PSPICE-Compatible Equivalent Circuit of Thermoelectric Coolers

Simon Lineykin and Sam Ben-Yaakov*

Power Electronics Laboratory
Department of Electrical and Computer Engineering
Ben-Gurion University of the Negev
P. O. Box 653,Beer-Sheva 84105, ISRAEL, Phone: +972-8-646-1561,Fax: +972-8-647-2949
Email: sby@ee.bgu.ac.il, Website: www.ee.bgu.ac.il/~pel

Abstract-the objective of this work was to develop a PSPICE-compatible equivalent circuit of a thermoelectric cooler (TEC). Equivalent circuits are convenient tools for power electronics engineers since they help in presenting a problem in electronic circuit terms and can assist in the design of power stages and the control circuitry and algorithms.

A methodology is developed for extracting the parameters of the proposed model from manufacturers' data of TECs. The present model is compatible with PSPICE or other electronic circuit simulators. An important feature of the model is its ability to generate small-signal transfer functions that can be used to design feedback networks for temperature control applications.

Several examples of successful utilization of the model are presented. Data of many different manufacturers were examined and the model parameters were extracted. In all cases, the model was found to reproduce accurately the performance of commercial TECs. The accuracy of the model was also verified by experiments.

I. INTRODUCTION

A thermoelectric cooler (chiller) (TEC) is a solid-state energy converter (Fig. 1). It normally consists of an array of pellets from dissimilar semiconductor material (p and n type), which are thermally joined in parallel and electrically in series. The thermoelectric module (TEM) can be used for cooling, heating, and energy generation [1] - [3]. The objective of this work was to develop a SPICE-compatible equivalent circuit of a TEC. An equivalent circuit is a convenient tool for electronic engineers. It helps in presenting the problem in electronic circuit terms and understanding its functionality, and it facilitates the solution of cooling or power-generation problems without the need for expertise in thermal engineering. A SPICE-compatible model is especially useful when dealing with a non-linear devices such as a TEC and incorporating it in a closed-loop system. In such cases a SPICE-compatible model can help in obtaining the transfer functions needed to design feedback circuitry.

II. PRINCIPLES OF OPERATION

Five energy-conversion processes take place in a thermoelectric module: conductive heat transfer, Joule heating, Peltier cooling/heating, Seebeck power generation and the Thompson phenomenon. All these processes account for the interrelations between thermal and electrical energies. Following the first law of thermodynamics, one can express the energy equilibrium at both sides of the thermoelectric module that are defined as the absorbing (a) and emitting (e) junctions. For the absorbing side:

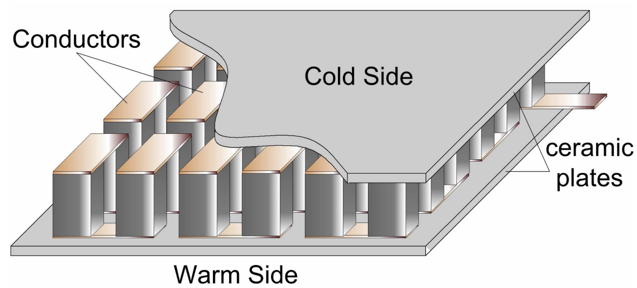


Fig. 1 Single-stage thermoelectric module construction

$$q_a = \frac{\Delta T}{\Theta_m} + \alpha_m T_a I - \frac{I^2 R_m}{2} \tag{1}$$

For the emitting side:

$$q_e = \frac{\Delta T}{\Theta_m} + \alpha_m T_e I + \frac{I^2 R_m}{2} \tag{2}$$

$$\alpha_m = \alpha N \tag{3}$$

$$R_m = RN \tag{4}$$

$$\Theta_m = \Theta / N \tag{5}$$

where q_a is heat absorbed at the a-side, q_e heat emitted at the e-side, N number of couples, T_a and T_e temperatures of (a-) and (e-) sides in K, Θ thermal resistance of the couple in the direction of the heat flow, R electrical resistance of the couple, α Seebeck coefficient, and $\Delta T=(T_e-T_a)$.

It is conventional to leave out the effect of the Thompson phenomena because it is negligibly small.

