Unified Control Strategy Covering CCM and DCM for a Synchronous Buck Converter

Dirk Hirschmann, Sebastian Richter, Christian Dick, Rik W. De Doncker
Institute for Power Electronics and Electrical Drives
RWTH Aachen University
Jägerstr. 17/19, D - 52066 Aachen, Germany
Phone: +49 241 80-96975, Fax: +49 241 80-92203
Email: hi@isea.rwth-aachen.de

Abstract—In general, transfer functions of switch-mode converters are treated differently when operating in continuous-conduction mode (CCM) or discontinuous-conduction mode (DCM). This complicates the development of a control algorithm for a converter operating in both operating modes. To solve this problem, a control algorithm is developed, which is applicable for CCM and DCM. Since the developed control algorithm is based on linear equations instead of differential equations, it is easy to implement and its calculation effort is minor. It is based on equating the volt-seconds across the inductor for both operating modes and adjusting the duty cycle $d$. All necessary equations are derived and the functionality of the developed algorithm is shown by means of simulation and measurements.

I. INTRODUCTION

The synchronous buck converter is a variation of the well known and widely used buck converter, where the freewheeling diode is replaced by another power MOSFET, which—in most applications—is always turned on when the top switch is turned off (complementary switching). There are three advantages over the traditional buck converter. At first, it allows bidirectional power flow. Secondly, in low voltage applications the efficiency can be increased, because the on-state voltage drop of the switch is less than the forward drop of the diode [?]. Finally, the control can be simplified.

Controlling the buck converter, one has to distinguish between continuous-conduction mode (CCM) and discontinuous-conduction mode (DCM). This distinction is necessary because the transfer function between duty cycle $d$ and the output voltage $v_{out}$ or between input voltage $v_{in}$ and output voltage $v_{out}$ becomes also dependent on load condition in DCM. In the complementary switched synchronous buck converter the current can reverse within the switching period $T_s$. Thus, it always operates in CCM. The main disadvantage of this operating mode is that the switches are always operated even if no power is transferred. Apart from the losses in the switches and the inductor, this causes additional ripple in the output voltage $v_{out}$.

In the given application, the synchronous buck converter is used to drive a piezo actuator. The corresponding circuit diagram is given in Fig. 1, where the effective resistance $R_e$ accounts for various series resistances in the actual circuit. The piezo actuator is used to correct the cutting edge position of a high precision drilling tool with high dynamics within the micrometer range [?]. Hence, the piezo actuator has to be driven over its full operating range (0–800 V) up to its natural frequency $f_0_{mech}$ (approximately 1 kHz with attached mechanics) with a high degree of accuracy. This accuracy requires a low ripple in the output voltage $v_{out}$, which can be achieved by using a high switching frequency $f_s$ and/or a large inductance $L$. If $f_s \gg f_0_{mech}$, the mechanical system cannot follow the ripple of the output voltage which are generated by the switching. Hence, in this case the accuracy is not reduced by the switching [?]. Nevertheless, these voltage ripples generate internal forces in the piezo actuator which have to be minimized because they degrade the piezo actuator and decrease its lifetime [?]. In addition, if no resistance is connected in parallel, the piezo actuator, which electrically behaves like a capacitor, can keep its voltage for a couple of minutes. Therefore, if the deflection of the piezo actuator is not to be changed, switching can be avoided. One can easily see that in the given application the synchronous buck converter should not always operate in CCM and a control strategy which covers CCM and DCM is needed.

In order to model the system, also the switches have to be modeled. Already in [?] and [?] circuit models were proposed that are based on state-space averaging and strictly distinguish between CCM and DCM. In [?] it is stated that for DCM the order of the state-space model is reduced by one. This
is revised in [?] and [?] and it is shown that the transfer function of a buck converter in DCM still contains two poles, where the second pole moves to higher frequencies when the system enters deeper into DCM. Therefore, if the changes in the transfer function are taken into consideration, it is possible to develop one single control algorithm for a system operating in CCM as well as in DCM. Since in this application the switching frequency is much higher than the natural frequency of the low-pass filter \( f_s \gg f_{\text{Mech}} \), state-space averaging can be applied to the synchronous buck converter [?].

In the following a state-space model and a control algorithm for the synchronous buck converter will be derived. Subsequently, the extension of this algorithm to DCM and its preconditions are described in detail. Finally, the approximations will be validated by means of simulation and experimental results.

