where $S_i=\mathscr{R}_{int\,i}/\mathscr{R}_{sh\,i}$ are the shielding factors for individual shells. $\mathscr{R}_{g\,i,j}/\mathscr{R}_{int\,j}$ in (13) and (14) represent the reluctances of air spaces between corresponding shells that are normalized to the reluctances of the shielded areas.

Reluctances $\mathscr{R}_{g\,i,j}$ in (14) can be easily calculated according to (6) and (8), taking into account the previously made assumption about the uniformity of the magnetic flux distribution within air gaps between the shells and assuming that the thicknesses of the shells (see Fig. 6) are small in comparison with their diameters and air gaps between them (practical case):

$$\frac{\mathscr{R}_{g\,ij}}{\mathscr{R}_{int\,j}}=\frac{\bar{l}_{g\,ij}/A_j}{\bar{l}_{int\,j}/A_j}\approx\frac{V_j-V_i}{V_j}\approx 1-\frac{V_i}{V_j}. \tag{15}$$

For multiple concentric cylinders, (15) can be rewritten as follows, considering that $V_i=A_i\times\varDelta$ and $V_j=A_j\times\varDelta$ (see Fig. 5(b)):

$$\frac{\mathscr{R}_{g\,ij}}{\mathscr{R}_{int}}\approx 1-\frac{A_i}{A_j}. \tag{16}$$

Finally, by considering (15), (16) and the recursion in (13) and (14), a general solution for n concentric cylinders can be written:

$$\begin{aligned}S_{tn}\approx{}&1+\sum_{i=1}^{n}S_i+\sum_{i=1}^{n-1}\sum_{j>i}^{n}S_iS_j\left(1-\frac{A_{j-1}}{A_j}\right)\\&+\sum_{i=1}^{n-2}\sum_{j>i}^{n-1}\sum_{k>j}^{n}S_iS_jS_k\left(1-\frac{A_{j-1}}{A_j}\right)\left(1-\frac{A_{k-1}}{A_k}\right)\\&+\sum_{i=1}^{n-3}\sum_{j>i}^{n-2}\sum_{k>j}^{n-1}\sum_{l>k}^{n}S_iS_jS_kS_l\\&\times\left(1-\frac{A_{j-1}}{A_j}\right)\left(1-\frac{A_{k-1}}{A_k}\right)\left(1-\frac{A_{l-1}}{A_l}\right)+\cdots.\end{aligned} \tag{17}$$

Equation (17) exactly reproduces (5). We can conclude therefore, that the magnetic circuit approach enables both qualitative and qualitative descriptions of magnetic shielding with single and multiple concentric cylinders.

The developed equivalent magnetic circuits clearly demonstrate an advantage of introducing air spaces between the shield shells. For a shield with no air gaps (Fig. 2(a)), elements representing in Fig. 2(b) reluctances of the shielded area, $\mathscr{R}_{int}$, and the shield shell, $\mathscr{R}_{sh\,1}$, are connected in parallel to the 'flux source' Φ_{ext}. For a relatively large value of the $\mathscr{R}_{int}/\mathscr{R}_{sh\,1}$ ratio, magnetic flux, Φ_{int}, flowing through $\mathscr{R}_{int}$ is in direct proportion to $\mathscr{R}_{sh\,1}$ (see (7)). According to (6), $\mathscr{R}_{sh\,1}$ is in inverse proportion to the permeability and the shield thickness. Therefore, for a solid shield, with no air gaps, the shielding factor, Φ_{ext}/Φ_{int}, is proportional to the μ and t (see (8), (9)). Dividing a solid shell into two shells and introducing an air gap between them (Fig. 3(a)) principally changes the shield's equivalent scheme (Fig. 3(b)). If the air gap in Fig. 3(a) is relatively narrow and its reluctance, $\mathscr{R}_{g\,1,2}$, is comparable to that of the shells, $\mathscr{R}_{sh\,1}$ and $\mathscr{R}_{sh\,2}$, then these two latter reluctances can be considered as not decoupled by the $\mathscr{R}_{g\,1,2}$. In this case, the behavior of the equivalent scheme in Fig. 3(b) resembles the behavior of the scheme in Fig. 2(a), and the effect of the permeability and thickness of the two shells on the shielding factor is additive. Increasing the gap width decouples the $\mathscr{R}_{sh\,1}$ from the $\mathscr{R}_{sh\,2}$ and causes magnetic flux, $\Phi_{sh\,1}$, flowing through $\mathscr{R}_{sh\,1}$ to be in direct proportion to the $\mathscr{R}_{sh\,2}$. Magnetic flux flowing through $\mathscr{R}_{int}$ is in direct proportion to the $\mathscr{R}_{sh\,1}$, as was mentioned above. Hence, the total effect of the $\mathscr{R}_{sh\,1}$ and the $\mathscr{R}_{sh\,2}$ on the Φ_{int} is multiplicative. Therefore, an introduction of spaces between the shield's shells allows one to save an amount of the magnetic material, keeping a constant shielding factor or *vice versa*, it allows one to increase the shielding factor, keeping a constant amount of the shielding material.

4. Conclusions

The usefulness of the magnetic circuit concept for illustrating magnetic shielding mechanism has been demonstrated. Equivalent magnetic circuits for both qualitative and quantitative descriptions of single and multiple cylindrical shields are developed and analyzed. The analysis resulted in formulas describing the transverse shielding factors in terms of the reluctances of the shielded area, the shield shells, and the spaces between the shells. In order to simplify further analysis, it is assumed that flux density is distributed uniformly throughout the shield parts, including the shielding material and the spaces between the shells. This allows a simple and straightforward evaluation of the corresponding reluctances. It is interesting that the finally developed equations exactly reproduce well-known approximate analytical shielding formulas. The described approach, assists in a simple and comprehensive illustration of magnetic shielding. It also anables a deeper and more straightforward physical insight into the shielding problem. The proposed simple physical model can assist in a more conscious and effective employment of numerical methods.

References

1) Della Porta: Magiae Naturalis, Sive de Miraculus Rerum Naturalium (Libri VII, 1589).
2) A. W. Rucker: *Phil. Mag.*, **23**, 225 (1887).
3) Du Bois: *Wied. Ann.*, **63**, 348 (1897).
4) Du Bois: *Wied. Ann.*, **65**, 1 (1897).
5) Du Bois: *The Electrician*, **40**, 1 (1898).
6) A. P. Wills: *Phys. Rev.*, **9**, 193 (1899).
7) A. J. Mager: *Z. Angew. Phys.*, **23**, 381 (1967).
8) A. J. Mager: *IEEE Trans. Magn.*, **6**, 67 (1970).
9) D. U. Gubser, S. A. Wolf, and J. E. Cox: *Rev. Sci. Instrum.*, **50**, 751 (1979).
10) D. Dubbers: *Nucl. Instrum. Methods A*, **243**, 511 (1985).
11) T. J. Sumner, J. M. Pendlebury, and K. F. Smith: *J. Phys. D: Appl. Phys.*, **20**, 1095 (1987).
12) A. K. Thomas: *IEEE Trans. Electromagnetic Compatibility*, **10**, 142 (1968).

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