Extending Blackman's Formula to Feedback Networks With Multiple Dependent Sources

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***Abstract*—The Blackman's formula for the impedance seen from a pair of arbitrary terminals of a linear feedback network with a single dependent source is extended to the case of linear feedback networks with multiple dependent sources. To reach this aim, we revisit the proof of the Blackman's formula, proof it in a similar manner for feedback networks with two dependent sources, and then extend to the case of feedback networks with multiple dependent sources.**

***Index Terms*—Blackman's formula, circuit and systems fundamentals, effect of feedback on network impedances, impedance evaluation, linear feedback networks, multiple dependent sources, networks with single, double, triple, and multiple dependent sources, return ratio.**

I. Introduction

To EXAMINE the effect of negative feedback [1]–[19] on impedances seen from a pair of arbitrary terminals of a linear feedback network, it is important to describe them analytically as a function of feedback partial gains, such as return ratios.

The impedance seen from a pair of arbitrary terminals of a linear feedback network with a single dependent source can be described by the Blackman's formula [3]

$$Z_t = Z_t|_{a=0}\frac{1+T_{\rm sc}}{1+T_{\rm oc}} \tag{1}$$

where Z_t is the closed-loop impedance seen by a test source v_t [see Fig. 1(a)], a is the dependent source gain, $Z_t|_{a=0}$ is the open-loop impedance, $T_{\rm sc}$ is the return ratio for the short-circuited terminals, and $T_{\rm oc}$ is the return ratio for the open-circuited terminals

$$T_{\rm sc} \equiv -\frac{s_\varepsilon|_{v_t=0}}{s_\varepsilon} \tag{2}$$

$$T_{\rm oc} \equiv -\frac{s_\varepsilon|_{i_t=0}}{s_\varepsilon}. \tag{3}$$

As seen from (2) and (3), the return ratio is defined as the contribution of a dependent source to its control signal, normalized to the total control signal s_ε appearing in the dependent source value $a\ s_\varepsilon$ (see Fig. 1).

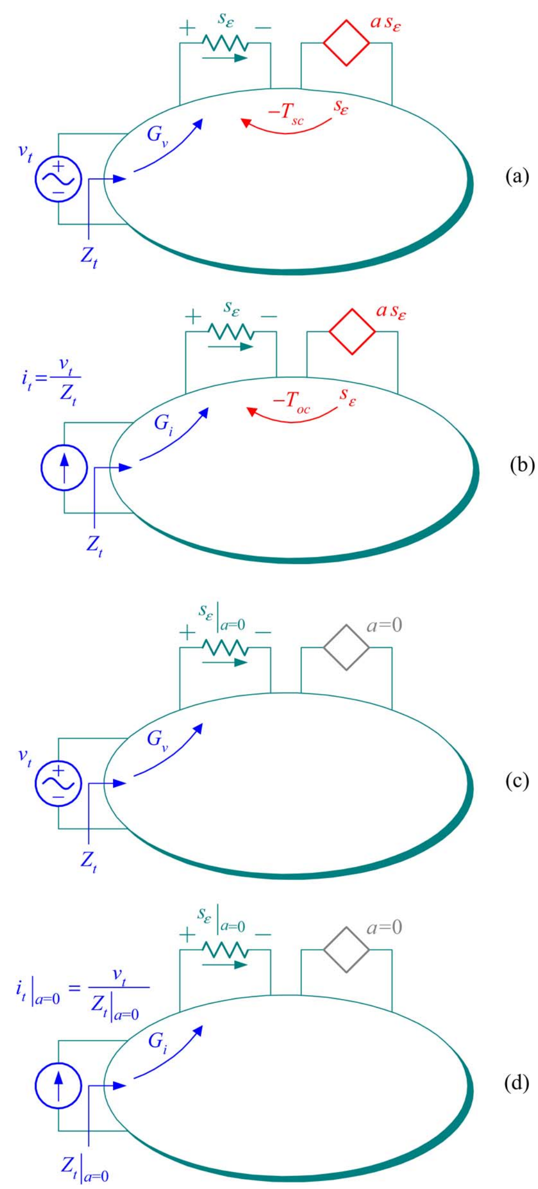

Fig. 1. Finding the impedance seen by a test source connected to arbitrary terminals of a generic feedback network with a single dependent source.