II. AVERAGED CONVERTER MODEL

To develop a state-space model of the synchronous buck converter the model can be divided in two sets of state equations, one for \( S_0 \) being in on-state or \( D_0 \) conducting \( v_{LC} = v_{in} \) and one for \( S_1 \) being in on-state or \( D_1 \) conducting \( v_{LC} = 0 \) V as it was done in [?]. Here the circuit was divided in two parts: the input stage and a second order low-pass filter. It is assumed that the natural frequency of the converter \( f_{0,\text{Mech}} \) is much smaller than the switching frequency \( f_s \) and that the voltage \( v_{LC} \) across the second order low-pass filter can be averaged over a switching period \( T_s \). Thus, if the voltage \( v_{LC} \) is averaged, only one set of state equations is needed. The basic equations are given in (1) and (2).

\[
\dot{x} = Ax + Bu \\
\dot{y} = Cx + Du
\]

where

\[
x = \begin{bmatrix} i_L \\ v_{out} \end{bmatrix}, \quad \ddot{u} = \bar{v}_{LC}
\]

and

\[
A = \begin{bmatrix} \frac{-R_e}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix},
\]

\[
C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad D = [0].
\]

The transfer function of the second order low-pass filter is:

\[
\frac{v \cdot \omega_0^2}{s^2 + 2\sigma \omega_0 s + \omega_0^2}
\]

where \( v \) is the gain, \( \sigma \) is the damping ratio and \( \omega_0 \) is the natural angular frequency. Using state feedback, \( \sigma \) and \( \omega_0 \) can be chosen arbitrarily. This was done in a way that the damping ratio \( \sigma = 1/\sqrt{2} \), which provides a good trade-off between settling time and overshoot [?]. \( \omega_0 \) was increased until the input variable, in this case the input voltage \( v_{in} \), became the dynamic limiting element. To consider the dead time, which is introduced by the switching period respectively sampling period \( T_s \), the state-space model as well as the transfer function have been discretized. Next, the feedback gain matrix \( K \) was computed to convert the system given in (1) and (2) to the system of (3). To provide unity gain \( v \), the steady-state gain of the feedback loop was computed and its reciprocal value was used as prefilter \( R \). Inverted center-aligned switching mode was used as switching pattern.

So far, standard control theory was applied to a continuous system of which the input variable is discontinuous and is calculated once per switching period. Here the input variable \( v_{LC} \) was averaged to \( \bar{v}_{LC} \). This can be done using the assumption that the dead time of the input variable is very short compared to reciprocal of the natural frequency of the system. In order to explain how the control algorithm was extended to DCM, first the hardware will be described in detail.

III. THE HARDWARE

The system consists of two MOSFETs, one inductor, and the piezo actuator. As mentioned above, the ripple in the output voltage is scaled by the inductance \( L \), which also scales the bandwidth of the second order low-pass filter. To provide a high bandwidth while smoothing the output voltage sufficiently, the inductance was selected in a manner that the cut-off frequency of the low-pass filter \( f_{0,\text{Mech}} \) lies in the geometric mean between the natural frequency \( f_{0,\text{Mech}} \) of the actuator and the switching frequency \( f_s \). Due to thermal limitations, the switching frequency \( f_s \) could not be increased above 50 kHz. The dc-link voltage was chosen 10% above the maximum actuator voltage and a dc-link capacitance of \( 10 \times \) the actuator capacitance \( C_{\text{Piezo}} \) was installed [?].

To control the system the rapid control prototyping system XCS2000 was used [?]. This system provides up to 32 analog simultaneous sampling AD/DA channels. The PWM is generated by a FPGA, where the duty cycle \( d \), which is the on-time interval of the upper switch \( S_0 \), is updated once per switching period \( T_s \). The AD-conversion is triggered at the beginning of each switching period (\( t = 0 \)). Each time the input voltage \( v_{in}(0) \), the output voltage \( v_{out}(0) \), and the inductor current \( i_L(0) \) are measured.

Since the implemented control algorithm is closely related to the switching pattern, the used switching pattern will be explained briefly. Two basic switching patterns can be distinguished: Center-aligned switching mode, where the on-state interval is aligned to the center of the switching period \( T_s \) and edge-aligned switching mode, where the on-state interval starts at the beginning of each interval. Both patterns can be inverted, like it is depicted for center-aligned switching mode in Fig. 2. This switching pattern, inverted center-aligned switching mode, was used in the synchronous buck converter. Thus, analyzing the current wave shapes, the switching period \( T_s \) has to be divided into three intervals. The first interval where the switch is in on-state, the second interval where the
switch is in off-state and the third interval where the switch is once more in on-state.

