Power Converter Steady-State Computation Using The Projected Lagrangian Method

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Abstract - In this paper, a new computation technique is presented for determining the steady-state solution of a switching network. This technique is based on the formulation of the steady-state conditions as a constrained optimization problem which is then solved by use of the projected Lagrangian method. A comparative analysis, validated by the steady-state simulation of various power converters, illustrates the efficiency of the projected Lagrangian method. The direct and fast computation of the steady-state operation, without any previous knowledge of the operating sequences, is a very efficient tool for the design and optimization of a power converter.

I. Introduction

In the case of a switching system, the voltages and currents during the steady-state operation take the form of periodic time-varying waveforms. Hence, the steady-state solution of a switching system cannot be solved by simply replacing capacitors and inductors by open- and short-circuits, as is the usual case of a continuous system.

Two basic approaches have been proposed for direct computation of the steady-state solution, avoiding a heavy time-consuming simulation with zero initial conditions of the circuit state variables. The first approach is based on the Newton-Raphson method, and the other on the unconstrained optimization method.

A new computational technique of the steady-state solution is presented in this paper. This new approach is based on the formulation of the steady-state conditions as a constrained optimization problem which is then solved by the projected Lagrangian method. This approach can be considered as a bridge between the two previous techniques: the optimisation aspect of this approach is ensuring the convergence of the iterative process even with a starting point far from the steady-state solution, while the constraining aspect is to ensure that the iterative process takes place in the hyperplane corresponding to the steady-state operation constraints.

A comparative analysis, validated by the simulation of the steady-state operation of various power converters, is given in this paper, to illustrate the efficiency of the projected Lagrangian method. The direct and fast computation of the steady-state operation, without any previous knowledge of the operating sequences, is a very efficient tool for the optimal design of a power converter.

II. Previous techniques for the computation of the steady-state solution

The basic conversion function of a power converter is achieved by repetitive switching between a finite number of sequences. The state variables, denoted by the vector \( X \), are continuous when the switching process from one sequence to another occurs. So, the steady-state operation conditions of a power converter can be expressed mathematically as:

\[
C_{ST}(X_0) = X_T(X_0) - X_0 = 0
\]

(1)

Where \( X_0 \) and \( X_T \) are respectively the vectors of the state variables at the beginning and the end of the period of operation \( T \), and \( C_{ST} \) is the vector of the steady-state constraint functions. Generally, the vector \( X_T \) as well as the steady-state constraint functions \( C_{ST} \) are nonlinear vector functions of the initial condition vector \( X_0 \). This non-linearity occurs in converters with nonlinear components or in converters with linear components which are operating with soft-commutation. Two main techniques have been proposed to find the solution \( X^{*0} \) of the non-linear steady-state equations (1): the Newton-Raphson method [2] and the unconstrained optimization method [1].

With the Newton-Raphson method, the steady-state constraint functions \( C_{ST} \) are approximated, around any starting point \( X_0(K) \) of the iterative process, by the first order Taylor series expansion:

\[
C_{ST}(X_0(k + 1)) = C_{ST}(X_0(k)) + A(k)(X_0(k + 1) - X_0(k))
\]

(2)

A(k) is the Jacobian matrix of the constraint vector functions \( C_{ST} \). The point \( X_0(k+1) \) is a local solution of the non-linear
equation (1) around the starting point $X_0(k)$. By equating the right-hand side of (2) to zero, we obtain:

$$X_0(k+1) = X_0(k) - (A(k))^{-1}C_{ST}(X_0(k))$$

(3)

The iterative process given by (3) generates a sequence $X(k)$ which converges to the steady-state solution $X^*$.

The second approach for solving the non-linear steady-state constraints system (1) is based on the use of an objective function defined by:

$$\Phi_{ST}(X_0) = C_{ST}(X_0)C_{ST}^T(X_0)$$

(4)

The minimum of this function is equal to the steady-state solution $X^*$. The gradient method [6] is the most common unconstrained minimization technique used to find the minimum of $\Phi_{ST}(X_0)$.

### III. Projected Lagrangian method for the computation of the steady-state solution

In the method proposed in this paper, the steady-state equation (1) is solved as a constrained optimization problem of the form:

$$\begin{align*}
\text{Minimize} & \quad \Phi_{ST}(X_0) \\
\text{Subjected to} & \quad C_{ST}(X_0) = X_T(X_0) - X_0 = 0
\end{align*}$$

(5)

With the projected Lagrangian method, the objective function $\Phi_{ST}(X_0)$ is taken equal to the common augmented Lagrangian function:

$$\Phi_{ST}(X_0) = -\frac{\lambda^T}{2}C_{ST}(X_0) + \frac{\rho}{2}C_{ST}(X_0)^T C_{ST}(X_0)$$

(6)

