An operations research approach to the triple jump

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Abstract: There are wide-ranging views among track and field experts regarding the optimal strategy to use in the triple jump. The controversy pertains to the effort which should be expended in each of the hop, step, and jump phases. In this paper, we develop a dynamic programming approach for finding the optimal strategy. The results of our model support a recent shift in opinion in favour of a jump-dominated technique. However, the improvement is marginal, and hence, the best strategy may in the end be athlete specific.

Keywords: dynamic programming; track and field; triple jump.


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Shaul Ladany is a Belgrade-born, Hungarian speaking, Bergen-Belsen concentration-camp survivor, living in Israel since 1948. Although he missed three years of elementary schooling and the remaining years were accomplished by having to study in five different languages, he succeeded in obtaining his BSc and MSc in Mechanical Engineering from the Technion, a graduate diploma in Business Administration from the Hebrew University, and a PhD from Columbia University. During his study years he also managed to acquire practical experience in managing and design engineering. Upon graduating from Columbia University he joined the staff of Tel-Aviv
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His research interests are mainly in the areas of economic quality control optimisations and operations research applications, especially applications to the leisure-time industry. His major contributions to science are in the field of optimal yield management (optimal price discrimination and market segmentation in the hotel and aviation industries), the derivation of models and solution procedures for optimal tourist (bus) tours, and the initiation and the leading of applications of operations research to sports and the sports industry.

In between his extensive full-time academic activities, he was and is still engaged in his hobby of race-walking. He won the 100km walk World Championship, set the yet unbroken 50-mile walk World Record, won seven US National Championships, has dominated the world ultra long-distance scenery for over ten years, participated in two Olympic Games, and survived the Munich massacre. He won his first London-to-Brighton race in 1970, and still managed to compete in its Jubilee edition in 2003.

1 Introduction

The triple jump is one of the more complicated events in track and field competition. As in the long jump, the athlete begins with a run-up to the takeoff board in order to gain speed. The run-up is followed by three successive phases: a hop, a step, and a jump. In the hop, the athlete takes off and lands on the same foot; the step requires landing on the opposite foot; and finally the jump is terminated by landing on both feet in a sand pit. The three phases require a high degree of coordination and balance. The aim is to maximise the total length measured from the takeoff line to the nearest contact point in the sand.

There is a difference of opinion among elite coaches and athletes as to the best triple jump strategy. By strategy we address the question of how to partition the total energy possessed by an athlete at the takeoff line among the three phases of the event. For much of the last 50 years the triple jump event has been dominated by Eastern European countries. Two diverging philosophies have developed there: the Russian school has favoured a strongly hop-dominated technique, while the Poles have emphasised a balanced technique. Referring to Hay [1-3] a phase-dominated technique implies that a given phase percentage (phase distance expressed as a percentage of the total actual distance) is at least 2% greater than the next largest phase percentage. If this condition is not satisfied, the technique is referred to as balanced.

In recent years, a third school of thought has emerged in support of a jump-dominated technique. Hay [3] notes that the current world record of 17.97 metres held by the American Willie Banks was in fact set using a jump-dominated technique (respective phase percentages of 35, 28, and 37 for the hop, step and jump). Hay develops qualitative arguments which deal with the dynamics of the event to conclude that there are several
inherent advantages in a jump-dominated technique which make this strategy a serious contender to the established philosophies. For example, he argues that the forces exerted in a hop-dominated technique may compromise the athlete’s takeoffs in the later phases, and also affect the athlete’s ability to keep his balance. Empirical observations suggest that a jump-dominated strategy allows higher initial velocities to be used entering the hop. Other practical considerations such as the nature of the surfaces (hard landings for the hop and step versus landing in the sand for the jump) also are presented by Hay in support of a jump-dominated technique.

The biomechanics approach used by Hay [1-3] analyses the dynamics of motion and forces acting on the athlete in each phase. However, it does not provide a global or systems view of the event. It is clear that if an athlete expends more energy in one phase, there will be less available for the others. Our objective is to quantify this relation. This leads in the next section to a dynamic programming model, which may be used to determine an optimal triple jump strategy.