Since the network independent sources are suppressed while finding return ratios, it is necessary to substitute a dependent source with an equivalent independent one having the same value $a\ s_\varepsilon$ to find the contributions $s_\varepsilon|_{v_t=0}$ and $s_\varepsilon|_{i_t=0}$ of the dependent source to its control signal s_ε.

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The aim of this present work is to extend the Blackman's formula to the case of feedback networks with multiple dependent sources. To reach this aim, we revisit the proof of the Blackman's formula for feedback networks with a single dependent source, proof it in a similar way for feedback networks with two dependent sources, and then show how it can be extended to the case of feedback networks with multiple dependent sources.

In feedback networks with two or more dependent sources, each dependent source contributes in a general case not only to its own control signal but also to the control signals of other dependent sources. For the sake of consistency, we refer in this work to such cross contributions, normalized to the total control signal appearing in the dependent source value, as cross return ratios. Although the signal may return to the control terminals of not the same source, but to the control terminals of other dependent sources, it still belongs to the contributions of the active part of the network, represented by all the dependent sources.

II. Feedback Networks With a Single Dependent Source

Let us consider a linear feedback network with a single dependent source (see Fig. 1). To find the impedance seen from a pair of arbitrary terminals, we connect to them a voltage test source in Fig. 1(a) and find the control signal by applying superposition

$$s_\varepsilon = v_t G_v - s_\varepsilon T_{\rm sc} \tag{4}$$

where the input transmission for the voltage test source

$$G_v \equiv \frac{s_\varepsilon|_{a=0}}{v_t}. \tag{5}$$

From (4), v_t can be obtained as

$$v_t = \frac{s_\varepsilon(1+T_{\rm sc})}{G_v}. \tag{6}$$

Let us now replace in Fig. 1(b) source v_t with i_t, such that $i_t = v_t/Z_t$, to keep the same conditions of the test source branch. Keeping the same branch voltage and current leaves the signal s_ε unchanged. As a result

$$s_\varepsilon = i_t G_i - s_\varepsilon T_{\rm oc} \tag{7}$$

where the input transmission for the current test source

$$G_i \equiv \frac{s_\varepsilon|_{a=0}}{i_t}. \tag{8}$$

From (7), i_t can be obtained as

$$i_t = \frac{s_\varepsilon(1+T_{\rm oc})}{G_i}. \tag{9}$$

Hence

$$Z_t = \frac{v_t}{i_t} = \frac{G_i}{G_v}\frac{1+T_{\rm sc}}{1+T_{\rm oc}}. \tag{10}$$

G_v and G_i in (10) can be found from Fig. 1(c) and (d)

$$G_v = \frac{s_\varepsilon|_{a=0}}{v_t} \quad \text{for an arbitrary } v_t \tag{11}$$

$$G_i = \frac{s_\varepsilon|_{a=0}}{i_t|_{a=0}} \quad \text{for} \quad i_t|_{a=0} = \frac{v_t}{Z_t|_{a=0}}. \tag{12}$$

Note that $i_t|_{a=0} = v_t/Z_t|_{a=0}$ in (12) keeps the same conditions of the test source branch for both v_t and $i_t|_{a=0}$ sources and, as a result, $s_\varepsilon|_{a=0}$ in (11) and (12) has the same value.

According to (11) and (12), $G_i/G_v = Z_t|_{a=0}$. Thus, considering (10), the Blackman's formula (1) can be obtained for feedback networks with a single dependent source.

III. Feedback Networks With Two Dependent Sources

Following the approach introduced in the previous section, we extend in this section the Blackman's formula to feedback networks with two dependent sources (see Fig. 2).