A linear approximation of the non-linear constraints is made around each starting point $X_0(k)$ of the iterative process:

$$C_{ST}(X_0(k+1)) = C_{ST}(X_0(k)) + A(k)(X_0(k+1) - X_0(k))$$

(7)

The vector $\lambda^T$ is the vector of the Lagrange multipliers and $\rho$ is the penalty parameter [6]. The steady-state problem is then reduced, around each starting point $X_0(k)$, to the resolution of the sub-problem:

$$\begin{align*}
\text{Minimize} & \quad \Phi_{ST}(X_0(k+1)) \\
\text{Subjected to} & \quad X_0(k+1) = -(A(k))^{-1}C_{ST}(X_0(k)) + X_0(k)
\end{align*}$$

(8)

This sub-problem, which generates a sequence $X_0(k)$, converging to the steady-state solution $X^*_0$, is a linearly constrained optimization problem. For each starting point $X_0(k)$, the linear constraints insure that the local solution $X_0(k+1)$ is located in the hyperplane corresponding to a zero of the linearized approximation of the steady-state nonlinear constraints $C_{ST}$ at $X_0(k)$. Hence, the projected Lagrangian method ensures that the steady-state solution $X^*_0$, is the minimum of the augmented Lagrangian function, whose minimization occurred only within the subspace defined by the steady-state constraints. For a switching network, this property of the projected lagrangian method is crucial for the convergence procedure of the computation of the steady-state operation.

The flowchart in fig.1 illustrates the proposed procedure for the computation of the steady-state solution. The determination of the steady-state operation is a non-linear constrained optimization problem which is solved by an optimization routine. The constraints are evaluated in the user-supplied subroutine. For each starting point $X_0$ proposed by the optimization routine, the simulation of the switching network is performed over a period of the steady-state operation by a specific simulation tool. The result of this simulation is the state variables vector $X_T$ at the end of the period of operation $T$. The vectors $X_T$ and $X_0$ are transferred to the user-supplied subroutine, where the steady-state constraints $C_{ST}$ are computed and returned to the optimization routine.

### IV. Comparative analysis

The three techniques discussed above for the computation of the steady-state solution of a power converter have been implemented in the SUPRA simulator [5] (this simulator operates without any previous knowledge of the operation sequences). Many examples have been treated, for a comparative evaluation of the efficiency and accuracy of each technique. Five examples of power converters are illustrated: a buck-converter (fig.2), a multi-level converter [7] (fig.3), a quasi-resonant buck converter (fig.4), a three-phase thyristor rectifier (fig.5) and a series resonant converter (fig.6). The table 1 presents the values of the computation time (CPU time), obtained with each technique on the same workstation. The three methods are also compared to a conventional technique of simulation of the whole transient operation until the steady-state operation is obtained, with zero initial conditions of the state variables.
Optimization Program
Computation of the initial condition vector $X_0$ of the state variables

Simulation Tool
Computation of the state variables vector $X_T$ at the end of the period of operation $T$

User-Supplied Subroutine
Computation of the steady-state constraint functions $C_S(X_0) = X_T - X_0$

Fig. 1: Flowchart of the constrained optimization method for the computation of the steady-state solution

First, it can be noticed that the computation time is much greater in the case of the Newton-Raphson method and in the case of the unconstrained optimisation method. In addition, the latter methods do not converge, from the starting point $X_0=0$, in the case of the quasi-resonant converter and the three-phase thyristor rectifier. In general, this failure occurs in the case of power converters which are operating with a soft-commutation process.

With the Newton-Raphson method, the computation time required to obtain a steady-state solution is often greater than the time associated with the heavy brute force simulation method. This is true for the second example and it has been tested with other topologies. This situation, which is illustrated in the computation of the steady-state solution of the multi-level converter, especially occurs when the starting point $X_0$ of the iterative process is not sufficiently close to the steady-state solution $X_0^*$. For each iteration of the unconstrained optimization method, the search direction towards the steady-state solution is not constrained to be in the hyperplane defined by the steady-state constraints. The iterative process can take a direction where the power converter has no possible mode of operation. This situation can be observed for example in the quasi-resonant converter with negative values of the resonant inductor current and the resonant capacitor voltage. In this case, the unconstrained optimisation method may diverge or converge to a point that is not a solution of the steady-state problem.

As mentioned previously, the constrained optimization technique based on the projected Lagrangian method can be considered as a bridge between the two above techniques: the optimization aspect of this approach ensures the convergence of the iterative process, even with a starting point $X_0$ far from the steady-state solution, while the constraint aspect ensures the iterative process to occur in the hyperplane corresponding to the steady-state constraints. Consequently, the projected Lagrangian method offers the advantages of both previous techniques: an efficient convergence to the steady-state solution from any feasible starting point.