2 The model

Consider the triple jump as a decision process. The athlete is assumed to arrive at the takeoff line with a given initial horizontal speed or total kinetic energy. He must decide how much of this energy to expend in the first phase of the event, the hop. Once the hop is completed, he must decide how much of the remaining energy will be used in the second phase, the step. After the step, the athlete will be out of decisions in the sense that his leftover energy for the final jump is already fixed. Hence, we can reduce the problem to two decision variables: $L_1$, the length of the hop, and $L_2$, the length of the step. Once these decisions are made, the length of the jump, $L_3$, is established, as well as the total length, $L_T = L_1 + L_2 + L_3$.

The nature of the problem lends itself to a dynamic programming formulation with two decision stages. We will refer to the step as stage 1 and the hop as stage 2. The state associated with each stage is the athlete’s horizontal speed or input velocity at the start of the given stage. Thus, $v_1$, the takeoff speed at the start of the triple jump is the state variable for stage 2. Depending on the decisions, $L_1$ and $L_2$ the respective input velocities, $v_2$ and $v_3$ for the step (stage 1) and the jump are determined. The principle of optimality requires that an optimal decision be made at each stage. However, in order to proceed, we must first quantify the relationships between decision and state variables.

3 Functional relations

Consider the jump first. In order to maximise the length ($L_3$), the athlete must produce an appreciable takeoff angle at the end of the step. That is, for a given horizontal speed, the length of the jump increases for higher trajectories, since the athlete will stay in the air longer. However, physical limitations resulting from the forces exerted on the athlete suggest that the takeoff angle of the jump is typically around $20^\circ$. The angles measured at 11 trials of different athletes at various international-level competitions [4] varied from $17.20^\circ$ to $22.73^\circ$. Thus, it seems a reasonable approximation to assume a constant takeoff angle for a given athlete. From a simple trajectory analysis, it then follows that the length
of the jump $L_3$ varies as the square of the horizontal speed $v_3$ at the start of the jump, that is,

$$L_3 = a_0 v_3^2$$

where $a_0$ is a parameter of the individual athlete. For the Hay data set of 11 trials, a mean value of 0.1164 was calculated for $a_0$, where length is measured in metres and speed in metres/sec. The variations about the mean were small (of the order of 10%), validating the use of this value in the subsequent calculations.

Next we consider the step. Here there are an input horizontal speed $v_2$, a decision to expend energy to obtain a step length $L_2$, and a resulting terminal speed $v_3$, which is the input for the subsequent phase of the jump. The dynamics of the step imply that there are decreasing marginal returns on $L_2$. We mean that each unit increase of $L_2$ should come at an increasing cost on $v_3$. This relation is modelled by the following formula:

$$v_3 = v_2 - a_1 L_2^c$$

where $a_1$ and $c$ are parameters. In order that $v_3$ be a concave function of $L_2$, we must have $c>1$. The following conditions are used to obtain estimates of the two parameters:

1. the preceding equation holds when mean values are substituted for $v_3$, $v_2$, and $L_2$;
2. the maximum step length, $L_2^\text{max}$, is obtained using the same relation as for the jump, that is:

$$L_2^\text{max} = a_0 v_2^2$$

At this step length, $v_3 = 0$.

Using the same 11 trials and the preceding two conditions (distances in metres and speeds in metres/sec) gives the following results: $a_1 = 9.69 \times 10^{-5}$ and $c=5.58$. The exponent $c$ calculated from our data set implies that the functional relation between $v_3$ and $L_2$ is strongly concave. For values of $L_2$ in the vicinity of 0, the curve (with $v_2$ constant) is very flat. However, further away, as the athlete becomes more greedy in the step phase, there are rapidly-increasing losses incurred on the jump. The same conclusion may be reached using an intuitive argument. A longer step requires a higher trajectory. Apart from the loss in kinetic energy to develop the required vertical momentum, the athlete’s ability in the jump phase will be severely affected on touching down at the end of the step where he will have to absorb much larger impact forces. The high value of $c$ quantifies these factors.