![Fig. 2. Different switching modes: (a) edge-aligned switching mode (b) center-aligned switching mode and (c) inverted center aligned switching mode]

IV. THE CONTROL ALGORITHM

The control algorithm is straightforward and easy to understand. First the state variables, i.e. inductor current \( i_L \) and output voltage \( v_{out} \), are calculated. The controller has to distinguish between CCM and DCM. Then the required duty cycle \( d \) is calculated. Finally, for DCM, the duty cycle has to be corrected.

The control algorithm can be divided into five steps:

A. Detecting DCM in the Current Switching Period
B. Calculating the Inductor Current \( i_L \)
C. Determining the Desired Input Voltage \( \bar{v}_{LC} \)
D. Detecting DCM in the Following Switching Period
E. Calculating the Duty Cycle \( d \)

Instead of solving the differential equations, the linear equations are used and it is assumed that the output voltage \( v_{out} \) does not change significantly during one switching period \( T_s \). As will be shown in the simulation results, this introduces small variations compared to theoretical results. However, the calculation effort is reduced dramatically making this approach reasonable.

In buck mode only the top switch \( S_0 \) and in boost mode only the bottom switch \( S_1 \) is operated. Buck or boost mode can be detected by comparing the reference voltage \( v_{ref} \) to the output voltage \( v_{out} \). If \( v_{ref} > v_{out} \), the boost mode is selected. When \( v_{ref} < v_{out} \), the buck mode is selected. If \( v_{ref} = v_{out} \) no switching action takes place. Since buck and boost mode are analogous, the control algorithm is discussed for buck mode only.

A. Detecting DCM in the Current Switching Period

The formulas to calculate the inductor current depend on the system state. There are 5 different system states that can be distinguished depending on the measured inductor current \( i_L(0) \). The current waveforms for center-aligned switching mode are depicted in Fig. 3.

![Fig. 3. Example current waveforms of different system states]

1) \( i_L(0) > 0 \text{A} \) and will not become 0 A within this switching period
2) \( i_L(0) > 0 \text{A} \) and will become 0 A within this switching period
3) \( i_L(0) < 0 \text{A} \) and will become positive, before it goes to 0 A
4) \( i_L(0) < 0 \text{A} \) and will become 0 A within this switching period
5) \( i_L(0) < 0 \text{A} \) and will not be discontinuous within this switching period

Due to the high dynamics of the system, state 5 can be excluded. Thus, the states can easily be distinguished between continuous (state 1) and discontinuous (state 2-4) states. If the measured current \( i_L(0) \) is negative, the current of this period is discontinuous. If the measured current \( i_L(0) \) is positive, it has to be determined if the system state is continuous or discontinuous. This is done by evaluating (4).

\[
i_L(t_{off}) - (1 - d) \cdot T_s \cdot \frac{v_{out}}{L} < 0 \text{A}, \quad (4)
\]

where

\[
i_L(t_{off}) = i_L(0) + \frac{d \cdot T_s}{2} \cdot \frac{v_{in} - v_{out}}{L}. \quad (5)
\]

B. Calculating the Inductor Current \( i_L \)

For the two operating modes, CCM and DCM, different current values are calculated.

Typically, in steady state CCM the average inductor current \( \bar{i}_L \) can be easily determined by measuring the current value in the center of the switching period. Due to the high dynamic operation of the converter also for CCM this may lead to a high degree of inaccuracy. Hence, in CCM the inductor current at the end of the switching period \( i_L(T_s) \) is taken as actual state variable. This should be done because this current sample contains the latest information available before the new duty cycle has to be determined.

The inductor current \( i_L(T_s) \) in CCM is calculated by:

\[
i_L(T_s) = i_L(0) + (d \cdot v_{in} - v_{out}) \cdot \frac{T_s}{L} \quad (6)
\]
This estimator tries to eliminate the delay between measurement and calculation of the next duty cycle $d$. Since this value is predicted over a whole switching period, it is important to exactly know the system quantities. If no estimator is used, an additional system state is created due to the delay and the order of the system increases to third order. In simulation this delay can be modeled by a first-order hold element. Since this state cannot be measured, it cannot be used for the feedback control and the system is less controllable.

For DCM, where the current $i_L(T_s) = 0$ A, the average current $\bar{i}_L$ of the current switching period is calculated. This average current $\bar{i}_L$ is calculated by determining the transferred charge of the three intervals and dividing it by the duration of the switching period.

