A New Calibration Procedure for Magnetic Tracking Systems

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In this study, we suggest a new approach for the calibration of magnetic tracking systems that allows us to calibrate the entire system in a single setting. The suggested approach is based on solving a system of equations involving all the system parameters. These parameters include: 1) the magnetic positions of the transmitting coils; 2) their magnetic moments; 3) the magnetic position of the sensor; 4) its sensitivity; and 5) the gain of the sensor output amplifier. We choose a set of parameters that define the origin, orientation, and scale of the reference coordinate system and consider them as constants in the above system of equations. Another set of constants is the sensor output measured at a number of arbitrary positions. The unknowns in the above equations are all the other system parameters. To define the origin and orientation of the reference coordinate system, we first relate it to a physical object, e.g., to the transmitter housing. We then use special supports to align the sensor with the edges of the transmitter housing and measure the sensor output at a number of aligned positions. To define the scale of the reference coordinate system, we measure the distance between two arbitrary sensor locations with a precise instrument (a caliper). This is the only parameter that should be calibrated with the help of an external measurement tool. To illustrate the efficiency of the new approach, we applied the calibration procedure to a magnetic tracking system employing 64 transmitting coils. We have measured the systematic tracking errors before and after applying the calibration. The systematic tracking errors were reduced by an order of magnitude due to applying the new calibration procedure.

Index Terms—Calibration, magnetic tracking system, solving a system of equations, system parameters, systematic tracking errors.

I. INTRODUCTION

The maximum possible accuracy of a magnetic tracking system cannot be achieved without an efficient calibration procedure. However, it is a nontrivial task due to a large number of parameters to be measured. These parameters include the magnetic positions of the transmitting coils (see Fig. 1), their magnetic moments, the magnetic position of the sensor, its sensitivity, and the gain of the sensor output amplifier.

According to the conventional approach [1], all the above parameters are calibrated individually. This is not efficient because individual calibrations cause the accumulation of errors. Another principal disadvantage of the conventional approach is in its inapplicability to the assembled system. Thus, it does not consider the assembling tolerances. A large number of individual calibrations also complicate the calibration procedure due to employing a number of different types of calibration setups [2].

In this study, we suggest a new approach that allows us to calibrate the entire system in a single setting. We calibrate all the parameters simultaneously by solving a system of equations (calibration model). The constants in these equations are the parameters that define the origin, orientation, and scale of the reference coordinate system and the sensor output acquired at different sensor positions. The unknowns are all the other system parameters. The only parameter in the above system of equations to be calibrated with the help of an external measuring tool is the scale of the reference coordinate system. This allows us to use a single individual calibration to find all the system parameters.

\begin{equation}
V_{ij} = C_i^3 \left( n_i' \cdot n_j' \right) \left( n_i' \times n_j' \right) d - \operatorname{sgn}(n_i' \cdot n_j' \times n_0) \left| \Delta r_{ij} \right| d
\end{equation}

Index $i = 1, \ldots, N$ in (1) is the transmitting coil number, and index $j = 1, \ldots, M$ is the number of the sensor po-
sition. \( V_{ij} \) is the sensor output voltage at the \( j \)th sensor position. This is generated by the \( i \)th transmitting coil. \( \mathbf{n}_i' = (\cos \varphi_i', \cos \theta_i', \sin \varphi_i', \cos \theta_i') \) is the unit vector describing the magnetic axis of the \( i \)th transmitting coil. \( \Delta \mathbf{r}_{ij} = (x_{s_j} - x_i', y_{s_j} - y_i', z_{s_j} - z_i') \) is the distance between the sensor and the \( j \)th transmitting coil. \( (x_{s_j}', y_{s_j}', z_{s_j}') \) is the location of the sensor magnetic center. \( (x_i', y_i', z_i') \) is the location of the transmitting coil magnetic center. \( \mathbf{n}_{s_j}' = (\cos \varphi_{s_j}', \cos \theta_{s_j}', \sin \varphi_{s_j}', \cos \theta_{s_j}') \) is the unit vector describing the magnetic axis of the sensor. “\( \cdot \)” denotes the dot product, and the upper case \( T \) denotes the vector transposition.

The gain coefficient \( C_i \) in (1) represents a number of parameters of the tracking system

\[
C_i = \mu_0 f G A_s I_i A_i / 4 \pi \tag{2}
\]

where \( \mu_0 \) is the magnetic permeability of vacuum, \( f \) is the transmitter excitation frequency, \( G \) is the gain of the sensor amplifier, \( A_s \) is the sensor effective area, and \( I_i \) and \( A_i \) are the excitation current and the effective area of the \( i \)th transmitting coil.

Variable \( d \) in (1) is the distance between the \( j \)th and \( i \)th sensor locations. This distance defines the scale of the coordinate system \( X'Y'Z' \). To accurately define the scale, \( d \) should be measured with a precise tool such as an interferometer, encoder, vernier caliper, etc. Such a measurement can be done, for example, by moving the sensor along a straight line with the help of two supports (see Fig. 3) and measuring the translation of the sensor housing.