The electrical part of the module is described as an electrical resistance R_m and an electrical potential difference V:

$$V = \alpha_m T_e - \alpha_m T_a = \alpha_m \Delta T \tag{6}$$

* Corresponding author

III. EQUIVALENT SCHEME

It is common practice in one-dimensional heat transfer problems to apply an equivalent electrical circuit scheme [4]. This approach was adopted in this study to describe the TEM system in which several energy types exist. All non-electrical processes are described in terms of electrical analogies, and transformers (or dependent sources) represent their interconnections. In this way, the equivalent circuit of the thermo-electrical system of a TEC can be built as a pure electrical circuit.

Table 1 shows the physical parameters of the thermal system and corresponding parameters of the equivalent electric circuit.

This system of analogies permits the equivalent circuit of the thermo-electrical system of the TEC to be constructed as an electrical network. Fig. 2 shows the equivalent circuit of the TEC using the analogies from Table 1, which are based on equations (1), (2), and (6) for a- and e-junctions [5].

The scheme consists of the Cauer (C-Θ_m-C) network, which is normally used in equivalent circuits to represent conductive heat transfer in solids [6], supplemented by current sources. The sources shows Joule heating of the TEC, q_j, Peltier cooling on the heat-absorbing side of the TEC, q_{pa}, and Peltier heating on the heat-emitting side of the TEC, q_{pe}. The electrical part consists of the voltage source V_s and electrical resistance R_m. All capacitors have the initial charge IC = T_{amb}.

A modified equivalent circuit topology of the model, based on the circuit of Fig. 2, is shown in Fig. 3 with two dependent sources instead of three and lumped parameters instead of distributed ones. This new representation is clearly closer to the intuitive understanding of active cooling.

IV. CALCULATION OF THE PARAMETERS OF THE MODEL FROM MANUFACTURERS' DATASHEETS

Manufacturers of TECs (Kryotherm [7], Hui Mao [8], Marlow [9], and others) use the following parameters to specify their product: **ΔT_{max}** is the largest temperature differential (K) that can be obtained between the hot and cold ceramic plates of a TEC for a given level of T_h (temperature of the hot side), **I_{max}** is the input current (A) which will produce the maximum possible **ΔT** across a TEM, **V_{max}** is the dc voltage (V) that will deliver the maximum possible **ΔT** at the supplied I_{max}, **Q_{max}** is the maximum amount of heat (W) that can be absorbed at the TEC's cold plate at I_{max} and at a **ΔT** equal to 0. Note that Q_{max} is not the maximum possible amount of heat that can be handled by the TEC, rather the heat flow corresponding to the current I_{max}. Q_{opt} is the maximum amount of heat that can be absorbed at the TEC's cold plate for a **ΔT** equal to 0. Q_{opt} is larger than Q_{max}. Some manufacturers apply the notation Q_{max} instead of Q_{opt}, so one needs to carefully read the description given in the datasheets.

TABLE I
Thermal to electrical analogy

Thermal quantities	Units	Analogous Electrical Quantities	Units
Heat, q	W	Current, I	A
Temperature, T	K	Voltage, V	V
Thermal Resistance, Θ	K/W	Resistance, R	Ω
Heat capacity, C	J/K	Capacity, C	F
Absolute zero temperature	0 K	Ground	0 V

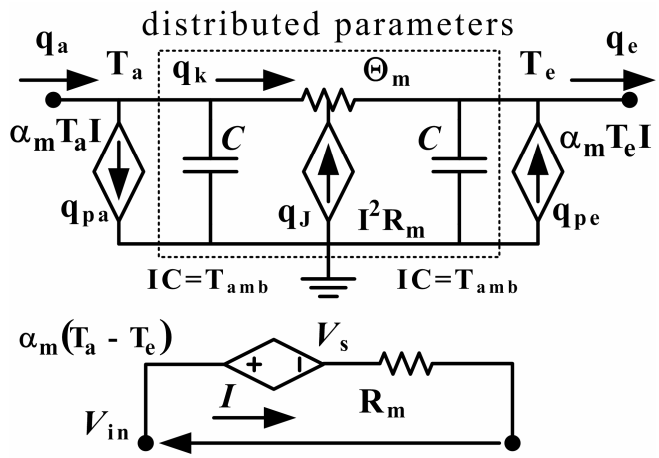


Fig. 2. The equivalent circuit of the TEC. The scheme is based on a Cauer-type network for describing heat transfer in a solid with internal heat sources (q_J). q_{pa} and q_{pe} (Peltier cooling and heating). The V_s voltage source describes Seebeck power generation. IC is the initial temperature of the device, T_{amb}.