Hence, the transferred charge of the three intervals is determined. For the first interval this is $(i_L(t_{off}))$ has already been calculated in (5):

$$q_{\text{first interval}} = \frac{d \cdot T_s}{2} \cdot \frac{i_L(0) + i_L(t_{off})}{2}$$  (7)

To calculate the transferred charge of the second interval, first it has to be determined when the current $i_L$ becomes 0 A. Thus, $\varepsilon$ is calculated, where $\varepsilon$ is the fraction of the switching period where the switch is in off-state and the current $i_L$ remains $\neq 0$ A.

For $i_L(t_{off}) > 0$ A (state 2 & 3) this is done in (8).

$$\varepsilon = \frac{i_L(t_{off}) \cdot L}{v_{out} \cdot T_s}$$  (8)

If $i_L(t_{off}) < 0$ A (state 4) (8) has to be slightly changed to

$$\varepsilon = \frac{i_L(t_{off}) \cdot L}{(v_{in} - v_{out}) \cdot T_s}.$$  (9)

For the states 2 - 4 the transferred charge of the second interval $q_{\text{second interval}}$ can then be determined by

$$q_{\text{second interval}} = \frac{i_L(t_{off}) \cdot \varepsilon \cdot T_s}{2}$$  (10)

For the third interval the transferred charge is

$$q_{\text{third interval}} = \frac{d \cdot T_s}{2} \cdot \frac{i_L(T_s)}{2},$$  (11)

where the current $i_L(T_s)$ is calculated by (12).

$$i_L(T_s) = \frac{d \cdot T_s}{2} \cdot \frac{v_{in} - v_{out}}{L}$$  (12)

Finally, the average current

$$\bar{i}_L = \frac{q_{\text{first interval}} + q_{\text{second interval}} + q_{\text{third interval}}}{T_s}$$  (13)

C. Determining the Desired Input Voltage $\bar{v}_{LC}$

The calculated inductor current $\bar{i}_L$ or $i_L(T_s)$ respectively and the output voltage $v_{out}(T_s)$ are taken for the actual state vector $\bar{x}$. For this reason, the output voltage $v_{out}(T_s)$ has to be calculated from the measured output voltage $v_{out}(0)$. Since the output voltage does not change significantly in one switching period $T_s$, this is done in a simplified manner by (14).

$$v_{out}(T_s) = v_{out}(0) + \frac{i_L(T_s) \cdot T_s}{C_{\text{piezo}}}$$  (14)

Using the feedback gain matrix $\bf{K}$ and the reference output voltage $v_{ref}$, the desired input voltage $\bar{v}_{LC}$ can be determined easily. First, CCM is assumed and the new duty cycle $d_{\text{new}}$ of the top switch $S_0$ is calculated to:

$$d_{\text{new}} = \frac{\bar{v}_{LC}}{V_{in}}$$  (15)

This value is taken and it is checked if it leads to DCM in the following period. In case of DCM, the duty cycle $d_{\text{new}}$ has to be changed.

D. Detecting DCM in the Following Switching Period

To detect DCM in the following period (4) can be applied once more, where $i_L(t_{off})$ of the following period has to be calculated differently. If in the current period DCM was detected, $i_L(t_{off})$ is calculated by (16). Otherwise, the calculation of the new $i_L(t_{off})$ becomes more complicated and is accomplished by evaluating (17).

$$i_L(t_{off}) = \frac{(d + d_{\text{new}}) \cdot T_s}{2} \cdot \frac{v_{in} - v_{out}}{L}$$  (16)

$$i_L(t_{off}) = i_L(0) + \left(\frac{d + d_{\text{new}}}{2}\right) \cdot T_s \cdot \frac{v_{in} - v_{out}}{L} - \left(1 - d\right) \cdot T_s \cdot \frac{v_{out}}{L}$$  (17)

This prediction is over a long time scale but the result is only binary (DCM or CCM). If DCM is detected, the duty cycle $d_{\text{new}}$ has to be adapted to DCM.