The number of sensor positions should be large enough to provide the unique solution of (1) for a given number of the transmitting coils. Measuring the sensor output at \( M \) different positions yields \( NM + 1 \) equations. Given five degrees of freedom (DOF) of the sensor and six DOF of the transmitting coils, the total number of variables is \( 5M + 6N \). Among these variables, six coordinates are known. They include the \( x \)-, \( y \)-, and \( z \)-coordinates of the transmitting coil defining the origin of the coordinate system \( X'Y'Z' \) [see Fig. 2(a)], \( y \)- and \( z \)-coordinates of another transmitting coil defining the \( X' \)-axis, and \( z \)-coordinate of the third transmitting coil defining the \( x'z' \)-plane. Therefore, the total number of unknowns is as follows:

\[
U = 5M + 6N - 6. \tag{3}
\]

Considering the above number of equations and unknowns, the uniqueness condition can be written as follows:

\[
NM + 1 \geq 5M + 6N - 6. \tag{4}
\]

From (4), the minimum number of measurements that provides the uniqueness of (1) is

\[
M_{\text{min}} = (6N - 7) / (N - 5). \tag{5}
\]

Substituting \( M_{\text{min}} \) into (3) gives us the corresponding number of unknowns \( U_{\text{min}} \). \( M_{\text{min}} \) and \( U_{\text{min}} \) are shown versus \( N \) in Fig. 3. Fig. 3 shows that there is a tradeoff that can be found between the number of measurements, which defines the time consumption of the calibration procedure and the number of unknowns, which defines the complexity of the numeric solution.

B. Calibration in the Coordinate System Defined by the Transmitter Housing

The axes of the \( X'Y'Z' \) coordinate system [see Fig. 2(a)] are intangible. Therefore, we define another coordinate system \( XYZ \) related to a physical object, for example, to the transmitter housing. In this study, we consider that the sensor and
transmitter housings are rectangular prisms. (In the case of arbitrary-shaped housings, special rectangular-shaped holding fixtures can be employed in the calibration procedure.) To find the position of the transmitter housing in the coordinate system $X'Y'Z'$, we measure the sensor output at five different positions, as suggested in Fig. 4. We use special mechanical supports to align the sensor with the edges of the transmitter housing.

This provides us with the following system of equations:

$$
\begin{align*}
\mathbf{d}' &= \mathbf{j}' \times \mathbf{k}' \\
\mathbf{j}' &= \frac{\mathbf{r}_{s1}' - \mathbf{r}_{s1}}{||\mathbf{r}_{s1}' - \mathbf{r}_{s1}||} \\
\mathbf{k}' &= \frac{\mathbf{r}_{s2}' - \mathbf{r}_{s1}}{||\mathbf{r}_{s2}' - \mathbf{r}_{s1}||} \times \mathbf{j}' \\
\mathbf{r}' &= \mathbf{r}_{s1}' - \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} \\
&= \frac{1}{2} \begin{bmatrix} \mathbf{r}_{s1}' - \mathbf{r}_{s2}' \\ \mathbf{r}_{s1}' - \mathbf{r}_{s3}' \\ \mathbf{r}_{s1}' - \mathbf{r}_{s6}' \end{bmatrix} \cdot \begin{bmatrix} \mathbf{j}'^T \\ \mathbf{j}'^T \\ \mathbf{k}'^T \end{bmatrix}
\end{align*}
$$

where $\mathbf{d}'$ is the location of the origin of the coordinate system $XYZ$ (see Fig. 4), $\mathbf{i}', \mathbf{j}', \text{ and } \mathbf{k}'$ are the base vectors of the coordinate system $XYZ$ translated to the origin of the coordinate system $X'Y'Z'$, $\mathbf{r}_{s1}', \mathbf{r}_{s2}', \mathbf{r}_{s3}', \mathbf{r}_{s4}', \text{ and } \mathbf{r}_{s5}'$ are the sensor locations in the coordinate system $X'Y'Z'$, $a', b'$, and $c'$ are the dimensions of the sensor housing, and $[\ldots]$ is the matrix notation.

Solving (6) together with (1) for $\mathbf{d}', \mathbf{i}', \mathbf{j}', \text{ and } \mathbf{k}'$ yields the magnetic position of the sensor in the coordinate system $XYZ$

$$
\begin{align*}
\mathbf{r}_{sj} &= (\mathbf{r}_{sj} - \mathbf{d}') \cdot [\mathbf{i}'^T \mathbf{j}'^T \mathbf{k}'^T] \\
\mathbf{n}_{sj} &= \mathbf{n}_{sj}' \cdot [\mathbf{i}'^T \mathbf{j}'^T \mathbf{k}'^T],
\end{align*}
$$

From (7), we can find the magnetic position of the sensor $(\mathbf{r}_{s1}, \mathbf{n}_{s1})$ relative to its housing.

### III. Calibration Example

To illustrate the efficiency of the suggested approach, we apply the new calibration procedure to a magnetic tracking system [3] employing an array of 64 transmitting coils [see Fig. 5(a)].

In our experiments, we used a simple induction coil sensor enclosed in a rectangular plastic housing (see Fig. 6). The sensing coil diameter is 3 mm and its length is 3 mm. The coil is wound with 250 turns of a 0.1-mm wire.

To most clearly illustrate the transmitter calibration, we moved one of the transmitting coils far away from its original position [see Fig. 5(a)]. We then measured the sensor output at
Fig. 7 shows the significant improvement of tracking accuracy due to the calibration. The systematic tracking errors were reduced by an order of magnitude. We relate the large tracking errors with no calibration to the difference between the magnetic and geometric positions of the transmitting coils [see Fig. 1(b)] and the difference between the theoretical and true magnetic moments. These differences are caused by the nonideality of the transmitting coils winding.

To find the position of the transmitter housing in the coordinate system \(X'Y'Z'\), we measured its output at five positions, as shown in Figs. 4 and 6. We then substituted the obtained measurements in (6) and found the transformation (7) of the sensor coordinates into the coordinate system \(XYZ\).

IV. Conclusion

A new approach has been suggested that allows the calibration of the entire magnetic tracking system in a single setting. The new approach minimizes the number of individual calibrations, simplifies the calibration procedure, and increases its accuracy. It is applied to the assembled system and does consider the assembling tolerances.

The suggested approach is generic and can be applied to many different types of tracking systems. All the system parameters can be calibrated not only by the system manufactures, but also by the users.

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References


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