Fig. 2(a) and (b) suggest the following equations for the control signals of the dependent sources

$$\begin{cases} s_{\varepsilon1} = v_t G_{1v} - s_{\varepsilon1}T_{1\rm sc} - s_{\varepsilon2}T_{21\rm sc} \\ s_{\varepsilon2} = v_t G_{2v} - s_{\varepsilon2}T_{2\rm sc} - s_{\varepsilon1}T_{12\rm sc} \end{cases} \tag{13}$$

$$\begin{cases} s_{\varepsilon1} = i_t G_{1i} - s_{\varepsilon1}T_{1\rm oc} - s_{\varepsilon2}T_{21\rm oc} \\ s_{\varepsilon2} = i_t G_{2i} - s_{\varepsilon2}T_{2\rm oc} - s_{\varepsilon1}T_{12\rm oc} \end{cases} \tag{14}$$

where

$$T_{1\rm sc} \equiv -\frac{s_{\varepsilon1}\big|_{\substack{v_t=0\\a_2=0}}}{s_{\varepsilon1}} \quad T_{1\rm oc} \equiv -\frac{s_{\varepsilon1}\big|_{\substack{i_t=0\\a_2=0}}}{s_{\varepsilon1}} \tag{15}$$

$$T_{12\rm sc} \equiv -\frac{s_{\varepsilon2}\big|_{\substack{v_t=0\\a_2=0}}}{s_{\varepsilon1}} \quad T_{12\rm oc} \equiv -\frac{s_{\varepsilon2}\big|_{\substack{i_t=0\\a_2=0}}}{s_{\varepsilon1}} \tag{16}$$

$$T_{2\rm sc} \equiv -\frac{s_{\varepsilon2}\big|_{\substack{v_t=0\\a_1=0}}}{s_{\varepsilon2}} \quad T_{2\rm oc} \equiv -\frac{s_{\varepsilon2}\big|i_{\substack{v_t=0\\a_1=0}}}{s_2} \tag{17}$$

$$T_{21\rm sc} \equiv -\frac{s_{\varepsilon1}\big|_{\substack{v_t=0\\a_1=0}}}{s_{\varepsilon2}} \quad T_{21\rm oc} \equiv -\frac{s_{\varepsilon1}\big|_{\substack{i_t=0\\a_1=0}}}{s_{\varepsilon2}}. \tag{18}$$

Equations (13) and (14) can be solved for v_t and i_t

$$v_t = \frac{1+T_{1\rm sc}+T_{2\rm sc}+T_{1\rm sc}T_{2\rm sc}-T_{12\rm sc}T_{21\rm sc}}{G_{2v}+G_{2v}T_{1\rm sc}-G_{1v}T_{12\rm sc}}s_{\varepsilon2} \tag{19}$$

$$i_t = \frac{1+T_{1\rm oc}+T_{2\rm oc}+T_{1\rm oc}T_{2\rm oc}-T_{12\rm oc}T_{21\rm oc}}{G_{2i}+G_{2i}T_{1\rm oc}-G_{1i}T_{12\rm oc}}s_{\varepsilon2}. \tag{20}$$

Dividing (19) by (20) gives

$$Z_t = \frac{G_{2i}}{G_{2v}}\frac{1+T_{1\rm oc}-\frac{G_{1i}}{G_{2i}}T_{12\rm oc}}{1+T_{1\rm sc}-\frac{G_{1v}}{G_{2v}}T_{12\rm sc}}\frac{1+T_{\Sigma_{\rm sc}}}{1+T_{\Sigma_{\rm oc}}} \tag{21}$$

where

$$\begin{aligned} T_{\Sigma_{\rm sc}} &= T_{1\rm sc}+T_{2\rm sc}+T_{1\rm sc}T_{2\rm sc}-T_{12\rm sc}T_{21\rm sc} \\ T_{\Sigma_{\rm oc}} &= T_{1\rm oc}+T_{2\rm oc}+T_{1\rm oc}T_{2\rm oc}-T_{12\rm oc}T_{21\rm oc}. \end{aligned} \tag{22}$$