E. Calculating the Duty Cycle $d_{\text{new,DCM}}$

In DCM for a certain time interval there is no voltage applied to the inductor. Therefore, applying the same duty cycle $d_{\text{new}}$, the average inductor voltage is not what it would be in CCM. Thus, the calculated duty cycle $d_{\text{new}}$ will not change the system state as intended. Using the fact that in DCM the applied average inductor voltage is larger than in CCM, the duty cycle $d_{\text{new}}$ can easily be corrected without running the risk of getting back to CCM. To correct the calculated duty cycle $d_{\text{new}}$, the volt-seconds applied in DCM and CCM are equated.

$$d_{\text{new,DCM}} \cdot (v_{in} - v_{out}) - \varepsilon_{\text{new}} \cdot v_{out} \cdot \frac{1}{L} = d_{\text{new}} \cdot v_{in} - v_{out}$$  (18)
Inserting (8) and (5) into (18) and reducing the equation to \( d_{\text{new, DCM}} \) leads to (19). If \( i_L(t_{\text{ref}}) < 0 \) A, then (9) and (5) have to be inserted into (18). This also leads to (19). Therefore, this formula is valid for all system states. This simple formula is needed to convert CCM quantities into DCM quantities and makes standard control theory applicable to a system operating in CCM as well as in DCM.

\[
d_{\text{new, DCM}} = 2 \cdot \frac{d_{\text{new}} \cdot v_{\text{in}} - v_{\text{out}} + i_L(T_s) \cdot \frac{L}{T_s}}{v_{\text{in}} - v_{\text{out}}}
\]  

(19)

For boost mode (19) changes to (20), where the duty cycle of the bottom switch \( S_1 \) can be easily determined by \( 1 - d_{\text{new, DCM}} \).

\[
d_{\text{new, DCM}} = -2 \cdot \frac{d_{\text{new}} \cdot v_{\text{in}} - v_{\text{out}} + i_L(T_s) \cdot \frac{L}{T_s}}{v_{\text{out}}}
\]  

(20)

Before implementing the given equations they should be simplified. To keep them comprehensible in this work they are written in their original form.

V. SIMULATION AND EXPERIMENTAL RESULTS

To test the control algorithm, a step response from 0 V to 700 V was simulated. Here the developed control algorithm was compared to the same control using complementary switching. Hence, for the complementary switching control the system is always in CCM and all additional calculation effort to detect DCM is not needed. The two voltage and current curves are given in Fig. 4. While the system is in CCM the developed control algorithm behaves exactly like the complementary switching control, as expected. According to control theory for the selected damping ratio there should be a voltage overshoot of approximately 4 % \[^3\] which would be 28 V for a 700 V step response. Here the voltage overshoot is less than 1 %. The reason for this difference is the linearization of the system equations. In CCM the system states at \( t = T_s \) are calculated. Based on this information the new duty cycle \( d_{\text{new}} \) is determined. Therefore, there is no delay but, as already mentioned, linear equations instead of the differential equations are used to calculate the system states. This small systematic error causes the system behaviour to differ from theory. Since the differences are small and the reduction of calculation time is high, this is acceptable.

As the difference in voltage between \( v_{\text{out}} \) and \( v_{\text{ref}} \) decreases, the inductor current \( i_L \) is reduced and the system enters DCM. As a result, the second pole moves to higher frequencies and the system is additionally damped. While the developed control algorithm reaches \( v_{\text{ref}} \), the complementary switching algorithm shows a constant voltage ripple of \( 5 \text{ V}_{\text{pp}} \) around the reference voltage. This shows the advantages of the developed control algorithm. For large voltage errors the system operates with a high degree of dynamics in CCM. When the voltage error is reduced, it automatically changes to DCM where ringing is eliminated. Thus, switching and conduction losses are reduced in areas where there is only little power transferred. As a result, the overall efficiency is increased.

The control algorithm was finally verified by experimental results. Fig. 5 shows the experimental results for the same load conditions as the simulation. Here CH1 is the reference voltage \( v_{\text{ref}} \), CH2 is the output voltage \( v_{\text{out}} \) and CH3 is the inductor current \( i_L \). One can see that the simulation results match the measurements fairly well. There are only small differences due to disturbances and the limited bandwidth of the current measurement. Thus, the overshoot is increased minimally. Rise time is approximately the same compared to the simulation. It should be mentioned that generating 0 V at the output of a synchronous buck converter is not a stable operating area. Already small disturbances may lead to oscillations. Therefore, this step response should not be repeated in a harsh environment.

VI. CONCLUSIONS

In this work a new control algorithm was introduced that can be applied for CCM and DCM of a synchronous buck
converter. It is easy to implement and is based on standard control theory. Hence, already available software tools can be utilized to develop the control parameters and to guarantee stability. In the developed control algorithm the duty cycle $d$, which is calculated for CCM, is changed for DCM to establish equal volt-seconds across the inductor $L$. The main advantages are the reduced switching losses. Especially, when no power has to be transferred, switching can be avoided completely. As a result, the voltage waveform is smoothed, which also improves the overall accuracy. Especially for piezo applications this is of paramount importance.