Using the relations (1), (2), and (6), the characteristic parameters of the TEC can be derived:

$$\Delta T_{max} = T_h + \frac{\left(1 - \sqrt{1 + 2T_h Z}\right)}{Z} \tag{7}$$

$$I_{max} = \frac{\sqrt{1 + 2T_h Z} - 1}{\alpha_m \Theta_m} \tag{8}$$

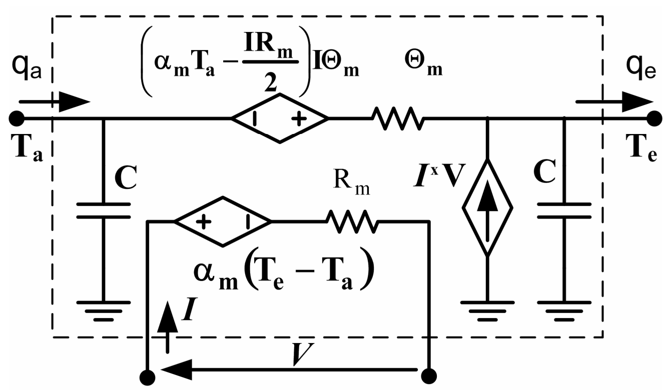


Fig. 3. Modified proposed equivalent circuit of a thermoelectric module.

$$V_{max} = \alpha_m T_h \qquad (9)$$

$$Q_{max} = \frac{\sqrt{1+2T_h Z}\left(\sqrt{1+2T_h Z}-1\right)^2}{2\Theta_m Z} \qquad (10)$$

$$I_{opt} = \frac{\alpha_m Th}{Rm} \qquad (11)$$

$$Q_{opt} = \frac{\alpha_m^2 Th^2}{2Rm} \qquad (12)$$

where Z is a figure of merit of the TEC, $Z=\alpha_m \theta_m / R_m$.

Applying (7) - (12), one can now use the set of data: $\mathbf{T_h}$, $\mathbf{\Delta T, V_{max}, I_{max}}$ for calculating the parameters of the proposed model:

$$R_m = \frac{V_{max}}{I_{max}} \frac{(T_h - \Delta T_{max})}{T_h} \; [\Omega] \qquad (13)$$

$$\Theta_m = \frac{\Delta T_{max}}{I_{max} V_{max}} \frac{2T_h}{(T_h - \Delta T_{max})} \left[\frac{K}{W}\right] \qquad (14)$$

$$\alpha_m = \frac{V_{max}}{T_h} \left[\frac{V}{K}\right] \qquad (15)$$

V. Steady-State Analysis

The TB-127-1.4-1.2 is one of the thermoelectric cooling modules available from Kryotherm [7]. From the manufacturer's datasheets: Under the T_h=300K condition ΔT_{max}= 70 K, I_{max}=7.6 A, V_{max}=15.9 V, and Q_{max}=75 W. Applying (13) – (15), one can calculate the model parameters: α_m=0.053 V/K, R_m=1.6 Ω, Θ_m=1.5 K/W. Fig. 4 shows the result of the application of the dc-sweep simulation that reconstructs the performance plot of the TEM TB-127-1.4-1.2. The dashed line is the simulation result. The original performance plot was copied from the software tool placed by Kryotherm on the Internet. The results obtained by the two methods are in close agreement.

VI. Comparison of Experimental to Model Time-Domain Response

The laboratory measurements of a physical TEC were compared with computer simulations that apply to the proposed model. The experiment was carried out using the TEC TB-127-1.4-1.2 (Kryotherm) TEC with dimensions of 40mmx40mm and ceramic plates 1mm thick on both sides. The module was thermally insulated. In the first phase of the experiment, a constant voltage was applied for several seconds to the electrical port. As a result, the temperature difference between the absorbing and emitting sides of the TEC was established. Then the time domain relaxation of the temperature difference to zero was observed by measuring the voltage of the open electrical port.

The capacitors C of the equivalent scheme of Fig. 3, determine the dynamic behavior of the model. The capacitors represent the lumped heat capacitance of the alumina ceramic plates and pellets of the TEC. The lumped heat capacitance of the TEC is C_{TEC} = 0.35 J/K and lamped heat capacitance of each one of the ceramic plates is C_c = 5.33 J/K. Data on thermal volumetric capacity are taken from [11], and the volumes of the ceramic plates and pellets from datasheets [7]. Thus $C = C_c + C_{TEC}/2$ = 5.68 F.