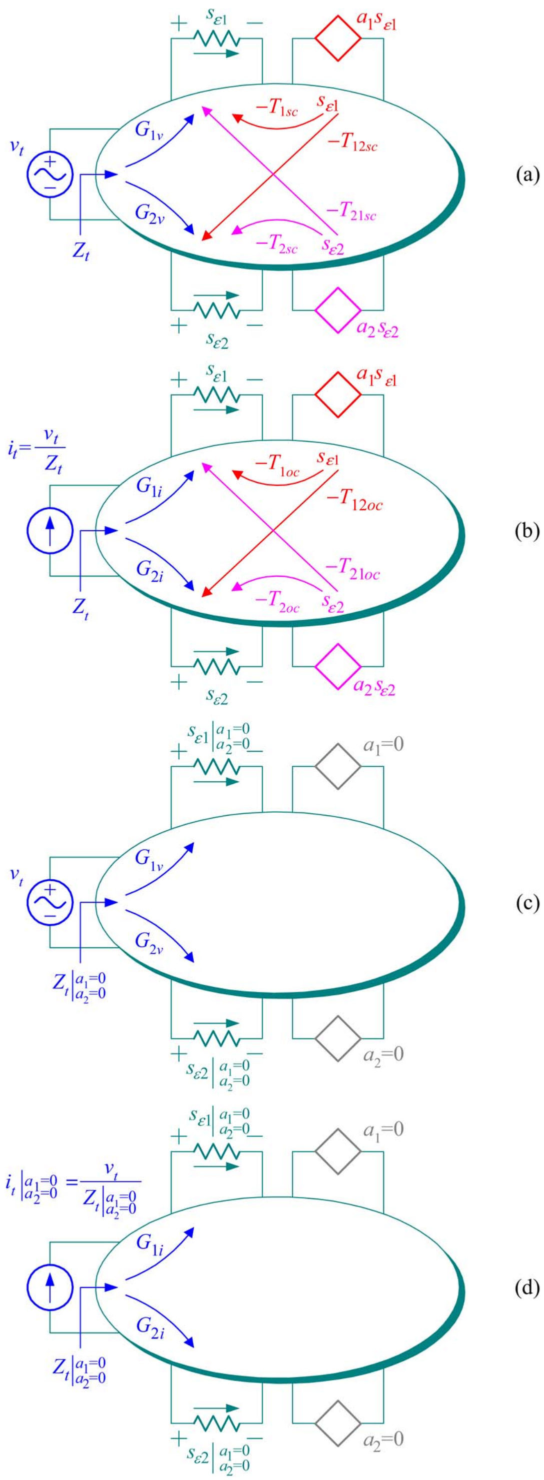

Fig. 2. Finding the impedance seen by a test source connected to arbitrary terminals of a generic feedback network with two dependent sources.

The first term in (21) represents the open-loop impedance seen by v_t

$$\frac{G_{2i}}{G_{2v}} \equiv \frac{s_{\varepsilon 2}|_{\substack{a_1=0\\a_2=0}}}{i_t|_{\substack{a_1=0\\a_2=0}}} \frac{v_t}{s_{\varepsilon 2}|_{\substack{a_1=0\\a_2=0}}} = \frac{v_t}{i_t|_{\substack{a_1=0\\a_2=0}}}$$
$$= Z_t|_{\substack{a_1=0\\a_2=0}} \quad \text{for} \quad i_t|_{\substack{a_1=0\\a_2=0}} = \frac{v_t}{Z_t|_{\substack{a_1=0\\a_2=0}}}. \tag{23}$$

