°C at an ambient temperature of $T_{\text{amb}} = 300\text{K}$ (27°C). That is, $T_{\text{car}} \approx 20\text{K}$. Fig. 3 shows a family S(I)-curves, each corresponding to a specific value of Θ_k . The solid d-line, starts at point (0, 20) and drops downward with angle coefficient of 40W. As evident from the graphical representation of the problem (Fig. 3), there are no real solutions for $\Theta_k \geq 0.8\text{K/W}$. For the case of $\Theta_k = 0.6\text{K/W}$, the d-line crosses at two points: $I \approx 2\text{A}$, and $I \approx 4\text{A}$. Operation at the lower current is obviously more desirable.

V. EXPERIMENTAL

The graphical analysis was verified by two different methods: a) Laboratory experiments that were carried out on a specific TEC, and b) Calculations according to proposed approach and comparison to experimental data given by manufacturers for different TECs.

The experimental setup (Fig. 4) included a TEC, a heat source, a heatsink with fan and aluminum plate with an embedded thermocouple for temperature measurements. A series of measurements of the steady-state difference of temperatures of T_1 (which is close to T_c) and T_{amb} under different heat dissipation conditions, and for different TEC current were carried out. The results of the measurements are shown in Table 1. Fig. 5 depicts S(I)-curve that corresponds to the data of the experimental TEC (TB-127-1.4-1.2) and the $\Theta_k = 0.67\text{K/W}$ of the heatsink and fan, at $T_{\text{amb}} = 294.8\text{K}$. The slope of the d-lines corresponds to the heat generated by the heat source. Each line starts at an S(I) points that correspond to each of the experimental TEC currents. The point of intersection between the d line and the "y" - axis is the estimated value of T_{ca} - the temperature of the cold side above the ambient temperature. The values of these model estimates are summarized in Table 1. Good agreement was found between the measured temperature differences and the ones estimated by the proposed graphical method (Table 1).

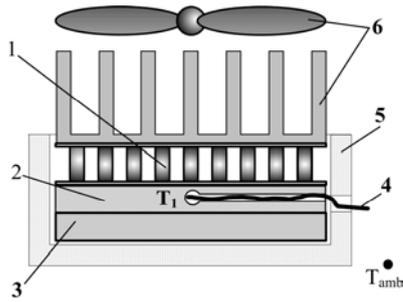


Fig. 4. Experimental setup: 1 - TEC, 2 - massive aluminum plate, 3 - heat source (q_c , W), 4 - thermocouple, measuring temperature T_1 , (K), 5 - thermal insulation, 6 - heatsink and a fan with thermal resistance $\Theta_k = 0.67$ K/W. The temperature of the ambient air during the experiment was $T_{amb} = 294.8$ K.

Applying the experimental data given by manufacturers, we have used the unified model to recalculate the value of Q_{max} also given in the commercial data sheets. Q_{max} is defined as the amount of heat (W) that can be pumped by the TEC for $I = I_{max}$ and ΔT equal to zero. To evaluate this case by proposed method, one needs to construct the $S(I)$ -curve using (14) and substituting $T_{amb} = T_h = 300$ K (standard temperature of the experiment) and $\Theta_k = 0$ (Fig. 6). This substitution means that the temperature of the hot side of the TEC is constant and equal to the temperature of the cold side. Now, the straight line from origin to the point on the S -curve that corresponding to I_{max} has a slope of Q_{max} . The tangential line from the origin to the S -curve (Fig. 6) has an angle of Q_{opt} - the maximum possible amount of heat for the experimental condition, which is also quoted by some manufacturers.

Fig. 7 shows the distribution of the error in the recalculated values of Q_{max} for 54 commercial TEC devices as compared to the independent data given by the manufacturers. In about 90% of the cases, the error in the recalculated Q_{max} by the proposed model, relative to the values given in the data sheets, was found to be less than 5%.

TABLE 1.
RESULTS OF THE EXPERIMENTAL MEASUREMENTS BY REFERENCE TO RESULTS ESTIMATED USING PROPOSED GRAPHICAL METHOD.

Measured values			Estimated T_{ca} from Fig. 5
q_c , W	I, A	T_{ca} , K	
24.75	1	30	30
25.2	2.9	7	7
24.5	4	1	0.9
25.2	5	2	2

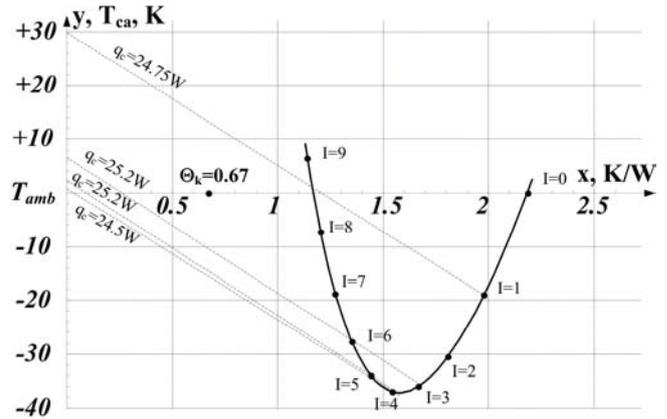


Fig. 5. The $S(I)$ curve of the experimental system of TEC and heatsink, showing the points that correspond to specific TEC drive currents, and graphical representations of the experimental conditions (d lines). The slope of each d line corresponds to the power of thermal load, (q_c , in W) of each experiment. The intersections of the d lines with the vertical axis (y) are the estimates of the temperature of the TEC cold side above the ambient temperature in K, for each experimental I.