Fig. 5 compares the results of the experimental TEC time response and the PSPICE transient simulation of the equivalent circuit. The figure shows a good fit of the simulation to the experimental data.

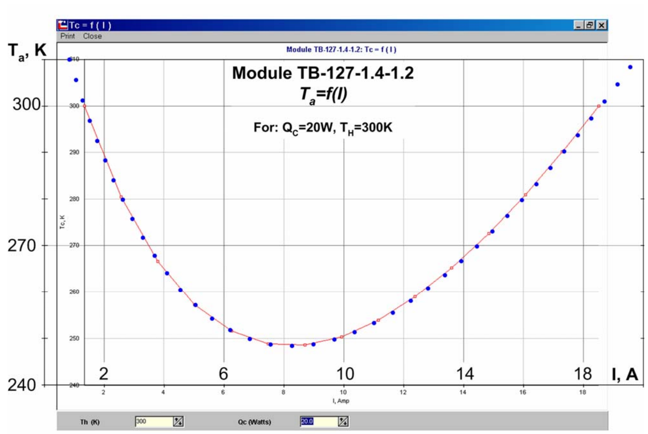


Fig. 4. Performance plot of TEM: TB-127-1.4-1.2. Temperature of the a-side (cold) vs. current under conditions: T_h=300, cooling power 20W. The dashed line is the simulation result obtained by the proposed model; the solid line is the performance plot published by the manufacturer.

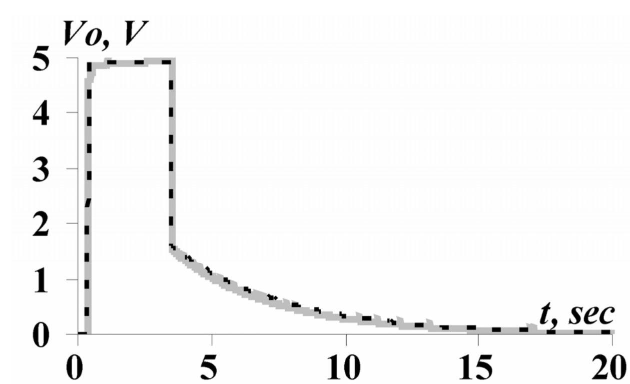


Fig. 5. Time response of the TEC. The gray thick line shows the result of the experiment on the physical module. The black dashed line is the result of the computer simulation using PSPICE (black line).

VII. SMALL-SIGNAL TRANSFER FUNCTION GENERATIONY USING EQUIVALENT CIRCUIT OF THE TEC

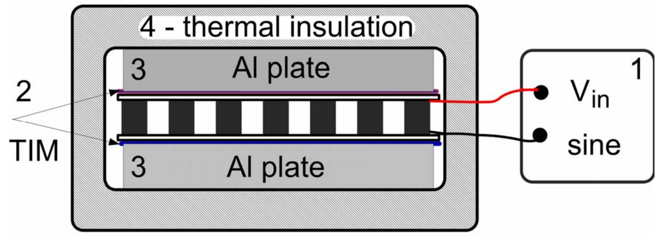
For a better controller design, one has to know the system frequency response (transfer function). An analytical method for calculating the poles and zeros of the transfer function of the TEC-based system is given in [12]. However, since the TEC is a non-linear system, the transfer function will be different for each operating (bias) point. The analytical derivation of the transfer function for all conditions of operation is thus a cumbersome process. The proposed model provides a simple way to get the transfer functions of the system directly from the large signal model by just carrying out a small-signal (ac) simulation of the cooling system by an electronic circuit simulator such as PSPICE.

Fig. 6(a) shows the experimental system of a TEC with a thermal load (two massive aluminum plates). The system is thermally insulated. There are two thermocouples inserted into the thermal load for temperature measurement. The setup permits the measurement of a response of the system to a sine wave voltage input. By making the measurements at different frequencies, one can get the frequency response of the system (the transfer function). Fig. 6(b) shows the equivalent circuit for the experiment simulation. The results of the simulations using the proposed equivalent circuit model are shown on Fig. 7. As one can see, the results of the small-signal (ac) simulation are in good agreement with those of the transient cycle-by-cycle simulation as well as with experimental results.