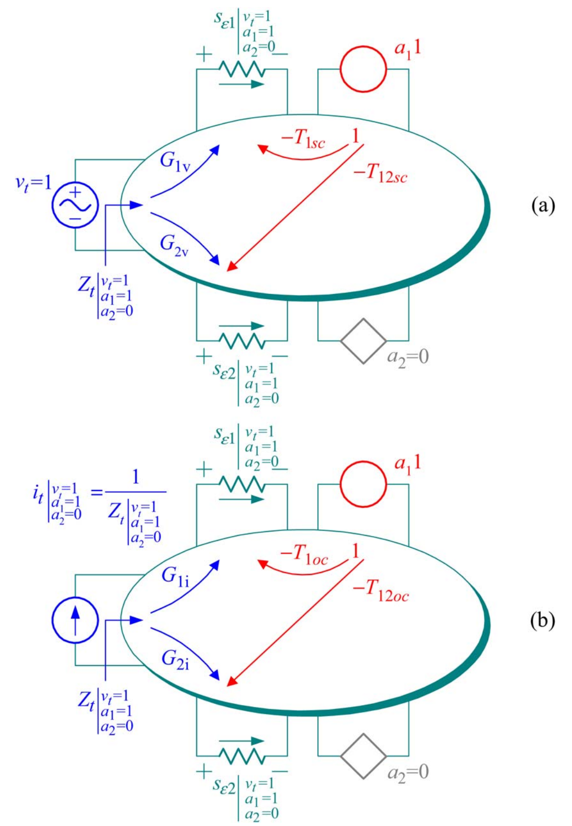

Fig. 3. Generic feedback network with two dependent sources, where $v_t = 1$, $s_{\varepsilon 1} = 1$, and $s_{\varepsilon 2} = 0$.

To find the second term in (21), let us consider Fig. 3, where $v_t = 1$, $s_{\varepsilon 1} = 1$, and $s_{\varepsilon 2} = 0$, and hence

$$\begin{cases} s_{\varepsilon 1}|_{\substack{v_t=1\\a_1=1\\a_2=0}} = G_{1v} - T_{1\text{sc}} \\ s_{\varepsilon 1}|_{\substack{v_t=1\\a_1=1\\a_2=0}} = i_t|_{\substack{v_t=1\\a_1=1\\a_2=0}} G_{1i} - T_{1\text{oc}} \end{cases} \tag{24}$$

$$\begin{cases} s_{\varepsilon 2}|_{\substack{v_t=1\\a_1=1\\a_2=0}} = G_{2v} - T_{12\text{sc}} \\ s_{\varepsilon 2}|_{\substack{v_t=1\\a_1=1\\a_2=0}} = i_t|_{\substack{v_t=1\\a_1=1\\a_2=0}} G_{2i} - T_{12\text{oc}}. \end{cases} \tag{25}$$

From (24) and (25)

$$T_{1\text{oc}} - \frac{G_{1i}}{G_{2i}} T_{12\text{oc}} - G_{2v}\left(\frac{G_{1v}}{G_{2v}} - \frac{G_{1i}}{G_{2i}}\right) = T_{1\text{sc}} - \frac{G_{1i}}{G_{2i}} T_{12\text{sc}}. \tag{26}$$

Considering that, in (26), in accordance with (23)

$$\frac{G_{1v}}{G_{1i}} = \frac{G_{2v}}{G_{2i}} = Z_t|_{\substack{a_1=0\\a_2=0}} \Rightarrow \frac{G_{1v}}{G_{2v}} = \frac{G_{1i}}{G_{2i}} \tag{27}$$

(26) can be rewritten as follows:

$$T_{1\text{oc}} - \frac{G_{1i}}{G_{2i}} T_{12\text{oc}} = T_{1\text{sc}} - \frac{G_{1v}}{G_{2v}} T_{12\text{sc}}. \tag{28}$$

Considering (21), (23), and (28), Z_t can eventually be obtained as

$$Z_t = Z_t|_{\substack{a_1=0\\a_2=0}} \frac{1 + T_{\Sigma_{\text{sc}}}}{1 + T_{\Sigma_{\text{oc}}}}. \tag{29}$$

IV. Feedback Networks With Multiple Dependent Sources

Following the approach given in the previous sections, the impedance seen from a pair of arbitrary terminals of feedback networks with multiple dependent sources can be obtained in accordance with (29).