An identical analysis may be used for the hop. We start off with the relation

$$v_2 = v_1 - a_2 L_1^d$$

where $a_2$ and $d$ are parameters. Using analogous conditions as above for the step, we obtain two equations to solve for the unknown parameters. The resulting estimates are: $a_2 = 4.734 \times 10^{-4}$ and $d=4.26$. Again observe a high exponent which indicates a strong concave relation between output speed ($v_2$) and the length chosen for this phase ($L_1$).
4 Finding the optimal strategy

The dynamic programming approach considers one stage of the decision process at a time. First we optimise stage 1 (the step and jump phases). The idea here is to find the optimal step length $L^*_2$ as a function of the input speed $v_2$. This value will maximise the combined length of the step and jump phases. Thus, we obtain the following problem:

$$\text{SPAN}_1(v_2) = \max_{L_2} [L_2 + L_1]$$

$$= \max_{L_2} [L_2 + a_0(v_2 - a_1L_2)^2]$$

Unfortunately, a closed-form solution $L^*_2(v_2)$ is not readily available for the empirically found value of the exponent $c$. However, a fast and simple numerical search may be used to derive this curve. For a given value of $v_2$, the search on $L_2$ is conducted over the range 0 to $a_0v_2^2$.

Using the principle of optimality, the stage 2 problem may be viewed as deciding on a hop length $L_1$ for a given starting speed $v_1$, and then choosing $L^*_2(v_2)$ for the resultant speed $v_2$ going into the step. Thus, the dynamic programming formulation for stage 2 becomes:

$$\text{SPAN}_2(v_1) = \max_{L_1} [L_1 + \text{SPAN}_1(v_2)]$$

$$= \max_{L_1} [L_1 + \text{SPAN}_1(v_1 - a_1L_1^2)]$$

Again, a numerical search quickly obtains an optimal solution $L^*_1(v_1)$ for stage 2.

5 Discussion

The results of the dynamic programming model are illustrated in Figure 1 for a typical input speed, $v_1 = 9.47 \text{ m/sec}$. The hop length $L_1$ is incremented here between values of 3.6m and 7.2m. These limits represent respective hop phase percentages of roughly 20% and 40%, which more than cover the range employed by elite athletes. Based on the results in Figure 1, the following observations may be made:

1. The optimal hop length (for the given input speed) should lie between 5 and 6 metres, giving a hop phase percentage of between 28% and 33%. More precisely, the curve of total length is maximised at $L^*_1 = 5.6m$, or 31%.

2. The optimal step length $L^*_2$ is surprisingly insensitive to different values of $L_1$. Irrespective of the strategy, the results indicate an almost constant step length around 4.3m, or about 24% for this phase. This strongly argues against a balanced technique employing equal levels of effort in the three phases. Our model thus recommends that the step, which is in the middle, receive a relatively low priority. Meanwhile the beginning and the last phase should receive the bulk of the energy package.
The total length is maximised for jump phase percentages between 43% and 48%. These figures are significantly higher than ones currently recorded in jump-dominated events. Our results, thus, clearly support a jump-dominated strategy.

Figure 1  Computational results

Perhaps the most important conclusion should be drawn from the flatness of the curve of total length. We note that there is only a marginal improvement in going from a hop-dominated to a jump-dominated technique. This may explain the ongoing controversy between the proponents of these two strategies. In the end, the best procedure may depend on the individual traits of the athlete. Thus, there may be no universally-optimal strategy for this event. Nonetheless, it must be emphasised that, according to our model, total length will increase as much as 4% by moving to a jump-dominated strategy. (This figure applies to a decrease in hop phase percentage from 40% to 30%.) In a competitive environment such as found in today’s track and field events, small gains often make a huge difference. Furthermore, coaches and athletes should be experimenting with even higher jump phase percentages than those currently in use in jump-dominated techniques.

In closing, we propose that a dynamic programming approach is a feasible decision tool for coaches and athletes in the triple jump event. The functional relations developed above may be custom-tailored to a given athlete, and then used in a dynamic programming model to show which strategic direction will improve the athlete’s performance.
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