Fig. 2: Buck converter ($E=100V, L_1=1mH, C_1=100\mu F, R=10K, f=10kHz, R=50\%$)

Fig. 3: Multi-level converter ($E=400V, L_1=85\mu H, C_1=45\mu F, C_2=5\mu F, R=10K, f=10kHz, R=25\%$)
Fig. 4: Quasi-resonant buck converter \((E=20\text{V}, L_1=6.4\mu\text{H}, L_2=7\text{mH}, R=17\Omega, C_1=16\mu\text{F}, C_2=1\mu\text{F}, f=400\text{kHz})\)

Fig. 5: Three-phase Thyristor converter \((E=170\text{V}, L_2=1\text{mH}, L_2=1\text{mH}, C_1=45\mu\text{F}, C_2=5\mu\text{F}, R=10\Omega, f=10\text{kHz})\)

Fig. 6: Series resonant converter \((E=250\text{V}, L=217\mu\text{H}, C_1=22\mu\text{F}, C_2=10\mu\text{F}, R=17\Omega, f=20\text{kHz})\)

**TABLE I**
Computation time (CPU Time) of the steady state solution with the main three techniques (starting point \(x_0=0\))

<table>
<thead>
<tr>
<th>Technique</th>
<th>Brute Force Method</th>
<th>Newton Raphson Method</th>
<th>Unconstrained Optimisation Method</th>
<th>Constrained Optimisation Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck Converter</td>
<td>13S</td>
<td>5.81S</td>
<td>4.12S</td>
<td>1.43S</td>
</tr>
<tr>
<td>Multilevel Converter</td>
<td>85.43S</td>
<td>112.55S</td>
<td>17.2S</td>
<td>3.24S</td>
</tr>
<tr>
<td>Quasi-Resonant Converter</td>
<td>25.34S</td>
<td>Failed</td>
<td>Failed</td>
<td>4.13S</td>
</tr>
<tr>
<td>3Φ Inverter</td>
<td>38.12S</td>
<td>Failed</td>
<td>Failed</td>
<td>5.21S</td>
</tr>
<tr>
<td>Series resonant converter</td>
<td>21.14S</td>
<td>Failed</td>
<td>Failed</td>
<td>2.76S</td>
</tr>
</tbody>
</table>
V. Extension of the method to study the design problem of a power converter

The design of a power converter cannot usually be restricted to the analysis of its circuit. A lot of compromises must be adopted between the electrical, thermal, and mechanical specifications of the application. The optimal design involves additional constraints on the design of the passive components like the inductors, the capacitors and the transformers, and on the design of the heat-sink system. The usual objective of the optimal design is the minimization of the power to weight ratio of the converter. The computation of the steady-state operation of a converter for a given set of its electrical components is a crucial tool of this process. The method which has been developed can be easily included in this kind of global optimization procedure to improve its efficiency.

The steady-state computation approach proposed in this paper is basically a non-linear constrained optimization technique based on the projected Lagrangian method, which can be formalized as:

\[
\text{Steady-state Problem} \begin{cases}
\text{Minimize } \Phi_{\text{ST}}(x_0) \\
\text{Subjected to } C_{\text{ST}}(x_0) = 0
\end{cases}
\] (9)

This steady-state problem can be easily extended to any optimal design problem of the power converter expressed in the form of a constrained optimization problem:

\[
\text{Design Problem} \begin{cases}
\text{Minimize } \Phi_{\text{D}}(x_0) \\
\text{Subjected to } C_{\text{D}}(x_0) = 0
\end{cases}
\] (10)

Where:
- \(x_0\): vector of design variables,
- \(C_{\text{D}}\): vector of design constraints,
- \(\Phi_{\text{D}}\): design objective function.

The combined steady-state and design optimization problem can be written as follows:

\[
\text{Minimize } \Phi(x) = \Phi_{\text{D}}(x) + \Phi_{\text{ST}}(x)
\] (11)

Subjected to \(C_{\text{D}}(x) = 0\) and \(C_{\text{ST}}(x) = 0\)

With this presentation, the steady-state equations are formalized as a part of the design optimization problem. This formalism offers many advantages when a simulator tool is used to compute the steady-state electric characteristics of the power converter. In the conventional process of global optimization, the steady-state simulation must be executed for each iteration of the optimization program. This method involves a heavy time-consuming design process with the conventional methods described in paragraphe II. With the proposed formalism, the conditions of the steady-state operation are expressed as an additional constraint in the design problem (11) and for each iteration the simulation is directly executed only over a switching period of the steady-state operation. This procedure is much more efficient.

VI. Conclusion

A new technique is presented in this paper for computing the steady-state solution of a power converter. This technique is based on the formulation of the steady-state conditions as a constrained optimization problem which is solved by the projected Lagrangian method. The efficiency of the method is validated by the steady-state simulation of various power converters. A comparative analysis has been presented to illustrate the advantage of the projected Lagrangian method with respect to the previous steady-state computation techniques. The extension of the method to study the design problem of a power converter has been also presented: this very efficient design approach is based on the combination of an optimization program with a simulation tool.

VII. References