For example, for a feedback network with three dependent sources, the control signals of the dependent sources

$$\begin{cases} s_{\varepsilon 1} = v_t G_{1_v} - s_{\varepsilon 1} T_{1\mathrm{cs}} - s_{\varepsilon 2} T_{21\mathrm{cs}} - s_{\varepsilon 3} T_{31\mathrm{cs}} \\ s_{\varepsilon 2} = v_t G_{2_v} - s_{\varepsilon 2} T_{2\mathrm{cs}} - s_{\varepsilon 1} T_{12\mathrm{cs}} - s_{\varepsilon 3} T_{32\mathrm{cs}} \\ s_{\varepsilon 3} = v_t G_{3_v} - s_{\varepsilon 3} T_{3\mathrm{cs}} - s_{\varepsilon 1} T_{13\mathrm{cs}} - s_{\varepsilon 2} T_{23\mathrm{cs}} \end{cases} \tag{30}$$

$$\begin{cases} s_{\varepsilon 1} = i_t G_{1_i} - s_{\varepsilon 1} T_{1\mathrm{oc}} - s_{\varepsilon 2} T_{21\mathrm{oc}} - s_{\varepsilon 3} T_{31\mathrm{oc}} \\ s_{\varepsilon 2} = i_t G_{2_i} - s_{\varepsilon 2} T_{2\mathrm{oc}} - s_{\varepsilon 1} T_{12\mathrm{oc}} - s_{\varepsilon 3} T_{32\mathrm{oc}} \\ s_{\varepsilon 3} = i_t G_{3_i} - s_{\varepsilon 3} T_{3\mathrm{oc}} - s_{\varepsilon 1} T_{13\mathrm{oc}} - s_{\varepsilon 2} T_{23\mathrm{oc}}. \end{cases} \tag{31}$$

Equations (30) and (31) can be solved for $Z_t = v_t/i_t$

$$Z_t = \frac{G_{3i}}{G_{3v}} \frac{B_{\mathrm{oc}}}{B_{\mathrm{sc}}} \frac{1 + T_{\Sigma_{\mathrm{sc}}}}{1 + T_{\Sigma_{\mathrm{oc}}}} \tag{32}$$

where

$$\begin{aligned} B_{\mathrm{oc}} =& 1 + T_{1\mathrm{oc}} + T_{2\mathrm{oc}} + T_{1\mathrm{oc}} T_{2\mathrm{oc}} - T_{12\mathrm{oc}} T_{21\mathrm{oc}} \\ &+ \frac{G_{i1}}{G_{i3}} (T_{12\mathrm{oc}} T_{23\mathrm{oc}} - T_{13\mathrm{oc}} - T_{2\mathrm{oc}} T_{13\mathrm{oc}}) \\ &+ \frac{G_{i2}}{G_{i3}} (T_{13\mathrm{oc}} T_{21\mathrm{oc}} - T_{23\mathrm{oc}} - T_{1\mathrm{oc}} T_{23\mathrm{oc}}) \end{aligned} \tag{33}$$

$$\begin{aligned} B_{\mathrm{sc}} =& 1 + T_{1\mathrm{sc}} + T_{2\mathrm{sc}} + T_{1\mathrm{sc}} T_{2\mathrm{sc}} - T_{12\mathrm{sc}} T_{21\mathrm{sc}} \\ &+ \frac{G_{v1}}{G_{v3}} (T_{12\mathrm{sc}} T_{23\mathrm{sc}} - T_{13\mathrm{sc}} - T_{2\mathrm{sc}} T_{13\mathrm{sc}}) \\ &+ \frac{G_{v2}}{G_{v3}} (T_{13\mathrm{sc}} T_{21\mathrm{sc}} - T_{23\mathrm{sc}} - T_{1\mathrm{sc}} T_{23\mathrm{sc}}) \end{aligned} \tag{34}$$