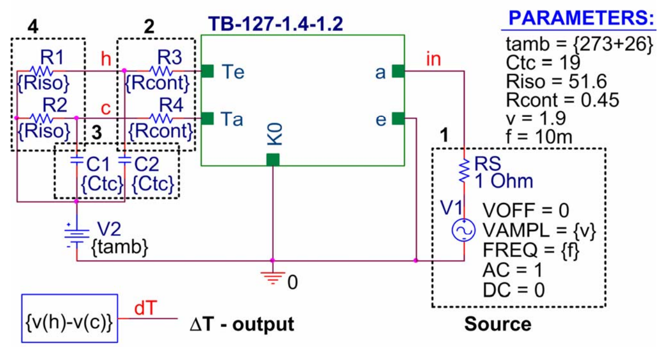
VIII. CONCLUSIONS

The study shows how the manufacturer's data for the thermoelectric cooler can be used to extract the parameters of the proposed model. The model could be helpful for analyzing the drive requirements of the TEC. Another important application of the proposed model is to analyze the performance of the TEC under specific conditions such as thermal leakage, non-ideal thermal insulation, etc. Using the model one can analyze not only existing modules, but also specify an optimal TEC for a specific problem. The present model is compatible with PSPICE or other electric circuit simulators for dc, ac, and transient simulation types and will thus be an excellent tool for solving problems of temperature control.

Several examples of successful utilization of the model are presented. The paper is based on data given by many different manufacturers that were used to reproduce accurately the performance of commercial TEMs. An important feature of the model is its ability to generate small-signal transfer functions that can be used to design a feedback network in temperature control applications.



(a)



(b)

Fig. 6. Measurement of the small-signal transfer function of the system: TEC sandwiched between two massive aluminum plates with built-in thermocouples. (a) Experimental setup. (b) PSPICE/OrCAD simulation scheme. The model of the TEC is the one shown in Fig. 3 with parameters α_m=0.053 V/K, R_m=1.6 Ω, Θ_m=1.5 K/W calculated above, 1: sine voltage source, 2: thermal interface material (TIM), 3: aluminum plates.

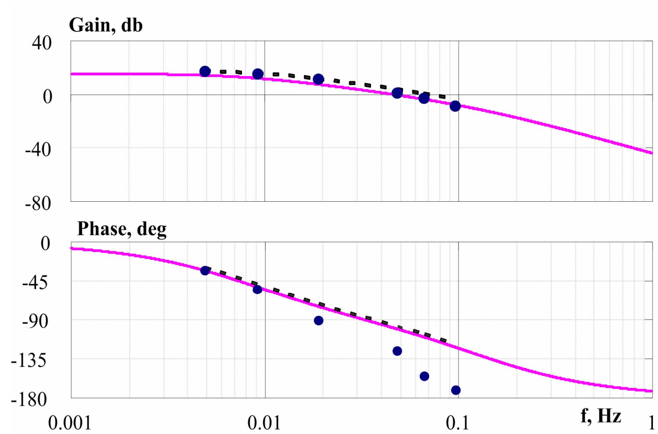


Fig. 7. Transfer function of the system of Fig. 6. Input variable is the input voltage and output is the temperature difference between the aluminum plates on both sides of the TEC. Dashed line is the result of cycle-by-cycle transient simulation of the system with sine input voltage (amplitude 1.3 V); Solid line is the small-signal (ac) simulation result, and points are data of the experimental measurements.

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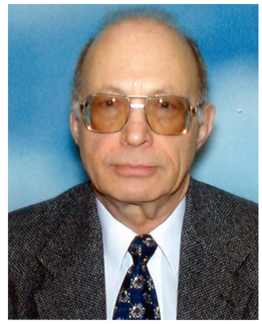

Shmuel (Sam) Ben-Yaakov received the BSc degree in Electrical Engineering from the Technion, Haifa Israel, in 1961 and the MS and PhD degrees in Engineering from the UCLA, in 1967 and 1970 respectively.

He is presently a Professor at the Department of Electrical and Computer Engineering, Ben-Gurion University of the Negev, Beer-Sheva, Israel, and heads the Power Electronics Group there. His current research interests include power electronics, circuits and systems, electronic instrumentation and engineering education. Professor Ben-Yaakov also serves as a consultant to commercial companies in the areas of analog and power electronics.

Simon Lineykin received the BSc degree in Mechanical engineering and MS degree in Electrical Engineering from Ben-Gurion University of the Negev, Israel. He is currently working toward his PhD degree in electrical engineering at Ben-Gurion University of the Negev. His research interests are modeling and emulation of the physical processes and active cooling systems using Peltier effect.