VI. DISCUSSION AND CONCLUSIONS

In some applications, thermoelectric active cooling systems can help maintain electronic devices at desired temperature conditions better than passive coolers. Active cooling is especially useful when the temperature of a device needs to be precisely controlled. This study proposes a user-friendly graphical method for calculating the steady-state operational point of a TEC based active cooling system. The method is simple and intuitive, and provides comprehensive information about the cooling system such as its feasibility, required heatsink, the TEC current, temperature of the cold side, and others. The method could help designers to examine and choose a thermoelectric module from catalogues to meet a specific cooling problem. To start using the method, designers need only the experimental TEC data provided by practically all manufacturers of such devices.

The proposed method is 'graphical' in the sense that it provides a clear graphical representation and hence an intuitive understanding of the cooling process by a TEC. It is based on the very basic equivalent circuit of a generic cooling system (Fig. 1b) that includes a thermal source, an apparent thermal resistance and a temperature pump. The strength of the graphical approach is that it can help the designers to examine different TEC and heatsinks combinations in a very friendly way. In fact, once defining a given cooling problem by a 'd' line and drawing a family of 'S' curves for a range of heatsinks, one can immediately identify the design options. After a TEC is selected, the accurate calculation of the operating conditions of the system can of course be carried out by applying the

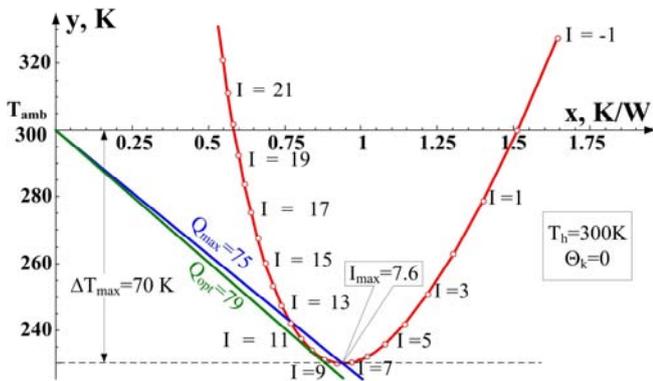


Fig. 6. Verification of the graphical method against experimental data given by manufacturers. The S-curve shown is for TB-127-1,2-1,4 with $T_{amb}=T_h=300\text{K}$, i.e. $\Theta_k=0$. The parameter Q_{max} is the slope of the straight line from $T_{amb}=300$ on the Y-axis, to the point of $I=I_{max}$ on the S-curve (the minimum of S-curve). The slope of the tangential line from the origin is Q_{opt} .

equations developed in this work. For example, the exact value of the required TEC drive current for a given case can be obtained by solving numerically (11) and (15) for I . Indeed, practically all the numerical results quoted in this paper were obtained by numerical solutions of the corresponding equations.

The proposed 'graphical' approach was verified against experimental results collected during this study and independent data given by manufacturers. The excellent agreement that was observed attests to the high accuracy of the proposed model and graphical approach. This good agreement also justifies the approximations that were done in the development of the proposed model (e.g. neglecting

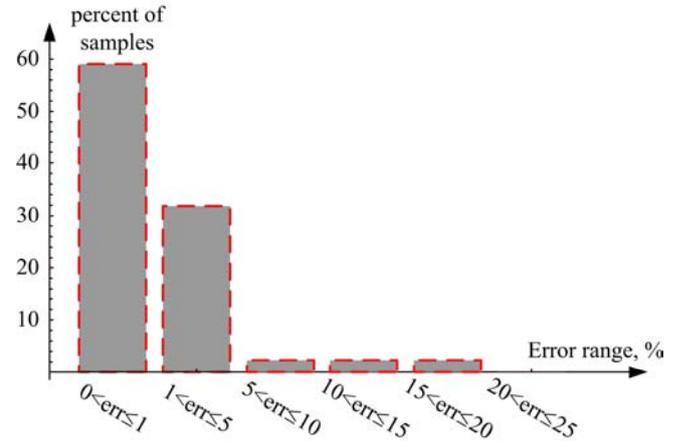


Fig. 7. Comparison of the recalculated Q_{max} by proposed model to the data given by TEC manufacturers' for 54 commercial TEC units.

the Thompson effect, variations of the parameters values with temperature, etc.).

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