$$\begin{aligned} T_{\Sigma_{\mathrm{sc}}} =& T_{1\mathrm{sc}} - T_{1\mathrm{sc}} T_{23\mathrm{sc}} T_{32\mathrm{sc}} + T_{2\mathrm{sc}} - T_{2\mathrm{sc}} T_{13\mathrm{sc}} T_{31\mathrm{sc}} \\ &+ T_{3\mathrm{sc}} - T_{3\mathrm{sc}} T_{12\mathrm{sc}} T_{21\mathrm{sc}} + T_{1\mathrm{sc}} T_{2\mathrm{sc}} + T_{1\mathrm{sc}} T_{3\mathrm{sc}} \\ &+ T_{2\mathrm{sc}} T_{3\mathrm{sc}} - T_{12\mathrm{sc}} T_{21\mathrm{sc}} - T_{13\mathrm{sc}} T_{31\mathrm{sc}} - T_{23\mathrm{sc}} T_{32\mathrm{sc}} \\ &+ T_{1\mathrm{sc}} T_{2\mathrm{sc}} T_{3\mathrm{sc}} + T_{12\mathrm{sc}} T_{23\mathrm{sc}} T_{31\mathrm{sc}} \\ &+ T_{13\mathrm{sc}} T_{21\mathrm{sc}} T_{32\mathrm{sc}} \end{aligned} \tag{35}$$

$$\begin{aligned} T_{\Sigma_{\mathrm{oc}}} =& T_{1\mathrm{oc}} - T_{1\mathrm{oc}} T_{23\mathrm{oc}} T_{32\mathrm{oc}} + T_{2\mathrm{oc}} - T_{2\mathrm{oc}} T_{13\mathrm{oc}} T_{31\mathrm{oc}} \\ &+ T_{3\mathrm{oc}} - T_{3\mathrm{oc}} T_{12\mathrm{oc}} T_{21\mathrm{oc}} + T_{1\mathrm{oc}} T_{2\mathrm{oc}} + T_{1\mathrm{oc}} T_{3\mathrm{oc}} \\ &+ T_{2\mathrm{oc}} T_{3\mathrm{oc}} - T_{12\mathrm{oc}} T_{21\mathrm{oc}} - T_{13\mathrm{oc}} T_{31\mathrm{oc}} - T_{23\mathrm{oc}} T_{32\mathrm{oc}} \\ &+ T_{1\mathrm{oc}} T_{2\mathrm{oc}} T_{3\mathrm{oc}} + T_{12\mathrm{oc}} T_{23\mathrm{oc}} T_{31\mathrm{oc}} \\ &+ T_{13\mathrm{oc}} T_{21\mathrm{oc}} T_{32\mathrm{oc}}. \end{aligned} \tag{36}$$

According to (23) and (27), the first term in (32) is

$$\frac{G_{3i}}{G_{3v}} = Z_t \Big|_{\substack{a_j = 0 \\ j=1,2,3}}. \tag{37}$$

As in the previous section, it can also be shown that the term $B_{\mathrm{oc}}/B_{\mathrm{sc}}$, in (32), equals unity. As a result, the impedance seen from a pair of arbitrary terminals of a linear feedback network with three dependent sources can be obtained by (29), where the open-loop impedance is given by (37) and $T_{\Sigma_{\mathrm{sc}}}$ and $T_{\Sigma_{\mathrm{oc}}}$ are given by (35) and (36), respectively.

V. Conclusion

It has been shown that the Blackman's formula can be extended to the case of feedback networks with multiple dependent sources. The obtained equations provide exact solutions for feedback networks with two and three dependent sources. Following the proposed approach, equations for a greater number of dependent sources can easily be developed.

Transistor small-signal models in many cases include only one dependent source. Double and multiple transistor feedback circuits are used widely. As a result, the proposed theory could serve a useful reference to do quick calculations by hand and, thus, to obtain a better intuitive insight into the effect of feedback on the circuit impedances.

In contrast to the canonical approach [11], [19], the described theoretical treatment involves no approximations to simplify the analysis.

To validate the proposed theory, theoretical results were compared against simulations and experimental measurements. In both the theoretical calculations and simulations, the transistors were modeled by equivalent small-signal circuits with a single dependent source $h_{fe} i_b$. A perfect matching has been obtained between the theory and simulations. The matching between the theoretical and experimental results was very close. The difference between these results is due to the approximate transistor models and the measurement uncertainties of the transistor small-signal parameters.

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